

SIMULATING SAMPLING EFFICIENCY IN AIRBORNE LASER SCANNING BASED FOREST INVENTORY

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ABSTRACT:

A simple simulator was developed to test whether airborne laser scanning can be used as a strip sampling tool for forest inventory purposes. The simulator is based on the existing two stages, grid based laser inventory procedure. A population of trees was created using an existing forest stand structure generator. Each tree was represented by means of its 3D-crown model derived from airborne laser scanning measurements and field measured parameters, i.e. total tree height, height of crown base and average crown diameter. Monte Carlo simulations were run to assess the efficiency of volume estimates obtained from airborne laser scanning and ground based inventory. The lowest RMSE for the laser based estimates was $5.1 \text{ m}^3\text{ha}^{-1}$ (2.0%) and the highest was $8.4 \text{ m}^3\text{ha}^{-1}$ (3.3%), while RMSE for the ground based estimates varied between $13.7 \text{ m}^3\text{ha}^{-1}$ (5.4%) and $18.4 \text{ m}^3\text{ha}^{-1}$ (7.2%). The LIDAR based estimation was on average 6.3 times more efficient in terms of MSE than ground based sampling. The RMSE of the volume estimates increased with increasing plot size, for a given sampling intensity. The results indicated that forest surveys over large areas carried out using airborne laser scanning as a strip sampling tool can provide accurate estimates, and can be more effective than traditional systematic ground plot based inventories.

1. INTRODUCTION

During the past two decades, remote sensing techniques have proven to meet some of the demand for environmental related data at fairly low cost. Among these techniques, small footprint LIDAR (LIght Detection and Ranging) has become one of the most common remotely sensed data sources for analyzing the canopy structure at the scale of operational forest management (Wynne, 2006).

Research has shown that profiling LIDAR can provide reliable biomass sampling based estimates at low costs (e.g. Nelson et al., 2006). The LIDAR based procedure consists of a two stage sampling scheme. LIDAR transects are taken by flying parallel flight lines separated by many kilometers over the area in question. Systematically distributed ground plots or ground transects are measured along the LIDAR transect. Ground based estimates are regressed against LIDAR measurements, and the resulting regression equation are used for prediction along the LIDAR transects across the entire sampled area (Nelson et al., 2004).

In contrast to profiling LIDAR systems which only collects a narrow line of data on the ground, commercial airborne laser scanners provide an accurately geolocated cloud of 3-dimensional observations, which can be related to ground measurements such as plots of various shapes and sizes. Scanning LIDAR is today used operationally for stand-based "wall-to-wall" inventories of forest stands in Norway (Næsset, 2004). For larger regions such as counties or nations, "wall-to-wall" inventories are not feasible. However, even scanning systems can be used in regional forest inventory, considering the flight lines as part of a strip sampling design by flying parallel, equally spaced strips over the study area and collecting sample plots only within strips, using for example systematic sampling schemes. Sampling applications are often relevant in areas with a size where it is not feasible to establish a ground

truth reference value. Consequently, designing an optimal inventory system has to rely on some kind of simulation, where different combinations of field and airborne data collection can be explored.

The aim of this study was to develop a prototype of a simple, small-scale simulator in order to assess the efficiency of laser scanning based volume estimates relative to the corresponding ground plot based estimates, when airborne laser scanning was used as a strip sampling tool. This simulator was based on the two stages, grid based sampling procedure developed and tested by Næsset & Bjercknes (2001) and Næsset (2002, 2004).

2. MATERIAL AND METHODS

Forest stand data and combinations of ground measurements of single tree parameters and airborne laser data were used to build 3D crown models for Norway spruce trees. Then, existing forest stand generator software and these models were employed to obtain a virtual forest as input for the simulations.

2.1 Stand data

The empirical stand data and single tree parameters were comprised in two datasets.

The first dataset (see Bollandsås & Næsset, 2007; Solberg et al., 2006) was collected in summer 2003. Twenty circular plots of 0.1 ha were collected from a boreal nature reserve located in south-eastern Norway. The forest was multilayered with a broad range of tree sizes and stand ages, and dominated by Norway spruce [*Picea abies* (L.) Karst.] and Scots pine (*Pinus silvestris* L.). The plots were established in subjectively selected spruce dominated sites. On each plot, all trees with height (d_{bh}) $\geq 3\text{cm}$ were callipered and tree heights were measured on trees selected with probability proportional to stem basal area. Mean

diameter was defined as diameter corresponding to mean stem basal area (d_{BA}) and mean height was defined as the average basal area weighted (Lorey's) height (h_L).

Both Global Positioning System (GPS) and Global Navigation Satellite System (GLONASS) were used to determine the planimetric coordinates (Euref89) of the plot centers. The average estimated accuracy of the plot coordinates was 10 cm.

For the first dataset, polar coordinates from the plot centre were registered for all trees with $d_{bh} \geq 3$ cm. Total tree height, height of crown base, crown radius in four cardinal directions, and average crown diameter were measured on trees selected from each plot. The final coordinates for all single trees were computed in Euref89, using plot centre coordinates and plot-wise polar tree coordinates.

The second dataset (see Næsset, 2004) comprised 60 large plots located in a productive forest area of approximately 5000 ha in the municipality of Krødsherad, south-eastern Norway. The forest composition was dominated by Norway spruce and Scots pine, while younger stands were dominated by deciduous species, mainly birch (*Betula pubescens* Ehrh.). The plot areas were from 3121 to 4219 m², with an average of 3739 m². Within each plot, all trees with diameter at breast height $d_{bh} \geq 4$ cm and $d_{bh} \geq 10$ cm were callipered in young and mature stands, respectively, using 2 cm diameter classes. Height measurements were taken from trees selected with probability proportional to stem basal area at breast height. For each plot, the mean height corresponding to Lorey's height was computed from the mean height of the individual diameter classes, weighted by total plot basal area for each diameter class.

2.2 Laser data

Laser scanner data were acquired during June 2005 (leaf-on canopy condition) from the same area as the first dataset, with an Optech ALTM 3100 sensor operating at 100 kHz laser pulse repetition rate and 70 Hz scanning frequency. The aircraft was flown approximately 750 m above ground with an average speed of 75 ms⁻¹. The maximum half scan angle was 10°, and the corresponding swath width was about 264 m. Pulses transmitted at scan angles that exceeded 8° were excluded from the final dataset. The average footprint size was about of 21 cm, with an average point density of 5.09 m⁻². First and last echo were recorded.

2.3 Laser-derived single tree models

Laser data and the ground measurements collected in summer 2003 from 0.1 ha stand plots comprised into the first dataset were used to obtain crown representation of Norway spruce trees. Laser pulse hits were related to tree crown projections by

the mean of planimetric coordinates, and then the resulted laser point clouds were considered as spatial crown models for Norway spruce trees. Laser pulses with heights below 2 m were considered as ground points.

The relationships between field and laser measurements were established for a total of 435 spruce trees. Hence, each of these trees were represented as unique combinations of diameter (d_{bh}), height (h), crown height (c_h), crown projection radius (c_r), stem volume (v) (Table 1), and the associated 3D crown models. For each of these trees, the volume was calculated by the means of functions for Norway spruce with bark (Vestjordet 1967). Further in this study, the trees were called "single tree models".

Metrics	Mean	S.D.	Min	Median	Max
d_{bh} (cm)	19.8	10.4	3.2	18.8	51
h (m)	15.8	6.1	3.6	16	29.5
c_h (m)	3.4	2.4	0.2	3.1	13.5
c_r (m)	1.3	0.4	0.6	1.3	2.9
v (m ³)	0.38	0.41	0.003	0.22	2.46

^a d_{bh} -diameter; h -height; c_h -crown height; c_r -crown radius; v -volume; S.D.-standard error

Table 1. Descriptive statistics for individual tree model parameters ^a.

2.4 Virtual forest

The program package SILVA 2.2 (Pretzsch et al., 2002) was used to generate a virtual forest. The stand generator provides a tree list with associated parameters. For each tree, the following information was recorded: tree species, diameter at breast height, total height and height of crown base, crown diameter and tree coordinates (x , y). To generate the tree lists, the input parameters were tree species, d_{BA} (cm), d_{max} (cm), h_L (m), and N , obtained from ground measurements. Only 13 plots from the first dataset and 25 plots from the second dataset provided acceptable combinations of input parameters which could be used to obtain tree lists by means of SILVA 2.2 (Tab. 2). The other plots were rejected due to inadvertencies between the test plot reference data and model calibration of the stand generator. Totally, a number of 38 tree lists were obtained and each of them was considered as a possible realization of a forest stand, given the ground-measured input parameters.

The virtual forest study area was defined in a 2D-local coordinate system with axes being multiples of 100 m, and the terrain was assumed to be flat. The frame of study area was considered to be a two-dimensional array, where each (i , j) position is a squared area representing a forest stand of 1.0 ha.

	Stand parameters															
	First dataset (0.1 ha plots)				Second dataset (large plots)											
	$d_{BA} s$	$d_{max} s$	$h_L s$	Ns	$d_{BA} s$	$d_{max} s$	$h_L s$	Ns	$d_{BA} p$	$d_{max} p$	$h_L p$	Np	$d_{BA} b$	$d_{max} b$	$h_L b$	Nb
Max	30.1	60.6	28.9	1040	27.6	51	22.2	904	36.3	49	23.6	622	23.9	47	20	286
Min	19.9	39.6	17.7	650	12	17	8.4	13	18.7	35	12.7	10	11.5	13	10.8	5
Mean	17.2	37.3	17	671	17.4	32.1	14.9	360	21.6	35.5	14.4	229	12.6	21.7	10.6	82

^a d_{BA} =basal area mean diameter (cm); d_{max} =maximum diameter (cm); h_L =basal area weighted mean height (m); N =stem number per ha; s =Norway spruce; p =Scots pine; b =deciduous trees (assimilated with birch).

Table 2. Summary of stand metrics for 13 selected plots from the first dataset and 25 plots from the second dataset ^a.

To create the population, one of the 38 virtual forest stands of 1.0 ha generated by means of SILVA 2.2 was randomly allocated to each (i, j) array position, and then the stand coordinates for each tree were translated according to the new location within the array. The neighborhood effects among forest stands were ignored. Thus, the spatial structure of each cell was supposed to be independent of the position in the array. The study area was defined as a square of 36 km².

Further, each tree from the tree list was substituted with a diameter-equivalent single tree model. Because of the relatively small number of single tree models (i.e. 435 trees) which could be derived from available dataset, only d_{bh} was used as key. The rest of the single tree model parameters, i.e. height, crown height, crown radius, laser pulse heights, and stem volumes, were then transferred to the corresponding diameter-equivalent trees from the tree list positioned at (x_i, y_i) coordinates in the study area. The matching results often consisted of more than one single tree model with equal diameters. In this situation, only one of these tree models was randomly selected to replace the tree at the position (x_i, y_i) from the generated forest stand. For the situations when diameter matching did not occur- which means that some trees from generated forest stands have diameters that were not among the diameters of single tree models, a single tree model with diameter closest to the missing value, either larger or smaller, was selected instead. Thus, the study area was re-populated with laser derived tree models, and the volume of the entire population was calculated as the sum of individual trees.

For other species than Norway spruce, i.e. Scots pine and birch, there were no available laser data for building 3D crown models. For this reason, diameter matching was done regardless of species, which means that trees of different species could be matched if they had the same diameter. After diameter matching, trees from the tree list generated by means of forest stand generator were replaced with diameter equivalent Norway spruce single tree models, regardless tree species.

Laser scanning data consist of clouds of laser hits related to tree crowns. In this study, each laser hit (first echo) has known x , y and z -coordinates, but in this analysis, the (x_i, y_i) coordinates of each laser hit were discarded. It was assumed that laser hits related to trees inside a grid cell fall inside the same cell where these trees are located, and that all the hits inside a tree crown projection belong only to that tree.

2.5 Simulator

The strip sampling simulation was based on the two-stage procedure described by Næsset & Bjerknes (2001) and Næsset (2002, 2004) and follows the approach proposed by Gobakken et al. (2006). In parallel, an estimation of mean volume by means of ground plot systematic sampling was done, as a kind of conventional inventory. The sampling units consist of equal strips containing the same number of grid cells. The total volume was estimated as the sum of predicted volume for all grid cells over all strips. Monte Carlo (MC) estimates of population mean volume and sampling error were derived running 50 iterations for each sampling scheme. Bias, standard deviation and RMSE for estimated mean values were used to assess the sampling estimates against the reference volume of the predefined population. The systematic samples of laser scanning strips and ground plots were treated as random samples. Relative efficiency of regression based estimates obtained from laser scanning strip sampling against ground

based systematic plot sampling estimates was assessed for each sampling scheme.

Computations

Multiple regression analysis was used to establish stratum-specific relationships between field measurements and laser derived metrics. Based on previous findings (e.g. Magnussen & Boudewyn, 1998; Næsset, 1997, 2002, 2004), two independent variables derived from first laser pulse returns were used for volume prediction within each grid cell: the percentile corresponding to the 9th quantile of laser canopy height (h_{90}) considering the lowest canopy height (≥ 2 m), and the canopy density corresponding to the proportion of the first pulse laser hits (d_0). Canopy density was defined as the proportions of first pulse laser hits above 2 m to total number of first pulse returns. To calculate the canopy density, it was necessary to find the total number of laser hits within each grid cell. Because the last echoes from initial laser scanning data were not available, it was assumed that each grid cell had a uniform coverage of laser hits. Thus, the total number of laser hits within a grid cell could be linearly extrapolated from the number of hits that fall inside the crown projection. Laser hits with heights below 2 m were considered as ground points as well.

The full second order regression model based on these variables was subject to stepwise variable selection to develop final models for prediction. Exploratory regression analysis was run to detect possible deviations from model assumptions. Various variance stabilizing transformations of the dependent variable (sample plot volume) were analytically assessed by the means of the Box-Cox method. Five regression models were finally proposed: (1) a multiplicative model, (2) a linear model without transformations, and three different models with transformed response variable: (3) $\log(y)$, (4) \sqrt{y} , and (5) $\text{asin}(\sqrt{y})$. For the multiplicative model, only two independent variables (h_{90} and d_0) were used, and consequently this model was not subject to stepwise selection.

For the other regression models, an empirical approach was used to obtain regression equations. Before each simulation, a number of 20 iterations were used to select the final regression models. First, a stripe sampling scheme was randomly generated over the study area, and the location of each stripe and correspondent sample plots were hold fixed. Initial sampling trials were run, and for each iteration a new population outcome was generated and sampled. Stepwise regression ($p_{in} = 0.05$, $p_{out} = 0.10$) was used for model selection, and each resulted subset model was registered. After running all iterations, the most frequently used model form for each regression model was selected as final model to be used for prediction during sampling simulations. Since serious multicollinearity problems occurred, best subsets regression models were also derived and compared to the stepwise regression subsets, in order to select unbiased regression models.

To estimate the population volume, Monte Carlo experiments were run to derive laser scanning and ground-based mean volume estimates. Initial tests showed that cumulated mean volume estimates over 50 iterations converged towards the value of MC estimates, while the sampling error decreased asymptotically. However, the number of iterations should vary with the study area, sample design, and population variability.

Squared sample plots of 200, 400, and 600 m² were used to provide ground estimates. Using squared plots significantly improves the computational performance during simulation.

Parallel laser strips with widths of 160, 180, and 200 m spaced at 1500 m were generated. The sampling intensity for different plot sizes was held almost constant around 0.6% of stripe sampling area, and the sample size varied with plots size. Compared to sampling intensities in ongoing research studies, which typically are less than 0.003% (Gobakken et al., 2006), the sampling intensity at stand plots level is much higher, but necessary to reach ground samples large enough to get reliable regression estimates.

Finally, the MC estimates for both laser strip and ground based systematic sampling were assessed by the means of a two-tailed t-test against the population value. Bias, standard deviation, and RMSE for the MC estimates of mean volume were then used to assess the sampling designs and regression models. Relative efficiency of regression based laser scanning estimates against corresponding ground based estimates was calculated as ratio of their respective MSE.

4. RESULTS

Except the multiplicative model, final regression equations were built using stepwise regression. A number of 45 mean volume estimates and their RMSE values were derived using five regression models (Table 4). In addition, for each sampling scheme, an estimate of mean volume and the corresponding RMSE were derived by ground based systematic plot sampling (Table 4). The reference value of mean volume per ha was 254 m³, i.e., total population volume of 914,400 m³ divided by the size of the study area of 3600 ha. The simulated study area included over 2.7 million trees. The number of iterations used for each simulation ensured convergence for both regression and ground plot based estimates. For mean timber volume estimates, the convergence occurred after ca 40-45 iteration for sampling schemes using ground plots of 200 m², ca 20-30 iterations for plots of 400 m², and after ca 15-25 iterations for plots of 600 m². As the number of iterations increased, the sampling error decreased asymptotically (Figure 1).

The regression models comprised two to five predictor variables. The most frequently used prediction variable was the interaction term, followed by squared height percentile and canopy density. Generally, the R² ranged between 0.79 and 0.96. The bias of mean volume estimates during iterations in each simulation ranged between -16.6 m³ha⁻¹ (6.5%) and 10.2 m³ha⁻¹ (4.0%) for regression estimates, while the bias of ground based estimates ranged from -34.1 m³ha⁻¹ (13.4%) to 31.8 m³ha⁻¹ (12.5%). MC estimates of mean volume derived by regression ranged between -5.7 m³ha⁻¹ (2.2%) and 0.3 m³ha⁻¹ (0.1%), and standard error between 5.0 m³ha⁻¹ (2.0%) and 7.4 m³ha⁻¹ (2.9%). For plot-based MC estimates, the range of bias was between -2.6 m³ha⁻¹ (1.0%) and 3.9 m³ha⁻¹ (1.5%), with a standard error between 13.7 m³ha⁻¹ (5.4%) and 18.4 m³ha⁻¹ (7.2%). The lowest RMSE for regression based MC estimates was 5.1 m³ha⁻¹ (2.0%) and the highest was 8.4 m³ha⁻¹ (3.3%). RMSE for ground plot MC estimates varied from 13.7 m³ha⁻¹ (5.4%) to 18.4 m³ha⁻¹ (7.2%). Among all regression models, only the multiplicative and linear models gave unbiased estimates ($p > 0.05$) under all sampling schemes. Ground based systematic plot sampling derived estimates provided unbiased estimates ($p > 0.05$) for all sampling designs. Relative efficiency of laser based estimates relative to ground plot estimates varied between 0.11 and 0.28, with an average of 0.16, which indicates efficiency in average 6.3 times higher for laser scanning strip sampling method (Table 5).

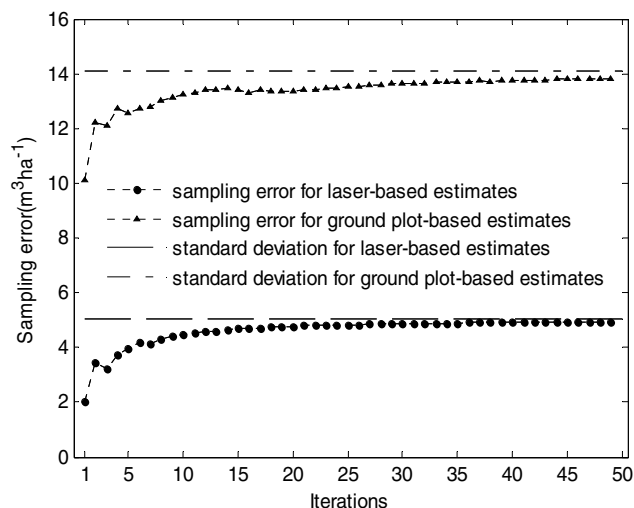


Figure 3. Example of sampling error estimation, for multiplicative regression model and ground plot base estimates using strip width of 180 m and plot size of 400 m².

5. DISCUSSION

The major findings of this study indicated that:

1) Laser scanning-based stripe sampling forest inventory can provide accurate and precise estimates of mean volume for relatively large forest areas. The LIDAR based estimation was on average 6.3 times more efficient in terms of MSE than ground-based sampling.

2) For both inventory methods, the inverse relationship between plot size and sample size seemed to be the dominant factors that led to a general increase of RMSE as the plot size increased.

3) For the ground-based systematic plot sampling method, the plot size was the dominant factor which led the overall trends for the MC estimates of mean volume. The RMSE of volume estimates increased by increasing plot size.

However, generalizations cannot be drawn from this study, since many assumptions were not realistic compared to real-world applications, i.e. small size of target area and small population variability. Another important issue is that all metrics derived from the population were considered to be "error free" and the effects of error propagation were neglected. As possible error sources could be mentioned errors concerning ground location of trees and ground plots, laser sampling and field measurements.

Nevertheless, we believe development and application of this first small-scale simulator has provided useful insight into some of the challenges we will have to face in the continued work to develop simulators that can operate on larger model forests where also spatial correlation and regional trends in the population value may be accounted for. Furthermore, a forest stand generator calibrated for Norwegian conditions should be developed, and there is also a need for building up an empirical database of laser derived individual tree models for all main tree species in Scandinavia.

Strip width (m)	Model	Plot area								
		200 m ²			400 m ²			600 m ²		
		bias	S.D	RMSE	bias	S.D	RMSE	bias	S.D	RMSE
Laser scanning based estimates (m ³ ha ⁻¹)										
160	1	-0.8 ^{ns}	5.5	5.6	-0.8 ^{ns}	5.7	5.8	-0.2 ^{ns}	6.4	6.4
	2	-2.2*	6.2	6.6	-1.3 ^{ns}	6.4	6.5	-0.8 ^{ns}	7.4	7.4
	3	-1.5*	5.2	5.4	-1.6 ^{ns}	5.7	5.9	-0.9 ^{ns}	6.5	6.5
	4	-5.5*	5.6	7.8	-3.6*	6.1	7.1	-4.3*	7.2	8.4
	5	-0.6 ^{ns}	5.5	5.5	-1.0 ^{ns}	5.7	5.8	-0.5 ^{ns}	6.3	6.4
180	1	-0.7 ^{ns}	5.5	5.6	-0.7 ^{ns}	5.0	5.1	-0.5 ^{ns}	5.6	5.7
	2	-3.0*	6.2	6.9	-0.4 ^{ns}	5.7	5.8	-0.8 ^{ns}	7.1	7.1
	3	-1.4 ^{ns}	5.3	5.4	-1.3 ^{ns}	5.1	5.3	-1.4 ^{ns}	6.0	6.2
	4	-5.7*	5.6	8.0	-3.6*	5.7	6.7	-4.1*	7.0	8.1
	5	-0.5 ^{ns}	5.4	5.4	-0.8 ^{ns}	5.0	5.1	-0.9 ^{ns}	5.5	5.6
200	1	-0.2 ^{ns}	5.4	5.4	-0.1 ^{ns}	5.1	5.1	-0.2 ^{ns}	6.0	6.0
	2	-2.5*	5.7	6.2	0.3 ^s	5.9	5.9	-0.7 ^{ns}	7.2	7.2
	3	-0.9 ^{ns}	5.3	5.4	-0.7 ^{ns}	5.0	5.1	-1.2 ^{ns}	6.3	6.4
	4	-5.2*	5.7	7.7	-3.2*	5.5	6.4	-3.1*	7.1	7.7
	5	0.0 ^{ns}	5.4	5.4	-0.2 ^{ns}	5.2	5.2	-1.0 ^{ns}	6.0	6.1
Ground plot based inventory (m ³ ha ⁻¹)										
160	-	-0.7 ^{ns}	13.7	13.7	3.9 ^{ns}	14.9	15.4	1.8 ^{ns}	18.4	18.4
180	-	-2.1 ^{ns}	14.8	15.0	2.2 ^{ns}	14.1	14.3	2.2 ^{ns}	17.5	17.6
200	-	-2.6 ^{ns}	14.5	14.8	2.2 ^{ns}	14.5	14.7	2.2 ^{ns}	18.2	18.4

^a significance level: * $p < 0.05$; not significant: $ns > 0.05$;
Models: 1–multiplicative; 2–log(y); 3–sqrt(y); 4–asin(sqrt(y)); 5–linear.

Table 4. Bias, standard error (S.D) and RMSE of mean volume estimates (m³ha⁻¹).

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Strip width (m)	Model	(MSE ^a / MSE ^b)		
		Plot size (m ²)		
		200	400	600
160	1	0.17	0.14	0.12
	2	0.23	0.18	0.16
	3	0.16	0.15	0.12
	4	0.32	0.21	0.21
	5	0.16	0.14	0.12
180	1	0.14	0.13	0.10
	2	0.21	0.16	0.16
	3	0.13	0.14	0.12
	4	0.28	0.22	0.21
	5	0.13	0.13	0.10
200	1	0.13	0.12	0.11
	2	0.18	0.16	0.15
	3	0.13	0.12	0.12
	4	0.27	0.19	0.18
	5	0.13	0.13	0.11

^a MSE of laser-based estimates

^b MSE of ground plot based estimates

Models: 1–multiplicative; 2–log(y); 3–sqrt(y); 4–asin(sqrt(y)); 5–linear.

Table 5. Relative efficiency of laser based against ground plot estimates.