QUALITATIVE SPATIAL REASONING ABOUT INTERNAL CARDINAL DIRECTION RELATIONS

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ABSTRACT:

A class of novel spatial relation, internal cardinal direction (ICD) relation, is introduced and discussed. Applying ICD-9 model, the characteristics and the simplification rule of ICD relations are discussed at first. Then the ICD related qualitative spatial reasoning is discussed in a formal way. Four composition cases are presented. They are 1) composing two nesting ICD relations and composing two coordinate ICD relations to deduce 2) conventional (or external) cardinal relations, 3) qualitative distance relations and 4) topological relations respectively. When ICD relations are taken into account, the container determines analysis scale and forms a positioning framework together with ICD relations. So the research on ICD related qualitative spatial reasoning would con-tribute to the representation and reasoning about survey knowledge.

1. INTERNAL CARDINAL-DIRECTION RELATIONS AND ICD-9 MODEL

1.1 Internal cardinal-direction relations

In fields of geographical information system (GIS), artificial intelligence (AI) and databases, qualitative spatial reasoning (QSR) has drawn lot of attention. Spatial relations, including topological relations, cardinal direction relations and metric relations, play essential roles in QSR. To the author's knowledge, without concerning temporal factors, the research of QSR mainly focuses on three aspects until recently. They are:

1. Formalizing one type of spatial relations and discussing their attributes, such as concept neighbourhood graph, computability, etc. (Egenhofer, 1992; Randell, 1992; Duckham, 2001; Skiadopoulos 2004).
2. Composing two or more spatial relations to obtain a previously unknown relation. This aspect includes composition of topological relations (Ligozat, 1999; Renz, 2002), composition of cardinal direction relations (Skiadopoulos, 2001; Isli, 2000), and composition of topological relation and metric relation (Giritli, 2003). Furthermore, the other SQR problems, such as path-consistency problem, minimal labels problem, can be solved based these compositions.
3. Determining a place's position according to provided spatial relations (Clementini, 1997; Isli, 1999; Moratz, 2003), where cardinal direction relations and metric relations are more often applied.

In this paper, internal cardinal-direction (ICD) relation related QSR is in discussion. Different from the other types of spatial relations, ICD is applied to represent the direction relations between an object and another area entity containing it. The ICD relations between the containee and the container depend on the containee's relative position in the latter.

It is well known that spatial knowledge development includes three stages, i.e. landmark knowledge, route knowledge and survey knowledge (Montello, 2001). In order to express and transfer survey knowledge, some base landmarks are usually selected and described using ICD relations at first. Then the other places are determined according to the base landmarks using topological, cardinal direction or qualitative distance relations. A typical statement to represent survey knowledge might be "*A* locates in the west-east of *B*, and *C* lies to the north of *A*". In (Mennis, 2000), a pyramid framework to model geographic data and geographic knowledge is developed based on geographic cognition (Fig. 1). According to this framework, geo-knowledge includes two parts, i.e. taxonomy (superordinate-subordinate relationships) and partonomy (part-whole relations). Obviously, ICD implies part-whole relations and conduces to the representation of partonomy knowledge.

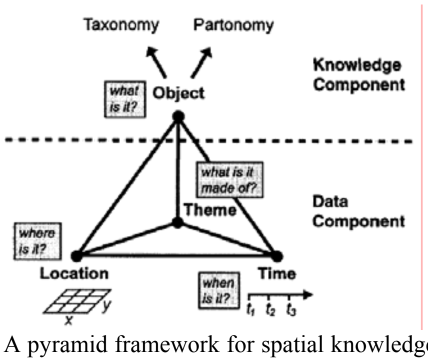

Figure 1. A pyramid framework for spatial knowledge (Mennis, 2000)

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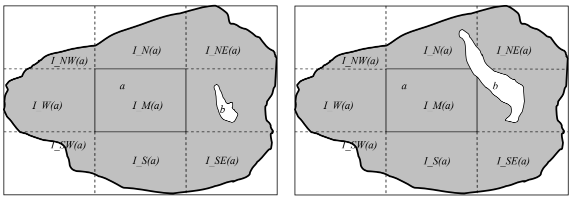
This paper is structured as follows. At first, ICD-9 model and its characteristics are described briefly. Then some fundamental concepts are defined. Based on these concepts, simplification rules of ICD-9 are established. In the third part of this paper, ICD-relation based qualitative spatial reasoning is in discussion. A series of composition tables are provided to demonstrate the QSR about ICD relations with ICD relations, ECD relations, topological relations and qualitative distance relations respectively. At last, we conclude the paper and present an agenda for future work.

1.2 ICD-9 model

In ICD-9, regions are defined as non-empty sets of points in $\mathbb{R}^2$. Let *a* be a region. For simpleness, the reference object, i.e. the container, is assumed to be connected region if for every pair of points in it there exists a line (not necessarily a straight one) joining these two points such that all the points on the line are also in it.

In order to partition *a* into ICD parts, the minimum bounding rectangle (MBR) of *a* is divided into 9 tiles averagely at first. As shown in Fig. 2, the tiles intersect *a* and form corresponding parts denoted by *I_N(a)*, *I_NE(a)*, *I_E(a)*, *I_SE(a)*, *I_S(a)*, *I_SW(a)*, *I_W(a)*, *I_NW(a)* and *I_M(a)*.



a. b.

Figure 2. ICD-9 model (a. atomic ICD relation b. complex ICD relation

If another geometry *b* locates within *I_E(a)*, then *b I_E a*, i.e. *b* is in the east of *a* (Fig.2-a). Similarly, the other ICD relations including *I_NE* (northeast), *I_E* (east), *I_SE* (southeast), *I_S* (south), *I_SW* (southwest), *I_W* (west), *I_NW* (northwest), *I_M* (middle) could also be defined. These ICD relations are called atomic relations. However, if *b* lies more than one part of *a*, using the method mentioned in (Skiadopoulos, 2001), it is complex and can be defined as $b\ R_1{:}\ldots{:}R_k\ a$, where $2 \le k \le 9$, and $R_i \in \{I\_N, I\_NE, I\_E, I\_SE, I\_S, I\_SW, I\_W, I\_NW, I\_M\}, 1 \le i \le k$

For example, the ICD relation shown in Fig. 2-b is denoted by *b I_N:I_NE:I_E:I_M a*. For the sake of defining complex ICD relation, a λ function is applied to combine a set of basic ICD relations to construct a complex one. For instance, λ(*I_N*;*I_NE*;*I_NW*:*I_N*)= *I_NW*:*I_N*:*I_NE*. If *b* is restricted to be connected, some disconnected cases, such as λ(*I_N*;*I_NE*;*I_S*), are impossible. λ function is also suitable for ECD relations.

Referring to the research suggested in (Skiadopoulos, 2001), we use the function δ as a shortcut to express a set of ICD relations. For arbitrary atomic cardinal direction relations R1; … ; Rk, the notation δ(R1; … ; Rk) is a shortcut for the disjunction of all valid basic cardinal direction relations that can be constructed by combining atomic relations R1, … , Rk. For instance, δ(I_SW;I_W;I_NW) stands for the disjunctive relation {I_SW, I_W, I_NW, I_SW:I_W, I_W:I_NW, I_SW:I_W:I_NW}. Obviously, δ(R1; … ; Rk) include 2k-1 basic relations.

Definition 1. Based on δ function, the set including all internal cardinal relations is denoted by ICD-9. ICD-9=δ(I_N; I_NE; I_E; I_SE; I_S; I_SW; I_W; I_NW; I_M). It has 511 elements.

Definition 2. ICD(b,a) is a function used to represent the ICD relation between b and a, i.e. $\mathrm{ICD}(a,b) = R \Leftrightarrow aRb$

Definition 3. Let b and a have the ICD relation of R, i.e. b R a, then we have $b \subseteq R(a)$.

If *R* is an atomic relation, then *R*(*a*) is corresponding to one partition cell of *a*. However, if *R* is complex relation that is represented as $R_1{:}\ldots{:}R_k$, then $R(a) = \bigcup_{i=1}^{k} R_i(a)$. *R*(*a*) has the following characteristic: $\forall p \in R(a)$, *p R a*, where *p* is a point geometry. In the context of ICD relations with *a*, *R*(*a*) forms an approximation of *b*. To be simple, if b is inside *a*, we could use the notation of $\bar{b}$ to denote the approximation based on ICD relation between *b* and *a*.

Theorem 1. Let *a*, *b* be two geometries and *b* might be disconnected. If $b \subseteq R(a)$ where $R = R_1{:}\ldots{:}R_k$ and $1 \le k \le 9$, then the set of all possible ICD relations is $\delta(R_1; \ldots ; R_k)$, i.e. $\mathrm{ICD}(b,a) \in \delta(R_1;\ldots;R_k)$. Briefly, $\delta(R_1; \ldots ; R_k)$ is written as $\delta(R)$ if $R = R_1{:}\ldots{:}R_k$.

Proof. Because *b* might be disconnected, it is simply a combination problem.□

However, if *b* is constrained to be connected, some disconnected combination cases should be excluded form $\delta(R)$. For example, the relation of *NE*: *SE* will not be reasonable any more. We use the symbol $\delta'(R)$ to denote the subset instead of $\delta(R)$. For example, δ'(*I_N*; *I_M*; *I_S*) = {*I_N*, *I_M*, *I_S*, *I_N*:*I_M*, *I_M*:*I_S*, *I*:*N*:*I_M*:*I_S*}, where relation *I_N*:*I_S* is excluded.

2. CHARACTERISTICS AND SIMPLIFICATION RULES OF ICD-9

Let R_1 and R_2 be atomic ICD relations, according to the relations between $R_1(a)$ and $R_2(a)$, the relations on ICD relations R_1 and R_2 could be determined. We have the following definitions.

Definition 4 R_1 and R_2 are **equal** if $R_1(a)=R_2(a)$. R_1 and R_2 are **neighboring** if $R_1(a)$ externally meet $R_2(a)$. R_1 and R_2 **disjoint** if $R_1(a) \cap R_2(a) = \varnothing$. Especially, R_1 and R_2 are **opposite** if R_1 and R_2 disjoint and centrally symmetrical.

The symbols ▽, ⚭, ⊠ and ⌀ are used to denote these three relations. For example, *I_N* ▽ *I_N*, *I_S* ⚭ *I_M*, *I_W* ⊠ *I_SE* and *I_NE* ⌀ *I_SW*. The relation of equal and opposite can also be extended to ECD relations if the ICD relations are quantified using the same approach. For example, *I_N* ▽ *E_N*, *I_NE* ⌀ *E_SW*, where *E_N* and *E_SW* are ECD relations north and south-west. What should be pointed out is that although the above relations are similar to topological relations, the prediction's objects are different. One is about ICD relations and another focuses on spatial geometries.

In order to define the above relations, a quantitative representation is introduced. Assume there is a Cartesian coordinate, the origin of which is middle part of the container. Then each tile is represented by an ordered pair $<Q_x,Q_y>$, where

$Q_x, Q_y \in \{-1,0,1\}$ (Fig. 3). Actually, -1, 0 and 1 stand for south, middle and north or west, middle and east respectively.

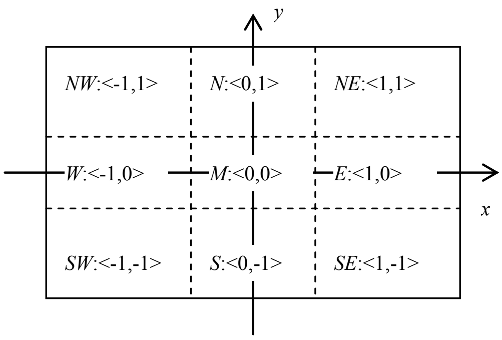


Figure 3. Quantification of atomic ICD relations

Theorem 2. If R_1 and R_2 are atomic and can be encoded as $<Q_{1x},Q_{1y}>$ and $<Q_{2x},Q_{2y}>$ respectively, then:

$R_1 \triangledown R_2$ iff $(Q_{1x} = Q_{2x}) \wedge (Q_{1y} = Q_{2y})$,

$R_1 \bullet\!\bullet R_2$ iff $\text{abs}(Q_{1x} - Q_{2x}) \le 1 \wedge \text{abs}(Q_{1y} - Q_{2y}) \le 1 \wedge \neg R_1 \, Q \, R_2$,

$R_1 \boxtimes R_2$ iff $\text{abs}(Q_{1x} - Q_{2x}) = 2 \vee \text{abs}(Q_{1y} - Q_{2y}) = 2$ and

$R_1 \varnothing R_2$ iff $Q_{1x} + Q_{2x} = 0 \wedge Q_{1y} + Q_{2y} = 0 \wedge Q_{1x} \ne 0 \wedge Q_{2x} \ne 0$.

This representation method can be extended to the case of complex ICD relations. If $R(a)$ is rectangle, then R can be described using the range of $R(a)$. For example, $I\_N{:}I\_NE{:}I\_M{:}I\_E$ could be denoted by <0~1,0~1>.

Applying the ordered pairs of atomic ICD relations, a complex ICD relation can be simplified. As shown in Fig. 4, the line object *b* and area object *c* have ICD relations with *a*. They are *b* $I\_NE{:}I\_E{:}I\_SE$ *a* and c $I\_N{:}I\_NE{:}I\_E$ *a*. But in practice, the more natural and geographical cognition accordant statements to describe these relations might be "*b* goes through east of *a*" and "*c* locates in the northeast of *a*". So a simplified method is necessary to simplify the complex ICD relations.

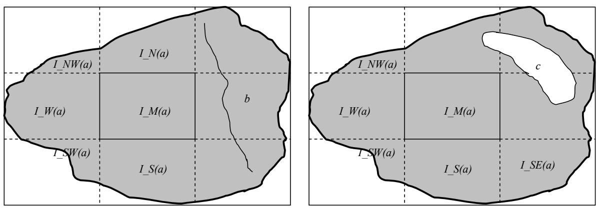


Figure 4. Complex ICD relation that can be simplified

Let $R=R_1{:}\ R_2{\dots}R_n$ be complex ICD relation, and $R_1{:}R_2{:}{\dots}{:}R_n$ be represented as $<Q_{1x},Q_{1y}>$, $<Q_{2x},Q_{2y}>\dots<Q_{nx},Q_{ny}>$. We define $<Q_{sx},Q_{sy}> = <\frac{\sum_{i=1}^{n} Q_{ix}}{n}, \frac{\sum_{i=1}^{n} Q_{iy}}{n}>$ as the result for simplification. For example, the pairs' value for ICD relations in Fig. 4 is <1,0> and <0.67,0.67> respectively. Then R could be simplified to an atomic ICD relation according to the minimum Euclidean distance from $<Q_{sx},Q_{sy}>$ to the pairs of every atomic relations. Following this rule, ICD(*b*,*a*) and ICD(*c*,*a*) are simplified to I_E and I_NE. This is accordant to commonsense geographical cognition. Especially, if ICD(*b*,*a*) is complex and the simplified result of is I_M, the size of *b* is usually comparative to *a*.

3. ICD RELATED QUALITATIVE SPATIAL REASONING

It is argued by (Goodchild, 2001) that many geographic attributes are scale-specific. An important characteristic of internal cardinal relations is that the container determines the analysis scale for describing the position of entities inside it. In order to position a place in the container, distance relations, cardinal direction relations and topological relations are all necessary besides ICD relations. So it is necessary to study the qualitative spatial reasoning about ICD relations.

Fig. 5 gives two categories of available ICD-involved composition. In Fig. 5, G_1, G_2 and G_3 are three spatial objects, and R_{ICD}, R_{ECD}, R_{QD}, R_{Topo} denote ICD relations, ECD relations, qualitative distance relations and topological relations respectively. The solid lines stand for known ICD relations. Meanwhile, the dash lines represent unknown spatial relations to be deduced.

1. Fig. 5-a represents the composition of nested ICD relations. This makes G_1, G_2 and G_3 be at different scale levels.
2. If G_2 and G_3 are within the same container G_1, then the external cardinal direction relations, qualitative distance relations and topological relations can be inferred according to their ICD relations to G_1. Fig. 5-b indicates such an instance.

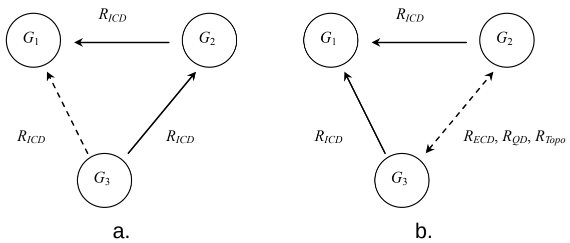


Figure 5. ICD-involved composition of spatial relations

The composition described in Fig.5-b is somewhat different from common relation compositions, which are defined as: $R_1 \circ R_2 = \{<x,y> \mid \exists t(xR_1t \wedge tR_2y)\}$, such as the case given in Fig.5-a. This is because ICD relations are not closed under inverse. For example, if *a* and *b* have a specific ICD relation, the relation between *b* and *a* will not be ICD any more. This nature of ICD is significantly difference from the other spatial relations (e.g. topological relations), i.e. container and containee play asymmetric roles. Because the container forms analysis context of the containees, it is more valuable to infer the relation between two objects which are within the same container based on their ICD relations. For making a standard composition, we still could inverse such a composition to predict the ICD relations according to an ICD relation and an ECD (or topological, qualitative distance) relation. The unclosed property makes that ICD relation itself could not form an integrated algebra system. Therefore, it is necessary to involve the other types of spatial relations in the composition.

3.1 QSR about nesting ICD relations

The qualitative spatial reasoning of two nesting ICD relations shown as the following theorem is somewhat straightforward.

Theorem 3. Let a, b, c be three geometries that have IDC relations of R_1 and R_2 respectively, i.e. *b* R_1 *a* and *c* R_2 *b*. Then $R_2 \circ R_1 \in \delta(R_1)$.

Proof. At first, according to definition 3,

$$bR_1a, R_1 \in \textbf{ICD\_9} \Rightarrow b \subseteq R_1(a),$$

$$cR_2b, R_2 \in \textbf{ICD\_9} \Rightarrow c \subseteq R_2(b) \Rightarrow c \subset R_1(a)$$

According to theorem 1, we have

$$\text{ICD}(c, a) \in \delta(R_1)$$

Therefore,

$$R_2 \circ R_1 \in \delta(R_1)$$

Theorem 3 indicates that $R_2 \circ R_1$ is unrelated to R_2. It is a natural result. But in practice we seldom use this reasoning process because of different spatial scale level. For example, if a city *b* locates in the north of a state *a*, meanwhile a building *c* is in the west of city *b*, we seldom tell that "building *c* locates in the north of state *a*" for the scale reason, although it is right.

3.2 QSR about ICD and ECD relations

Compared with ICD relations, the conventional direction relations when two geometries are disjoint are named as external cardinal direction (ECD) relations. In order to represent ECD-relations, some models were constructed. They include cone-based model, project-based model (Frank, 1991), double-cross model (Freksa, 1992) and MBR-based model (Goyal, 2000; Goyal, 2001; Skiadopoulos, 2001; Skiadopoulos, 2004), etc. (Fig. 6)

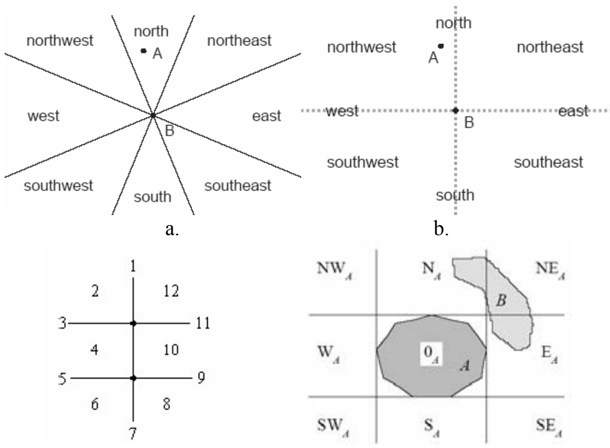


Figure 6. ECD models (a. cone-based model; b. project-based model; c. double-cross model; d. MBR-based mode)

To keep consistent with ICD-9 model, MBR-based model is applied to represent external cardinal relations (Fig. 7). There are mainly two representing approaches, i.e. matrix method (Goyal, 2000) and string method (Skiadopoulos, 2001).

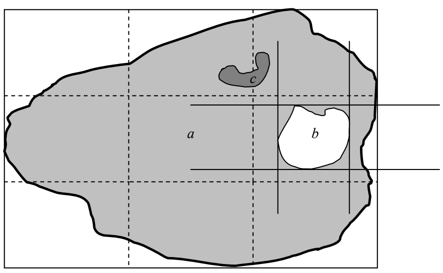


Figure 7. Composing ICD and ECD relations

When MBR-based ECD model is chosen, the middle part of the reference object in ICD relations can be treated as another reference object for ECD relations. Meanwhile, the other eight parts have corresponding external cardinal relations to the middle part. Therefore ICD relations have similar characteristics to ECD relations. The differences between them exist in two aspects:

1. ICD partitions only the MBR of the reference object, while ECD partitions the whole space of R^2.
2. The areas of every ICD parts usually comparative (depending on shape of the container). But ECD tiles have not such an attribute. With a given region, the area of middle part is fixed, and what of the other eight parts are infinite. Moreover, areas of *E_NE*, *E_SE*, *E_SW* and *E_NW* are higher order infinities than *E_N*, *E_E*, *E_S* and *E_W*. This makes MBR-based ECD model not suitable for relatively "small" reference object. As an extreme case, if the reference geometry degenerates into a point, then cone-based model should be applied instead of MBR-based model.

In the discussion on QSR of ICD and ECD relations, the containee objects are restricted to be connected. So the function *δ'* is used to represent the combination result. It can also be applied to external direction relations with the similar meanings. The reason for the restriction is that the spatial relations between disconnected or multiple geometries are usually complicated and less meaningful, although there are some papers discussing this questions, for example (Behr, 2001).

Let *b*, *c* be two geometries inside a region *a*, and assume they are all connected According to ICD_9 model, they occupy different parts of *a*, i.e. *b* R_1 *a* and *c* R_2 *a*, where R_1 and R_2 are basic ICD relations. If R_1 and R_2 are atomic, the composition relations can be deduced via table 1.

Table 1. Composition table of ICD and ICD to get ECD relations

	I_N	I_NE	I_E	I_SE
I_N	*	*δ'(E_E; E_NE; E_SE)*	*E_SE*	*E_SE*
I_NE	*δ'(E_W; E_NW; E_SW)*	*	*δ'(E_S;E_SW; E_SE)*	*δ'(E_S;E_SW; E_SE)*
I_E	*E_NW*	*δ'(E_N; E_NE; E_NW)*	*	*δ'(E_S;E_SW; E_SE)*
I_SE	*E_NW*	*δ'(E_N ;E_NE, E_NW)*	*δ'(E_N;E_NE; E_NW)*	*
I_S	*δ'(E_N;E_NE ; E_NW)*	*E_NE*	*E_NE*	*δ'(E_S;E_SW; E_SE)*
I_SW	*E_NE*	*E_NE*	*E_NE*	*δ'(E_E;E_NE; E_SE)*
I_W	*E_NE*	*E_NE*	*δ'(E_E;E_NE; E_SE)*	*E_SE*
I_NW	*δ'(E_E; E_NE; E_SE)*	*δ'(E_E;E_NE; E_SE)*	*E_SE*	*E_SE*
I_M	*δ'(E_N;E_NE; E_MW)*	*E_NE*	*δ'(E_E;E_NE; E_SE)*	*E_SE*

	I_S	I_SW	I_W	I_NW	I_M
I_N	*δ'(E_S;E_SW; E_SE)*	*E_SW*	*E_SW*	*δ'(E_W; E_NW; E_SW)*	*δ'(E_S;E_SW; E_SE)*
I_NE	*E_SW*	*E_SW*	*E_SW*	*δ'(E_W; E_NW;E_SW)*	*E_SW*
I_E	*E_SW*	*E_SW*	*δ'(E_W; E_NW;*	*E_NW*	*δ'(E_W, E_NW;*

			E_SW)		E_SW)
I_SE	δ’(E_W; E_NW; E_SW)	δ’(E_W; E_NW;E_SW)	E_NW	E_NW	E_NW
I_S	*	δ’(E_W; E_NW; E_SW)	E_NW	E_NW	δ’(E_N,E_NE; E_NW)
I_SW	δ’(E_E;E_NE; E_SE)	*	δ’(E_N ; E_NE; E_NW)	δ’(E_N ; E_NE; E_NW)	E_NE
I_W	E_SE	δ’(E_S;E_SW; E_SE)	*	δ’(E_N ; E_NE; E_NW)	δ’(E_E;E_NE; E_SE)
I_NW	E_SE	δ’(E_S;E_SW; E_SE)	δ’(E_S;E_SW; E_SE)	*	E_SE
I_M	δ’(E_S;E_SW; E_SE)	E_SW	δ’(E_W; E_NW; E_SW)	E_NW	*

In the above table, the symbol of “*” stands for universal relation. The table can be easily proved. Let the MBR of a, b and c be (x_{al}, y_{ab})-(x_{ar}, y_{at}), (x_{bl}, y_{bb})-(x_{br}, y_{bt}) and (x_{cl}, y_{cb})-(x_{cr}, y_{ct}), where (x_{al}, y_{ab}) and (x_{ar}, y_{at}) are the coordinates of two corner points (left-bottom and right-top) of a’s MBR. Considering b *I_N* a and c *I_NE* a, so we have:

$$x_{al}+\frac{1}{3}(x_{ar}-x_{al})<x_{bl}<x_{br}<x_{al}+\frac{2}{3}(x_{ar}-x_{al})$$

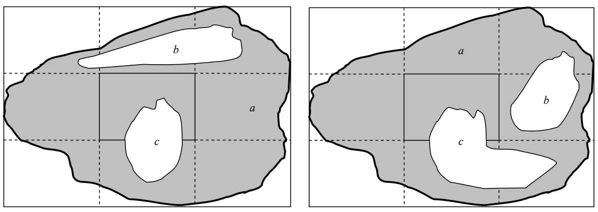
$$y_{ab}+\frac{2}{3}(y_{at}-y_{ab})<y_{bb}<y_{bt}<y_{at}$$

$$x_{al}+\frac{2}{3}(x_{ar}-x_{al})<x_{cl}<x_{cr}<x_{ar}$$

$$y_{ab}+\frac{2}{3}(y_{at}-y_{ab})<y_{cb}<y_{ct}<y_{at}.$$

The relation between b’s MBR and c’ MBR is $x_{bl}<x_{br}<x_{cl}<x_{cr}$, meanwhile, the relations between y_{bb}, y_{bt} and y_{cb}, y_{ct} is undetermined. According to MBR-based ECD model, the ECD relations between c and b might be c *E_E* b, c *E_NE* b, c *E_SE* b, c *E_NE*:*E_E* b, c *E_E*:*E_SE* b, c *E_NE*:*E_E*:*E_SE* b. They can be written as *δ’*(*E_E*; *E_NE*; *E_SE*).

When R_1 and R_2 are not atomic relations, i.e. b or c occupy more that one cell of the partition, the relation composition is a little complex. As shown in Fig. 8-a, according to b *I_NW*:*I_N*:*I_NE* a and c *I_M*:*I_S* a, the ECD relation between c and b could be determined as c *E_S* b.



a. b.

Figure 8. Composing complex ICD relations to get ECD relations

In order to discuss the composition of complex ICD relations, the quantitive representation of ICD relation should be applied.

Definition 5. Let R be complex ICD relation. $R= R_1: R_2 ... R_n$. Two ranges of R along x and y axes are respectively defined as:

$$[\min(Q_{1x},Q_{2x}...Q_{nx}),\max(Q_{1x},Q_{2x}...Q_{nx})]$$ and

$$[\min(Q_{1y},Q_{2y}...Q_{ny}),\max(Q_{1y},Q_{2y}...Q_{ny})]$$

Where $<Q_{ix}, Q_{iy}>$ is the quantification of R_i (i=1,2,…,n). For example, the ranges for *I_NW*:*I_N*:*I_NE*:*I_E* is [-1,1] and [0,1] respectively.

Based on this definition, the ECD relations can be inferred. Assume b R_1 a and c R_2 a, and at least one of them is complex ICD relation. In order to compute the ECD relation between c and b, each atomic ICD relation in R_2 and the ranges of R_1 should be considered. Let the ranges of R_1 be [F_x, T_x] and [F_y, T_y] and the atomic relations in R_2 denoted by $<Q_{ix}, Q_{iy}>$, i=1,…,n. Then Q_{ix} has 6 jointly exhaustive and pairwise disjoint (JEPD) possible relations to [F_{1x}, T_{1x}]. They are $Q_{ix}=F_x= T_x$, $Q_{ix}<F_x$, $Q_{ix}=F_x<T_{1x}$, $F_x<Q_{ix}<T_{1x}$, $F_x<Q_{ix}=T_x$ and $Q_{ix}>T_x$. These determine the ECD relations along horizontal axis. The relations along vertical axis are similar. According to their combinations, including 6*6=36 cases, the ECD relations between an object in $R_i(a)$ and b can be obtained, where R_i is an atomic relation in R_2 (Table 2).

Table 2. Composition of ECD relations along two axes

	$Q_{ix}=F_x= T_x$	$Q_{ix}<F_x$	$Q_{ix}=F_x<T_{1x}$	$F_x<Q_{ix}<T_{1x}$	$F_x<Q_{ix}=T_x$	$Q_{ix}>T_x$
$Q_{iy}=F_y= T_y$	*	δ’(E_NW;E_W; E_SW)	δ’(E_NW;E_W;E_SW;E_N;E_M;E_S)	δ’(E_N;E_M;E_S)	δ’(E_N;E_M;E_S;E_NE;E_E;E_SE)	δ’(E_NE;E_E;E_SE)
$Q_{iy}<F_y$	δ’(E_SW;E_N; E_SE)	E_SW	δ’(E_SW;E_S)	E_S	δ’(E_S;E_SE)	E_SE
$Q_{iy}=F_y<T_{1y}$	δ’(E_NW;E_N;_NE;E_W;E_M;E_E)	δ’(E_SW;E_W)	δ’(E_W;E_M; E_SW;E_S)	δ’(E_S;E_M)	δ’(E_M;E_E;E_S;E_SE)	δ’(E_E;E_SE)
$F_y<Q_{iy}<T_{1y}$	δ’(E_W;E_M;E_E)	E_W	δ’(E_W;E_M)	E_M	δ’(E_M;E_E)	E_E
$F_y<Q_{iy}=T_y$	δ’(E_W;E_M;E_E;E_SW;E_S;E_SE)	δ’(E_NW;E_W)	δ’(E_NW;E_N;E_W;E_M)	δ’(E_N;E_M)	δ’(E_N;E_NE;E_M;E_E)	δ’(E_NE;E_E)
$Q_{iy}>T_y$	δ’(E_NW;E_N;E_NE)	E_NW	δ’(E_NW;E_N)	E_N	δ’(E_N;E_NE)	E_NE

Through table 2, the relations of each tile of $R_2(a)$ to b can be looked up. Let they be $S_1,...,S_n$. Then we calculate the Cartesian product $S = S_1\times\ldots\times S_n$. Finally, the λ function is applied to combine the elements in the set of S and form the ECD relations in $R_1 \circ R_2$.

Let take Fig. 8-b as an example. The ranges of ICD(b,a) is [1,1] and [0,1], and ICD(c,a) include three atomic relations which are quantified as <0,0>, <0,-1> and <1,-1>. So we get three sets according to table 2: *δ’*(*E_W*;*E_SW*), {*E_SW*} and *δ’*(*E_S*;*E_SE*). The Cartesian product is {(*E_W*,*E_SW*,*E_S*), (*E_W*,*E_SW*,*E_SE*), (*E_W*,*E_SW*,*E_S*:*E_SE*), (*E_SW*,*E_SW*, *E_S*), (*E_SW*,*E_SW*, *E_SE*), (*E_SW*, *E_SW*,*E_S*:*E_SE*), (*E_W*:*E_SW*, *E_SW*, *E_S*), (*E_W*:*E_SW*, *E_SW*, *E_SE*), (*E_W*:*E_SW*,*E_SW*, *E_S*:*E_SE*)}. After applying λ function and removing duplicated relations, the result of composing R_1 and R_2 is {*E_W*:*E_SW*:*E_S*, *E_W*:*E_SW*:*E_S*:*E_SE*, *E_SW*:*E_S*, *E_SW*:*E_S*:*E_SE*}. Because c is assumed to be connected, some disconnected cases are excluded.

3.3 QSR about ICD and qualitative distance relations

In QSR, scale is an important concept, which refers to the size of the unit at which some problem is analyzed, such as at the county or state level (Montello, 2001). It is widely accepted that qualitative distance relations is scale-dependent (Clementini, 1997). When we said a place is "near" to another place in an urban scale, it might be much farther than the concept of "far" in a campus scale. Usually, we consider it is "far" when the metric distance between two objects is close to the analysis scale, while we believe it is "near" when the metric distance is much shorter compared with the scale size.

Compared with qualitative distance, quantitative distance has the following three axioms:

1. $d(x,x) = 0$ (reflexivity)
2. $d(x,y) = d(y,x)$ (symmetry)
3. $d(x,y)+d(y,z) \geq d(x,z)$ (triangle inequality)

Where $d(x,y)$ is the quantitative distance function from x to y. But when qualitative distance is taken into account, thses three rules will not be satisfied well any more. Usually qualitative distances are asymmetric (Egenhofer, 1995) and do not follow the triangle inequality rule.

Employing ICD relations, the container forms the background scale for determining qualitative distance. Adopting the distance measure method presented in (Goyal, 2001), distance is defined in ICD framework based on the shortest path when assume moving an object from one tile to another. In this paper, the qualitative distance is quantified into three distinctions: close (*Cl*), commensurate (*Cm*) and far (*F*). They have the order relation of *Cl*≤*Cm*≤*F*.

In ICD relation based framework, let *b*, *c* be two geometries inside *a*, they have atomic ICD relations with *a*. If the shortest path from $\bar{b}$ to $\bar{c}$ passes one tile ($\bar{b}$ and $\bar{c}$ are equal), then the QD relations is *close*. If it includes two parts, then the relation may be *commensurate* or *close*. At last, if more than two parts are involved, the possible relations are *far* or *commensurate*. The compositions are shown in table 3.

Table 3. Composition table of ICD and ICD to get qualitative distance relations

	I_N	I_NE	I_E	I_SE
I_N	*Cl*	*Cm, Cl*	*Cm, Cl*	*F, Cm*
I_NE	*Cm, Cl*	*Cl*	*Cm, Cl*	*F, Cm*
I_E	*Cm, Cl*	*Cm, Cl*	*Cl*	*Cm, Cl*
I_SE	*F, Cm*	*F, Cm*	*Cm, Cl*	*Cl*
I_S	*F, Cm*	*F, Cm*	*Cm, Cl*	*Cm*
I_SW	*F, Cm*	*F, Cm*	*F, Cm*	*F, Cm*
I_W	*Cm, Cl,*	*F, Cm*	*F, Cm*	*F, Cm*
I_NW	*Cm, Cl*	*F, Cm*	*F, Cm*	*F, Cm*
I_M	*Cm, Cl*	*Cm, Cl*	*Cm, Cl*	*Cm, Cl*

	I_S	I_SW	I_W	I_NW	I_M
I_N	*F, Cm*	*F, Cm*	*Cm, Cl*	*Cm, Cl*	*Cm, Cl*
I_NE	*F, Cm*	*F, Cm*	*F, Cm*	*F, Cm*	*Cm, Cl*
I_E	*Cm, Cl*	*F, Cm*	*F, Cm*	*F, Cm*	*Cm, Cl*
I_SE	*Cm, Cl*	*F, Cm*	*F, Cm*	*F, Cm*	*Cm, Cl*
I_S	*Cl*	*Cm, Cl*	*Cm, Cl*	*F, Cm*	*Cm, Cl*
I_SW	*Cm, Cl*	*Cl*	*Cm, Cl*	*F, Cm*	*Cm, Cl*
I_W	*Cm, Cl*	*Cm, Cl*	*Cl*	*Cm, Cl*	*Cm, Cl*
I_NW	*F, Cm*	*F, Cm*	*Cm, Cl*	*Cl*	*Cm, Cl*
I_M	*Cm, Cl*	*Cm, Cl*	*Cm, Cl*	*Cm, Cl*	*Cl*

If ICD(*b*,*a*) or ICD(*c*,*a*) are complex relations, it mean that $\bar{b}$ or $\bar{c}$ occupy more than one part. Assume $\bar{b}=\bigcup_{i=1}^{l} R_{bi}(a)$ and $\bar{c}=\bigcup_{i=1}^{m} R_{ci}(a)$, then

$$QD(b,c) = \min\{QD(R_{bi}, R_{cj}) \mid 1 \leq i \leq l, 1 \leq j \leq m\}$$

For example, let *b I_N:I_NE:I_M a* and *b I_S:I_SW a*, then QD(*b*,*c*) is *Cl* or *Cm*. A shortcoming of it is that the sizes of *b* and *c* are ignored.

3.4 QSR about ICD and topologic relations

Topological relations are related to the connection between spatial objects. Their important characteristic is that they remain invariant under topological transformations, such as rotation, translation, and scaling. There are two approaches to describe topological relations in a formalized fashion, i.e. region connection calculus (RCC) (Randell, 1992) and point set based intersection model (Egenhofer, 1991).

As mentioned above, if *b* and *c* are contained by *a*, then $\bar{b}$ and $\bar{c}$ can be regarded as an upper approximation of *b* and *c*. So the topological relation between $\bar{b}$ and $\bar{c}$ plays a filter role to exclude some impossible relations. When discussing topological relations, the point, line and area geometries should all be concerned. Considering that $\bar{b}$ and $\bar{c}$ are area geometries, we assume that *b* and *c* both are area geometries to be connected. So RCC-8 is applied to represent the topological relations in the following part.

According to RCC-8, there are eight jointly exhaustive and pairwise disjoint topological base relations between area geometries. They are *DC* (DisConnected), *EC(*Externally Connected*)*, *PO* (Partial Overlap), *EQ* (EQual), *TPP* (Tangential Proper Part), *NTPP* (Non-Tangential Proper Part), TPP^{-1} (converse TPP), and $NTPP^{-1}$ (converse TPP). (Fig. 9)

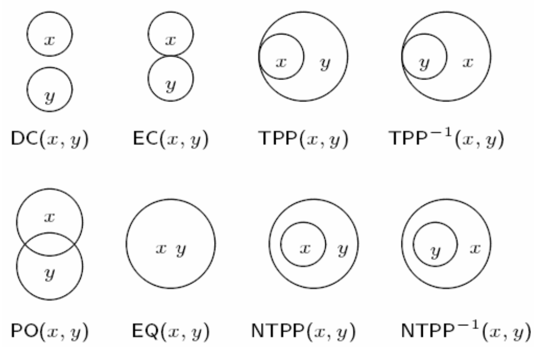


Figure 9. Eight base topologic relations of RCC-8

To judge topological relation between $\bar{b}$ and $\bar{c}$ is easy. If ICD(*b*,*a*) and ICD(*c*,*a*) are atomic ICD relations, then the relation between $\bar{b}$ and $\bar{c}$ is somewhat simple. There are only three possible topological relations, i.e. *DC*, *EC* or *EQ*. For example, *I_N*(*a*) *DC I_S*(*a*), *I_N*(*a*) *EC I_NE*(*a*) and *N*(*a*) *EQ N*(*a*). Otherwise, if they are complex, the relation can be determined according to their constituent parts. As shown in Fig. 10, the geometry *b* and *c* have ICD relations of *I_N:I_NE:I_M:I_E* and *I_S:I_SE:I_M:I_E* with *a* respectively, therefore we have

$\bar{b} = I\_N(a) \cup I\_NE(a) \cup I\_M(a) \cup I\_E(a)$ and

$\bar{c} = I\_S(a) \cup I\_SE(a) \cup I\_M(a) \cup I\_E(a)$ ·

Obviously the topological relation is $\bar{b}$ *PO* $\bar{c}$. That makes the possible relations between *b* and *c* include *DC, EC* and *PO*, meanwhile, the other 5 relations are excluded (Fig. 10).

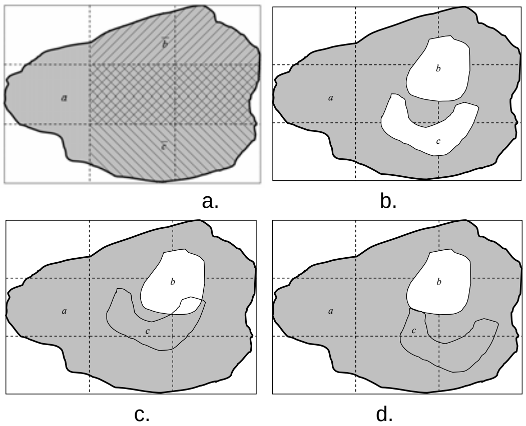
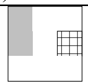
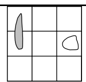
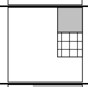
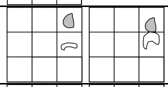
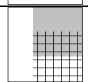
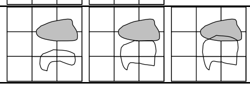
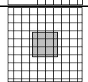
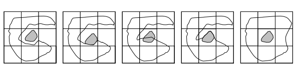
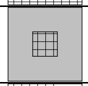
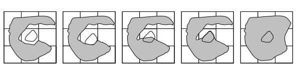
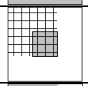
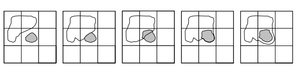
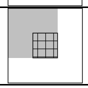
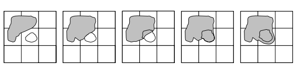
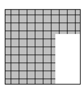
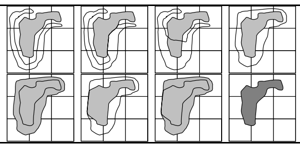


Figure 10. Filtering impossible topological relations according to ICD relations (**a.** $\bar{b}$ *PO* $\bar{c}$ **b.** *b DC c* **c.** *b PO c* **d.** *b EC c*)

Table 4 lists all cases of the topological relation between $\bar{b}$ and $\bar{c}$. In this table, the purpose of function TR is to determine the topological relation between two objects.

Table 4. Topological relations according to ICD relations

TR($\bar{b}$, $\bar{c}$)		TR(*b*,*c*)	
DC		*DC*	
EC		*DC,EC*	
PO		*DC,EC, PO*	
NTPP		*DC,EC, PO,TPP, NTPP*	
$NTPP^{-1}$		*DC,EC,* PO,TPP^{-1}, $NTPP^{-1}$	
TPP		*DC,EC, PO,TPP, NTPP*	
TPP^{-1}		*DC,EC,* PO,TPP^{-1}, $NTPP^{-1}$	
EQ		*	

The above discussion can be extended to some situations of upper approximation, such as MBRs instead of ICD cells. Some conclusions can be drawn:

1. Ordered by their capacities to exclude impossible topological relations, we have: $DC>EC>PO>NTPP=NTPP^{-1}= TPP=TPP^{-1}>EQ$. The *DC* relation gives the strongest constraint, while *EQ* has the weakest constraint, i.e. *EQ* can't exclude any relation.

2. Considering the relations after filtered, *DC* relation can be realized most easily, i.e. it can not be filtered. According to the times appearing in filtered relations, a sequence can also be created. It is $DC>EC>PO>NTPP=NTPP^{-1}= TPP=TPP^{-1}>EQ$, which is same to the above one.

To study these properties is beyond the inclusion of this paper, but what should be pointed out is that all spatial relations (not only topological relations) except for distance relations are possible if $\bar{b}$ *EQ* $\bar{c}$. Table 1 also supports this point. Such a conclusion is natural, for example, we could not deny any case of spatial relation between two objects if we only know that they locate in the same place.

4. CONCLUSIONS

Internal cardinal relation is a class of spatial relations should be emphasized on. The container object determines the analysis scale and the containee objects have part whole relation with the former. That makes ICD relation play an important role in the representation and transferring of spatial knowledge, especially survey knowledge.

Focusing on ICD-9 model, this paper summarized ICD-related qualitative reasoning. The main contribution is discussion on possible relations, including ECD relations, qualitative distance relations and topological relations, between two containee objects according to their ICD relations to the container in detail. Applying some tables, the composition of ICD relations is presented. Beside that, the simplification rule for complex ICD relations and the case of composing nesting ICD relations in deferent scale level are also described. As a conclusion, because the other spatial relations can be induced according to ICD relations, we can tell that the container and the ICD relations together form a positioning framework for spatial knowledge.

Focusing on ICD relations, planned further work includes:
1. studying on ICD-5 and ICD-13 models and corresponding QSRs;
2. developing some more complex and quantitative approaches of ICD relation and exploring the related QSR and computability.

Eventually, together with research on the other spatial relations, we wish the research lead to a formalized and computable way for survey knowledge.

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