ROBUST METRIC STRUCTURE FROM MOTION FOR AN EXTENDED SEQUENCE WITH OUTLIERS AND MISSING DATA

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ABSTRACT:

In this paper, we propose a robust metric structure from motion (SfM) algorithm for an extended sequence with outliers and missing data. There are three main contributions in the proposed SfM algorithm. The first is a novel jury-based preemptive LMedS procedure to achieve efficient outlier detection. The second contribution is a new iterative two-step scheme that consists of robust estimation techniques for projective structure from motion. The third contribution is a novel algorithm for robust metric upgrade by applying the M-estimator to the traditional linear constraints for metric upgrade. In addition, comparisons of the proposed algorithm with some previous methods through experiments on simulated data are shown to demonstrate the efficiency and robustness of the proposed algorithm
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1. INTRODUCTION

Structure from motion (SfM) has been one of the central problems in computer vision. Recent advances on multi-view geometry have been summarized in some representative books [1,2]. Since the outlier and missing data problems are inevitable during the process of automatic extraction and correspondence of feature points in practice, recent researches on SfM has focused on improving the robustness of SfM. In this paper, we proposed a novel algorithm to achieve the metric SfM for a long sequence with large missing data and outliers. We compare the proposed algorithm with previous methods through experiments on simulated data.

Some previous works on dealing with the missing data problem in SfM are briefly reviewed in the following. For the projective SfM, Fitzgibbon and Zisserman [4] proposed a solution based on trifocal tensor. Later, Martinec and Pajdla [3] proposed an algorithm that combines Sturm and Triggs' projective factorization method [5] and Jacob's fitting method [6] based on the subspace constraint. Note that this algorithm is used for comparison with the proposed method in the experimental results. On the other hand, several related works were developed under affine camera assumption, i.e. [6, 7], which simplifies the SfM to a linear system. This affine approximation of the SfM problem makes it equivalent to principle component analysis (PCA) with missing data [8], which is easier than the projective SfM in principle.

In addition, let us consider the other closely related issue - outlier problem. Up to now, there still exists no solution to handle outliers under projective SfM for a long sequence, though there were some previous methods developed based on pairwise or triplet views. For example, Torr [9] proposed the MAPSAC technique to estimate the fundamental matrix. Aanaes et al. [10] proposed to apply the robust M-estimators under the assumption of affine camera, thus leading to a linear system equivalent to the problem of robust PCA with outliers [11]. In this paper, we proposed a robust projective SfM algorithm to handle outliers in a long sequence.

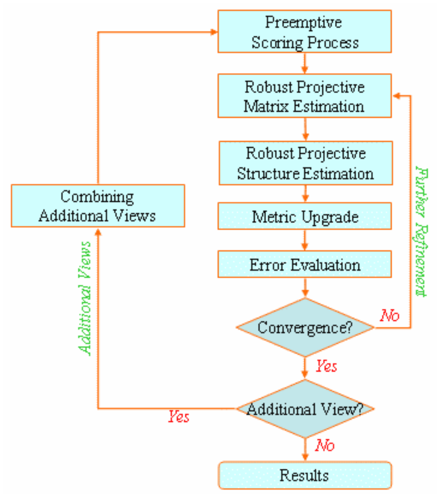


Figure 1. Flow diagram of the proposed metric SfM algorithm.

The main challenges in SfM come from the input data contaminated by missing features, mismatches, and false positions. It is obvious that the subspace / rank constraint on SfM can alleviate the influence due to Gaussian image noises. However, the subspace constraint from the measurement matrix cannot effectively handle outliers. In addition, the high degree of freedom in the projective matrices as well as the unknown projective depth makes the detection of outliers difficult.

The main goal in this work is to develop a robust algorithm for metric SfM from contaminated data without pre-setting any case-by-case parameters. The flow diagram of the proposed algorithm is shown in Figure 1, and the details are given in the next section. There are three main innovative ideas in the proposed SfM algorithm. First of all, we propose a preemptive jury-based consensus process, which dramatically improves the computational efficiency of LMedS estimation for outlier elimination. Secondly, an iterative projective reconstruction algorithm is developed to achieve the desired robustness. In this algorithm, each iteration involves first using the preemptive LMedS procedure to determine the projective matrices and then applying the robust M-estimator to optimize the projective structure as well as the projective depth with the projection matrices fixed. Thirdly, a robust metric upgrade process by using the iterative reweighted least squared approach is proposed. For self-calibration, in order to reduce the sensitivity of decomposing the projection matrix into camera calibration matrix and metric camera motion, we further take advantage of hard constraints on the calibration matrix to achieve a more robust solution.

The rest of this paper is organized as follows. An overview of the proposed algorithm is given in the next section. In section 3, we describe the proposed preemptive jury-based LMedS technique. Then the proposed iterative two-step projective SfM algorithm is described in section 4. Section 5 presents the robust metric upgrade process as well as a self-calibration process. Subsequently, we demonstrate the performance of the proposed algorithm on both simulated and real data. Finally, we conclude this paper in the last section.

2. SYSTEM OVERVIEW

The structure from motion problem is to recover camera motions as well as object structure from a given image sequence. To focus on the 3D reconstruction problem, we assume the feature point correspondences across different views in the video are given. Note that the given correspondences may include imperfect data, i.e. missing data and outliers. The camera information includes intrinsic and extrinsic parameters: the intrinsic parameters are represented by the camera calibration matrix, K; the extrinsic parameters determine the 3×3 rotation matrix, R, and a camera translation vector $\mathbf{t}$.

The flow diagram of the proposed algorithm is shown in figure 1. Started from the preemptive scoring process, we score each observation from the two-view geometry, i.e. fundamental matrix. The second stage is the robust projective factorization via an iterative two-step reconstruction algorithm. Then a robust estimation approach is applied to the upgrade the projective reconstruction into a metric one. The error evaluation, obtained from the residues between the data matrix and the reconstructed projection and structure matrices, provides information for further refinement. Followed by combining additional views, we return to the first stage until all views are integrated.

3. JURY-BASED PREEMPTIVE LMEDS

RANSAC [12] and LMedS [2] are two traditional robust techniques to eliminate outliers. However, these techniques are computationally expensive. Therefore, we proposed a more efficient procedure to speed up the computational process. Motivated by Nister's preemptive RANSAC [14], we developed the so-called preemptive jury-based LMedS.

Referred to Nister's literature [14], the preemptive scheme can be categorized into the depth first and breath first manners. The depth first manner, noted as an order rule in the preemption scheme, dominates the hypothesis generation. This rule selects the inliers with higher likelihood for hypothesis generation according to previous experiences. On the other hand, the breath first fashion, noted as the preference rule, efficiently evaluates the hypotheses on equal footing. Not all observations are used to score all the selected hypotheses, but this rule eliminates bad hypotheses in the scoring procedure.

In principal, Nister's breath first preemptive scheme has a potential problem that the final result strongly depends on the scoring series. In his algorithm, the measurement is not on equal footing because earlier selected observations possess greater power in the hypotheses elimination than the later selected observations. We can declare that the breath first preemptive scheme works well only when the outlier rate is relatively small. Some experimental results will be shown later to support this argument.

To overcome the above problem with the breath first scheme and to further improve the efficiency in the LMedS technique, we develop a jury-based preemptive scheme in conjunction with the LMedS process. Instead of a single observation as used in the breath first scheme of the preemptive RANSAC method, we select a set of observations into a jury. Under the assumption of random sampling, the outlier rate in jury is approximately the same as that in whole. Thus, we can approximate the median value efficiently. The proposed jury-based preemptive LMedS process is given as follows,

1. Generate the hypotheses indexed with h = 1,…, $f(1)$.
2. Randomly permute the observations and classify them into m juries.
3. Compute the scores $L_1(h)$ = median{ $\rho(j, h)$ | j belongs to jury 1} for $h = 1, . . . , f(1)$. Set $i = 2$.
4. Reorder the hypotheses so that the range h = 1, . . . , f(i) contains the best f(i) remaining hypotheses according to $L_{i-1}(h)$
5. If $i > m$ or $f(i) = 1$, quit with the best remaining hypothesis as the preferred one. Otherwise, compute the scores $L_i(h)$ = median{ $\rho(j, h)$ | j belongs to jury $1 .. i$,} for $h = 1, . . . , f(i)$, set $i = i + 1$ and go to Step 4.

Algorithm 1. Jury-based preemptive LMedS algorithm

Note that, in Algorithm 1, $f(i)=\left\lfloor M\, 2^{-\left\lfloor \frac{i}{B}\right\rfloor}\right\rfloor$, where M is the total number of hypotheses and B is the block size, denotes a decreasing preemption function that indicates how many hypotheses are to be kept at each stage. The scoring function, $\rho(j,h)$, gives a scalar value representing the log likelihood of the observation, j, given that the hypothesis, h, is the correct motion model. Note that observations are random selection of the input matches which are not used for building hypothesis. For more details of the theoretical derivation of the preemptive scheme, the readers can refer to Nister's original paper [14]. The notations in this section follow those used in [14]. We modify the scoring process in the proposed preemptive LMedS scheme and improve the computational efficiency. Some experiments on simulated data are demonstrated to show its performance in section 6.

We applied this procedure to the two stages of our algorithm. One is the computation of fundamental matrix in the preemptive scoring process; the other is the projective factorization in the stage of robust projection matrix estimation.

4. ROBUST PROJECTIVE STRUCTURE FROM MOTION

For projective SfM, the factorization approach can be formulated as follows,

$$\mathbf{W} \equiv \mathbf{D} \otimes \mathbf{U} = \mathbf{PX} = (\mathbf{PH})(\mathbf{H}^{-1}\mathbf{X}) = \tilde{\mathbf{P}}\tilde{\mathbf{X}} \quad (1)$$

where $\mathbf{W}$ is the measurement matrix formed by input data matrix, $\mathbf{U}$, and their corresponding projective depths, $\mathbf{D}$. The operator $\otimes$ denotes the scale (projective depth) multiplying its corresponding vectors (homogeneous image coordinates). The matrix form is as follows,

$$\begin{bmatrix} \lambda_1^1\mathbf{u}_1^1 & \lambda_2^1\mathbf{u}_2^1 & \cdots & \lambda_n^1\mathbf{u}_n^1 \\ \lambda_1^2\mathbf{u}_1^2 & \lambda_2^2\mathbf{u}_2^2 & \cdots & \lambda_n^2\mathbf{u}_n^2 \\ \vdots & \vdots & \ddots & \vdots \\ \lambda_1^m\mathbf{u}_1^m & \lambda_2^m\mathbf{u}_2^m & \cdots & \lambda_n^m\mathbf{u}_n^m \end{bmatrix} = \begin{bmatrix} \mathbf{P}^1 \\ \mathbf{P}^2 \\ \vdots \\ \mathbf{P}^m \end{bmatrix} \begin{bmatrix} \mathbf{X}_1 & \mathbf{X}_2 & \cdots & \mathbf{X}_n \end{bmatrix} \quad (2)$$

where $\mathbf{u}_j^i$ denotes the image position of the *j*-th point at the *i*-th view represented in a homogeneous 3-vector $(u_j^i, v_j^i, 1)^T$, X_j is the corresponding three-dimensional position of the *j*-th point represented in a homogeneous coordinate, $\mathbf{P}^i$ is a 3×4 camera projection matrix of the *i*-th view, and λ_j^i is the corresponding projective depth in the projective geometry. The projective 3D-to-2D transformation is written as $\lambda_j^i\mathbf{u}_j^i = \mathbf{P}^i\mathbf{X}_j$. Applying the singular value decomposition (SVD) to the measurement matrix enforces the subspace constraint, i.e. rank-4 constraint. Referred to [1], the algorithm iteratively tunes the projective depths with the subspace constraint to achieve the projective reconstruction. The convergence property is further discussed in [1].

4.1 Robust Determination of the Projection Matrices

Projective factorization is very sensitive to outliers. To overcome such challenges, robust techniques, such as RANSAC, can strategically be applied to the original algorithm to improve the results under outlier disturbance. The basic requirement for such robust techniques to eliminate outliers is that there exist more than necessary constraints, so that the reliability of each constraint can be consensually evaluated. For the projective factorization, the minimal reconstruction set requires 3 views with 6 corresponding points. The robust version for projective factorization is to apply the preemptive jury-based LMedS to the projective factorization [1]. Note that the feature selection is based on the Monte-Carlo process according to the preemptive scores.

4.2 M-estimator to Compute the Projective Structure

Given a set of 2D image points with the associated projection matrices, the corresponding three-dimensional feature points can be computed with a closed-from solution once the projective depths are known. However, the projective depths are generally unknown. Therefore, we carry out an iteratively approach to estimate the optimal three-dimensional structure by adjusting the projective depths appropriately. For the *k*-th iteration, we denote the current depth matrix as $\mathbf{D}^{(k)}$, and the closed-from solution can be formulated as follows,

$$\mathbf{X}^{(k)} = \mathbf{P}^{+}\left(\mathbf{D}^{(k)} \otimes \mathbf{U}\right) \quad (3)$$

where $\mathbf{P}^{\dagger}$ denotes the pseudo-inverse of the matrix $\mathbf{P}$. Each projective depth entry in $\mathbf{D}^{(k+1)}$ at the next iteration is then updated as $\lambda_i^{j(k+1)} = \left\|\mathbf{P}^i\mathbf{X}_j^{(k)}\right\| / \left\|\mathbf{u}_j^i\right\|$. However, to further improve the robustness, we apply a robust measure at the outer loop. Thus, the RLS (re-weighted least square) solution replaces the LS (least square) solution in equation (3). We make use of the robust ρ function, such as the Lorentzion (or Cauchy) function [14] commonly used in robust statistics, to develop the M-estimation for the projective factorization. The robust ρ function is defined [14] as follows

$$\rho(r) = \log(1 + \frac{r^2}{2\hat{\sigma}^2}) \quad (4)$$

The minimization of the robust energy function can be achieved by the iteratively RLS minimization. In this case, the weights are associated with the given projective matrices, and the residue is the norm of the error between the 2D image points and the obtained projective reconstruction which is determined at the inner-loop by adjusting the depths as described above. Thus the energy function to be minimized can be written as the following dynamic energy function, which is changing from iteration to iteration.

$$\mathbf{W}^{(k)}\mathbf{PX} = \mathbf{W}^{(k)}\left(\mathbf{D} \otimes \mathbf{U}\right) \quad (5)$$

where the weights associated with the residue is given as follows

$$w_i = \frac{2\hat{\sigma}^2}{2\hat{\sigma}^2 + r^2} \quad (6)$$

Note that $\hat{\sigma} = 1.4826\left(1 + 5/(n-p)\right)\sqrt{E_{med}}$ is the median absolute deviation (MAD) estimation [14].

1. Initialize all the weights to 1, i.e. W = ***I***.
   - 2a. Initialize all $\lambda_i^{j(0)} = 1$ for $\mathbf{D}^{(0)}$ and set $k = 0$.
   - 2b. Normalize the depths by multiplying each column of D with a constant factor.
   - 2c. Solve $\mathbf{X}^{(k)} = \mathbf{P}_{\mathbf{W}}^{+}\left(\mathbf{D}^{(k)} \otimes \mathbf{M}\right)$
   - 2d. Update $\lambda_i^{j(k+1)}$. Set $k = k + 1$
   - 2e. Exit the inner-loop if converged, else go to step 2b.
3. Update the weights by the M-estimator from eq(5)
4. Exit if converged, else go to the inner loop in step 2.

Algorithm 2. Robust M-estimation of the projective structure with projective matrices given.

With this modification, the pseudo-inverse of $\mathbf{P}$, turns from the

LS solution, $\mathbf{P}^{\dagger} = \left(\mathbf{P}^T\mathbf{P}\right)^{-1}\mathbf{P}^T$, to the RLS solution, $\mathbf{P}_{\mathbf{W}}^{\dagger} = \left(\mathbf{P}^T\mathbf{W}^2\mathbf{P}\right)^{-1}\mathbf{P}^T\mathbf{W}^2$. Given the projective matrix, the algorithm2 shows the robust estimation of the homogeneous three-dimensional structure. Note that step 2 in Algorithm 2 is the inner-loop in order to iteratively determine the projective depths; step 1, 3, and 4 is for evaluating the reliability of the input projective matrices for robust estimation of the projective structure.

5. ROBUST METRIC UPGRADE

To upgrade the projective reconstruction to a metric one, we have to determine the ambiguity matrix, $\mathbf{H}$ in equation (1). This has to employ additional constraints, which may come from the prior knowledge of the camera calibration matrix. According to the absolute quadric constraint [17], the projection of the absolute quadric in the image yields the dual image absolute conic. This formulation of the absolute quadric constraint is shown as follows,

$$\omega_i^* = K_i K_i^T \propto P_i \Omega^* P_i^T \tag{7}$$

The following assumptions provide linear constraints for the entries in a symmetric 4×4 rank-3 matrix Ω^*, i.e

$f_x = f_y$	$\mathbf{P}_i^{(1)}\Omega^*\mathbf{P}_i^{(1)T} = \mathbf{P}_i^{(2)}\Omega^*\mathbf{P}_i^{(2)T}$
$s = 0$	$\mathbf{P}_i^{(1)}\Omega^*\mathbf{P}_i^{(2)T} = 0$
(u_o, v_o)	$\mathbf{P}_i^{(1)}\Omega^*\mathbf{P}_i^{(3)T} = 0$ $\mathbf{P}_i^{(2)}\Omega^*\mathbf{P}_i^{(3)T} = 0$

(8)

Note that $\left(f_x, f_y\right)$ are the focal lengths along x- and y-axis, respectively, s denotes the skew factor, $\left(u_o, v_o\right)$ is the principle point or image center, and $\mathbf{P}_i^{(j)}$ denotes the *j*-th row of $\mathbf{P}_i$. The linear (closed-form) solution is referred to [15].

In order to obtain a more robust solution, we weight each constraint with the robust M-estimator, which is similar to the computation of the robust projective structure introduced in section 4.2. For each view, we have the following linear equations,

$$\begin{cases} w_i\left(\mathbf{P}_i^{(1)}\tilde{\Omega}\mathbf{P}_i^{(1)T} - \mathbf{P}_i^{(2)}\tilde{\Omega}^*\mathbf{P}_i^{(2)T}\right) = 0 \\ w_i\left(\mathbf{P}_i^{(1)}\tilde{\Omega}^*\mathbf{P}_i^{(2)T}\right) = 0 \\ w_i\left(\mathbf{P}_i^{(1)}\tilde{\Omega}^*\mathbf{P}_i^{(3)T}\right) = 0 \\ w_i\left(\mathbf{P}_i^{(2)}\tilde{\Omega}^*\mathbf{P}_i^{(3)T}\right) = 0 \end{cases} \tag{9}$$

In order to clarify the notation, we denote $\tilde{\mathbf{\Omega}}^*$ as the initial absolute quadric computed from the above equations, which may not be exactly rank-3. At the beginning, we set equal weights for each view, i.e. $w_i = 1$, to determine $\tilde{\mathbf{\Omega}}^*$. By enforcing the rank-3 constraint on $\tilde{\mathbf{\Omega}}^*$, we determine the absolute quadric $\mathbf{\Omega}^*$ via SVD. However, this step leads to additional errors in the linear system, thus we define the following residue for each projection matrix,

$$\begin{aligned} r_i^2 &= \left(\mathbf{P}_i^{(1)}\mathbf{\Omega}^*\mathbf{P}_i^{(1)T} - \mathbf{P}_i^{(2)}\mathbf{\Omega}^*\mathbf{P}_i^{(2)T}\right)^2 \\ &+ \left(\mathbf{P}_i^{(1)}\mathbf{\Omega}^*\mathbf{P}_i^{(2)T}\right)^2 + \left(\mathbf{P}_i^{(1)}\mathbf{\Omega}^*\mathbf{P}_i^{(3)T}\right)^2 + \left(\mathbf{P}_i^{(2)}\mathbf{\Omega}^*\mathbf{P}_i^{(3)T}\right) \end{aligned} \tag{10}$$

According to the residues, we re-adjust the weights as equation (6), i.e. Lorentzion (or Cauchy) function [14] mentioned in 4.2. The RWLS process, iteratively reducing the residues under the rank-3 constraint on $\mathbf{\Omega}^*$ through tuning the weights, turns out to be an M-estimator for robust metric upgrade.

6. EXPERIMENTAL RESULTS

In this section, we show some experimental results of the proposed algorithm in comparison with some previous methods on simulated data. We first show the experimental comparison of the proposed jury-based preemptive LMedS algorithm, followed by the experimental comparison for the proposed SfM algorithm.

We used 200 point correspondences with additive Gaussian noises (standard deviation = 1.5) in image as well as 5~40% gross outliers. We compared the proposed preemptive LMedS, which uses the block size $B=1$ and 10 observations in a jury, with Nister's preemptive RANSAC, with the block size $B=10$, LMedS, and MAPSAC algorithms with this experiment on fundamental matrix estimation with contaminated data._For a fair comparison, we used the same Torr's seven-point method for computing the fundamental matrix for all the above four algorithms. Furthermore, the four algorithms share the same set of hypotheses which were randomly generated from 1000 samples, so that we examined which of these four methods makes the best use of the hypothesis. The experimental results shown in Figure 2 indicate that the proposed scheme approximates the performance of the full-scoring procedures, i.e. LMedS and MAPSAC, and it reduces 90% of the full-scoring burdens. Thus, it shows the advantage in the computational efficiency of the proposed algorithm.

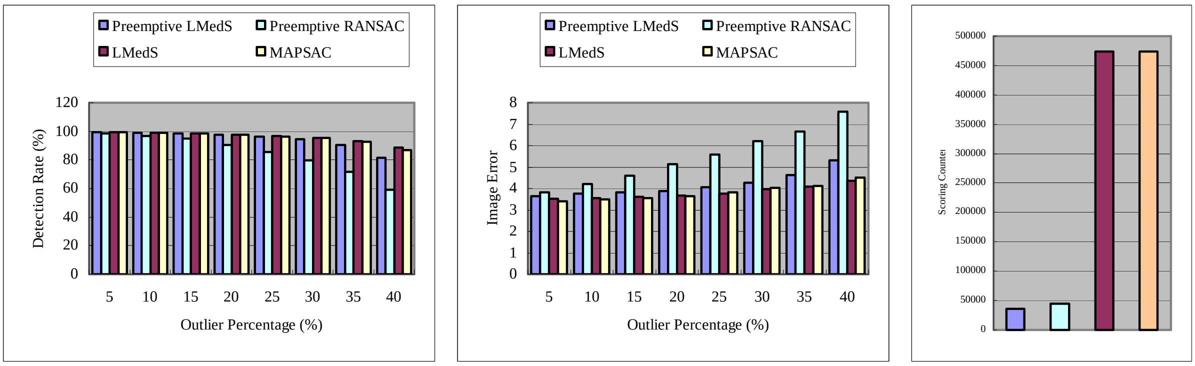
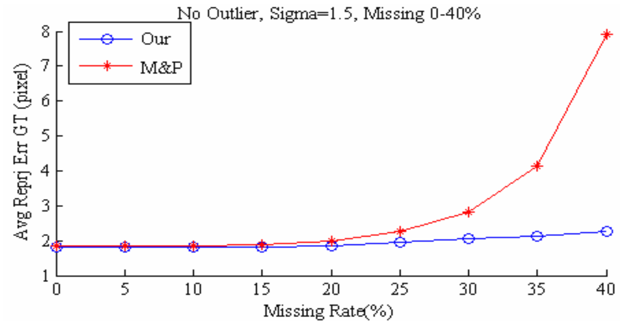


Figure 2. Robust computation for increasing outlier rate with the same set of hypotheses. Left: detection rate of the outliers; Middle: Image error that represents the correctness of fundamental matrix; Right: scoring counter.



(a)

(b)

Figure 3. (a) The reprojection errors and (b) the 3D errors at different missing rates for the proposed algorithm and Martinec and Pajdla's algorithm.

In the following, we show the experimental comparison of the SfM methods on simulated data. The simulations are designed to examine the performance under different data missing and outlier rates. We compare the proposed SfM algorithm with Martinec and Pajdla's algorithm [1]. Their code is available from the public domain. Note that their metric upgrade process is removed since it crashed for some simulated cases. So we upgraded their projective reconstruction according to the ground truth.

The simulation environment is as follows. First of all, we randomly generated 300 points within a 20 unit length squared box in 3D space, and its center is randomly located around the world center in the radius of 10 unit length. Thirty cameras are located in a circle of radius 100 unit length, and their viewing directions are the world center plus additional random rotation within 5 degrees in Euler angle. Calibration matrices are constant with focal lengths within 1500, and skew parameters 1.5, and image center (1000, 650). Each observation point is perturbed with Gaussian noises $\sigma = 1.5$, followed by the rounding operation. The outliers are randomly selected according to the simulated outlier rate. The farthest points to the current camera are selected as the missing points at that view according to a given missing rate. One hundred trials are made to obtain the final results.

We examine the reprojection error, 3D reconstruction error, and reconstruction rate at different data missing rates and outlier rates to compare the performance. The reprojection error evaluates the error between reconstructed and measured points in image space, and the 3D error is measured in RMS of the Euclidean distance of the simulated unit length. The reconstruction rate is the ratio of the reconstructed points to the total number of points.

In the first simulation, we examine the algorithms with different missing rates as shown in Figure 3. In the second simulation, we examine the robustness under different outlier rates with a constant missing rate as shown in Figure 4. Then, we test the performance with more views integrated as shown in Figure 5.

7. CONCLUSION

In this paper, we proposed a novel robust metric structure from motion algorithm for a long sequence with outliers and large missing data. The jury-based preemptive LMedS procedure was developed to achieve efficient outlier detection in the robust projective SfM. In addition, we also applied robust estimation techniques in the projective SfM as well as the metric upgrade processes. We demonstrate the robustness, accuracy and efficiency of the proposed SfM algorithm through experimental comparisons with previous methods.

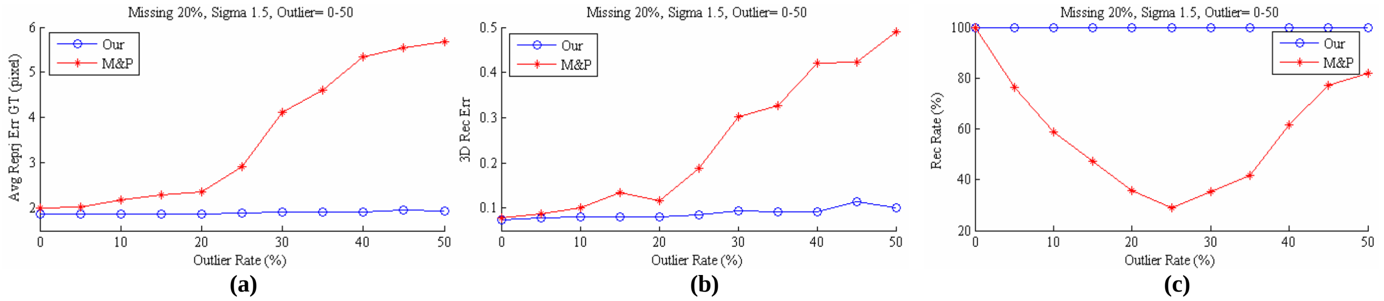


Figure 4. (a) The reprojection errors; (b) the 3D errors; (c) the reconstruction rates at different outlier rates for the proposed algorithm and Martinec and Pajdla's algorithm.

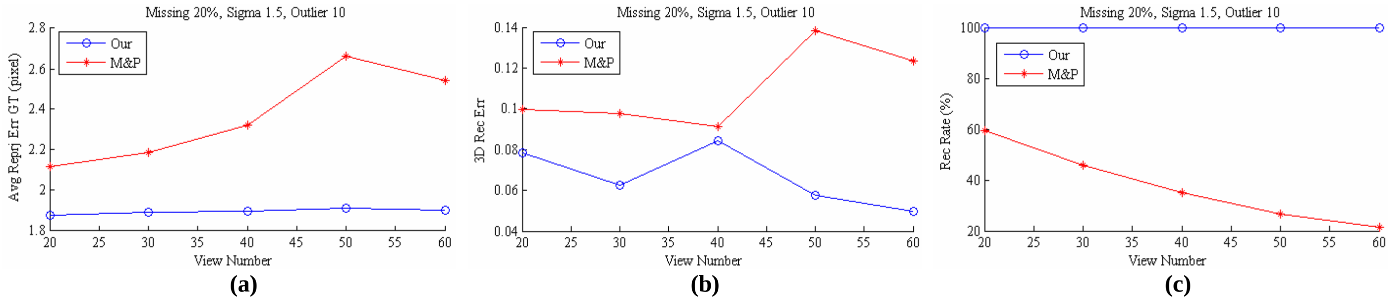


Figure 5. (a) The re-projection errors; (b) the 3D errors; (c) the reconstruction rates at different views for the proposed algorithm and Martinec and Pajdla's algorithm.

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