THE SIMPLICITY AND COMPLEXITY OF STRAIGHTS AND CURVES.

C. S. Fraser[a], A. M. Judd[b], F. J. Leahy[a]

[a] Department of Geomatics, University of Melbourne, Victoria 3010, Australia
[b] Geomatic Technologies Pty Ltd, 190 City Rd, Southbank, Victoria 3006, Australia

Commission VI, WG VI/4

KEY WORDS: railway mapping, asset management systems, transition curves, least squares, curve fitting.

ABSTRACT:
Much of the mobile mapping activity in Australia has been applied to the mapping of railways. Over the last decade the process has evolved from one narrowly focused upon establishing the position of track centrelines to one which includes the position and geometry of the track, imagery and detailed attributes of all trackside features. This broader perspective has been required as the mobile mapping processes have become a core requirement for the development of railway asset management systems. Positioning sensors installed on the mapping platform include GPS receivers (differential pseudo-range), shaft encoders for the measurement of travelled distance, direction sensors and tilt meters. Measurements from these sensors are taken at regular epochs and a Kalman filter is employed to integrate these and compute strings of 3D coordinates to locate the track centerline. The location of trackside features is achieved through photogrammetry, the number and configuration of the cameras being determined by the level of detail required. The preferred form of representing railway track geometry is as a continuous series of straights and curves with defined tangent points. Whilst the need for this form of geometry arises out of a business requirement for the management of the track as an asset, the representation of the track in this format has additional benefits. These include improving the accuracy of the track centreline position and providing a simple mechanism for combining a number of sets of track centreline data into a single spatial database. This paper reviews the processes used to map track centrelines and discusses the algorithms employed in reducing the coordinate strings to the required geometric format. In the horizontal plane, the string of coordinates provided by the Kalman filter is reduced to a series of straights, transition curves and circular curves. The vertical profiles are reduced to a series of straights and parabolas. The processes are essentially those of curve fitting but with added challenges of ensuring the smooth intersection of each component curve into a continuous path.

1. INTRODUCTION

1.1 Mobile Mapping of Railways in Australia

The term “route mapping” is now commonly applied to the mapping of any linear infrastructure. A decade or so ago the surveys were rather simple processes for the acquisition of strings of coordinates as a record of the centreline of a roadway or railway. The techniques have now evolved to a high level of sophistication and are more likely to be associated with production of a detailed spatial data model of both the linear feature itself and other features of interest appearing in the corridor. Data required to supplement the model include:

» high-resolution aerial photography as an additional record of the linear feature; and

» refined horizontal and vertical track geometry with tangent points, intersection points and radii of curvature.

As shown in Figure-1, the aerial photography provides a highly intuitive interface to the attributes of each facility and is a key dataset in the interface deployed with asset management systems. The geometric data is required to define maintenance and lifecycle requirements for the different sections of track. For example, wear of curved sections is largely dependent on the level of curvature. Being able to identify sections of high curvature greatly assists the scheduling of maintenance.

The drive for these increased requirements arises from recent changes in the management of assets of railways. In the last decade, in the Australian state of Victoria at least, much of the operation of the state’s railways has become the responsibility of private corporations. This necessarily means that the assets associated with such complex and extensive infrastructures as railways need to be recorded in detail. This is not only to allow a value to be placed on the facility, but also to aid in gauging the level of maintenance and improvement that will be required. As an accurate record of the state of the infrastructure is essential, there is a need for continuity in the route mapping.

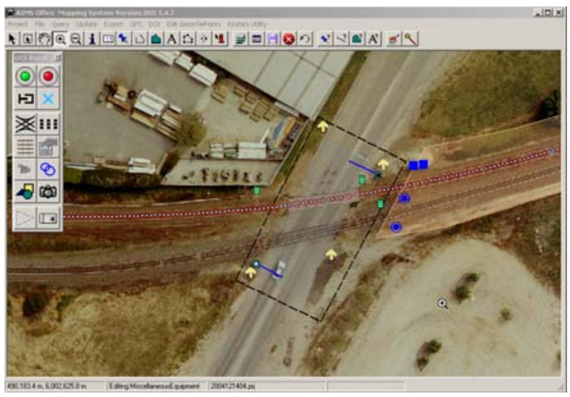


Figure 1. High resolution aerial photograph utilised as interface to asset attributes.

1.2 Spatial Data for Asset Management Systems

Early asset mapping projects involved working with individual files holding centreline data (in the form of coordinates) on the one hand and data (coordinates and attributes) associated with the trackside features on the other. Asset mapping projects now require delivery of all data in an integrated GIS data model including the refined track geometry and aerial imagery. This involves reducing the mass of coordinates acquired in the mapping of the track centreline to a condensed set of data consisting of coordinates of tangent points and parameters of curves. Apart from reducing the data to more manageable amount and presenting it in a format familiar to design engineers, it brings other advantages. In describing the data in the form of straights and curves, a most convenient mechanism for integrating neighbouring tracks into the spatial data model is provided. Moreover, experience has shown that the precision of the mapping is improved.

The major discussion in this paper is focused upon the algorithms employed in the reduction of coordinate data to the required geometric format. In the horizontal plane, the string of coordinates provided by a Kalman filter is reduced to a series of straights, transition curves and circular curves. The vertical profiles are reduced to a series of straights and parabolas. The processes are essentially those of curve fitting but with the added challenge of ensuring the smooth intersection of each component curve into a continuous path.

The main discussion is preceded by a summary of the data acquisition technology involved. The summary lists the type of sensors employed and the calibration processes adopted.

2. THE TECHNOLOGY

2.1 GPS

The precision required for asset mapping is generally to achieve a standard deviation of the coordinates defining the track centreline and the peripheral features to be better than 1.0m. The dominant sensor for the absolute positioning of points is GPS. Differential pseudo range GPS, when integrated with suitable "relative location sensors" (for instance, rate gyroscopes and running distance meters) is capable of achieving this level of precision. A base station to provide the differential corrections does not generally need to be established as the corrections are available from existing commercial services.

One essential contribution of the GPS is to provide an accurate time base for the logging of measurements from the other positioning sensors – gyros for direction, tilt meter for track grade, and cant and wheel encoder for running distance. Thus, measurements from the gyroscopes, distance meter and tilt meter are taken at epochs with 1-second intervals as defined by the GPS 1 PPS (pulse per second) interface. Imagery captured by the cameras is linked to the wheel encoder to allow accurate exterior orientation of the images during post processing.

2.2 Gyroscopes

A solid-state rate gyroscope is used to measure the change in horizontal direction of the mapping platform as it moves along the track. The measurements are converted into true bearings by repeatedly referencing the path generated from gyroscope and travelled distance measurements with the path represented by the string of GPS coordinates. This is in effect a routine to continuously determine the "index error" which when applied to the raw measurements converts them to true bearings. The process is part of a calibration routine that simultaneously determines the index error of the gyroscope measurements and the scale factor to be applied to the travelled distance measurements. The result of this process yields bearings with an accuracy of 0.1 deg (1 sigma). Details of the routine are given in Judd & Leahy (2005).

2.3 Tilt Meters

The tilts of the mapping platform (pitch and roll) are measured by way of a tilt meter. Measurements from this source contain a significant level of noise due to the acceleration and jerk of the mapping platform. However, as the tilt meter provides measurements at a frequency of 10 hertz, these can be summed over each 1 second period to produce an average tilt for that interval. This practice has been shown to be effective in reducing the noise to a workable level. Each measurement of pitch (tilt angle representing grade) is converted into the change in height of the mapping platform in moving from one epoch to the next. This is computed by multiplying the travelled distance by the sine of the pitch angle.

Any acceleration of the mapping platform will cause a bias in the measurement of pitch. As the distance travelled over each 1-second interval between epochs is measured, this provides a record of the velocity of the mapping platform. The acceleration computed from this record is in the direction of travel and is used to correct the pitch measurements. Both corrections and measurements contain significant noise which is effectively minimised through the use of a linear Kalman filter as a smoothing mechanism.

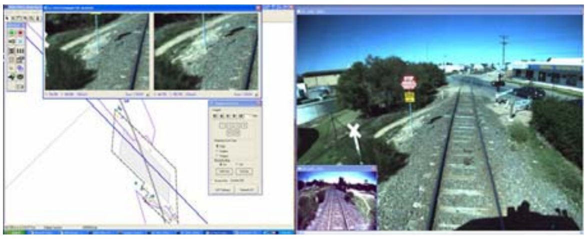
The measurement of pitch is also likely to have an index error due to the impracticability of mounting the tilt meter so that its axis is normal to the rail track. The index error in the pitch measurements is continuously determined by comparing the vertical profile of a track section derived from measurements of tilt and travelled distance with the profile derived from GPS measurements. A conformal transformation of the first onto the second determines a rotation that can be applied as the index error in the grade measurements. Further details can be found in Judd & Leahy (2005).

2.4 Pulse Counter

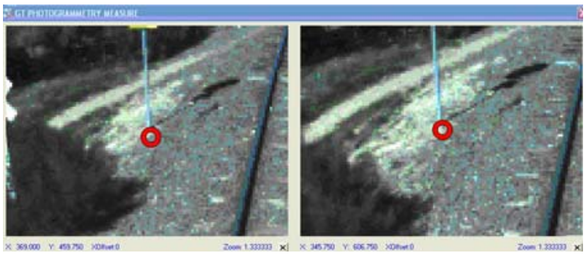
The travelled distance is measured by way of a pulse counter mounted on an axle of the mapping platform. The pulses are scaled so their count provides distance travelled in metres. In typical operations, the scale will vary with the condition of the track and the diameter of the wheels changing due to wear. As mentioned in section 2.2, a continuous calibration procedure has been integrated into the system. In this, the horizontal profile generated from the gyroscope and travelled distance measurements is compared to that mapped by GPS. Again, a conformal transformation of the first onto the second is used to determine the index error of both the gyroscope and the scale of the travelled distance measurements. The pulse counter is recorded at each 1-second epoch triggered by the GPS receiver. The measurement included in the Kalman filter is the distance travelled between epochs.

2.5 Cameras

In many projects, a single camera mounted with its axis in the direction of travel and angled down to the track will provide the precision required for the positioning of trackside features.



(a) Photogrammetric mapping screen.



(b) Single camera images utilised to locate trackside features.

Figure 2

Stereo photogrammetry is possible by using successive photographs as the platform moves down the track. The computer screen images shown in Figure-2b show two of a series of taken as the platform approaches the sign on the left . The principles of photogrammetric triangulation are employed in a post processing routine to compute the coordinates of this feature after moving a cursor to record the image coordinates of the base of the sign in each of the images in Figure 2b.

In other route mapping operations, the precision required for the 3D positioning of the trackside features is more demanding. A configuration frequently used has two cameras firmly mounted with the axes aligned to the direction of travel. The interior and exterior orientation of the cameras, with respect to the mapping platform, is determined beforehand, as described in Fraser & Judd (1999) and Fraser (1997).

2.6 Results

A Kalman filter is employed to integrate the data from all sensors to produce a string of coordinates that represent the mapping of the track centre line, as detailed in Leahy & Judd (1996; 1998). The state vector is computed at each 1-second epoch as triggered by the GPS receiver. Thus, for the measuring platform moving at 50kph, centre line coordinates are computed at intervals of approximately 14m along the track. The strings of coordinates acquired are reduced to the format used in the design of railway tracks, that is, a series of straights and transition and circular curves.

3. FITTING VERTICAL CURVES

3.1 Curve fitting producing discontinuities

When constructing railways, the vertical profile of the track passing over a rise is designed as a parabola fitted between a rising and falling straight. As vertical grades of railways are small, the curvature of a vertical profile is significantly less than that of the horizontal case. Thus, in the design of vertical curves, parabolas tangential to the straights can be used because the curvature is small and they automatically provide a transition between straight and curved sections.

The mapping of existing railways could be seen as the reverse of the process of design. That is, the fitting of straights and curves to the coordinates acquired in the mapping of the existing track. The initial problem is to locate the tangent points - that is, those points where the track moves from a straight into a vertical curved section. Initially, this can be achieved by examining a plot of the vertical profile. It is noteworthy that the location of tangent points is refined at a later stage. On completion of the curve fitting routine described in the next section, other points are tested by moving backwards and forwards along the track until an optimum is reached; the optimum being indicated by the size of the residuals resulting from the curve fitting.

Curve fitting can now be employed to fit straight lines to the "entering" and "exiting" straights and a parabola to the curve between. Unfortunately due to both the actual track having been deformed from the original design and noise in the mapping of the coordinates of the track centreline, curves fitted in isolation will invariably be non-continuous. A typical result of this approach to fitting is illustrated in Figure-3.

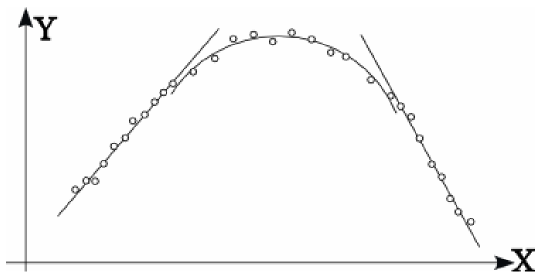


Figure 3. Discontinuities resulting from separate fitting of straights and parabola.

3.2 Constraining measurements to remove discontinuities

To obtain a smooth and continuous fitting of the straights and curves a least squares approach is adopted as follows:

» As straights are invariably longer than the vertical curves and therefore based on more coordinate data, these are fitted first and thereafter regarded as "fixed". Thus, in Figure-4 the two straights are represented by the lines $y = a_1x + b_1$ and $y = a_2x + b_2$. Hereafter, the coefficients a_1, b_1, a_2 and b_2 determined in the fitting of these straight lines are regarded as constants.

» The parabola is constrained to be tangential to the curve at the points A and B. The four coordinates of the tangent points, (x_a, y_a) and (x_b, y_b), become extra unknown parameters for the fitting of the parabola. Four constraining measurements are added to achieve the continuity of curves shown in Figure-4. The constraints are to achieve the following:

  - o the point A must be on both the straight and the curve,
  - o the gradient of the curve at A must be equal to the gradient of the straight through A,
  - o the point B must be on both the straight and the curve, and

- o the gradient of the curve at B must be equal to the gradient of the straight through B.

» The four constraints are applied by measurements which will be added to the standard least squares method of fitting a parabola to the mapped coordinates.

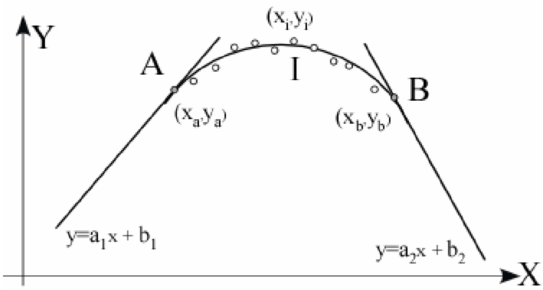


Figure 4. Constrained fitting of parabola to achieve continuity.

The observation equations for the curve fitting to the mapped data will be (for the point I)

$$y_i = a_p x_i^2 + b_p x_i + c_p \qquad (1)$$

The unknown parameters are the coefficients a_p, b_p and c_p .

The constraints for the straights to meet the parabola at point A and B are

$$a_1 x_a + b_1 = a_p x_a^2 + b_p x_a + c_p \qquad (2)$$

$$a_2 x_b + b_2 = a_p x_b^2 + b_p x_b + c_p$$

These two “measurement” equations can be recast as follows:

$$f_1 = 0 = a_p x_a^2 + b_p x_a + c_p - a_1 x_a - b_1 \qquad (3)$$

$$f_2 = 0 = a_p x_b^2 + b_p x_b + c_p - a_2 x_b - b_2$$

The constraints for the curve to have the same gradient as the straights at points A and B are

$$a_1 = 2a_p x_a + b_p \qquad (4)$$

$$a_2 = 2a_p x_b + b_p$$

A recasting of these two equations produces the form:

$$f_3 = 0 = 2a_p x_a + b_p - a_1 \qquad (5)$$

$$f_4 = 0 = 2a_p x_b + b_p - a_2$$

As the observation equations imposing the conditions are non-linear, all observations equations need to be linearised.

If a'_p, b'_p, c'_p, x'_a and x'_b are approximate values for the unknown parameters, the observation equation for the mapped position I can be written as

$$y_i - (a'_p x_i^2 + b'_p x_i + c'_p) = \frac{\partial y_i}{\partial a_p}\Delta a_p + \frac{\partial y_i}{\partial b_p}\Delta b_p + \frac{\partial y_i}{\partial c_p}\Delta c_p$$

where

$$\frac{\partial y_i}{\partial a_p} = x_i^2 \qquad \frac{\partial y_i}{\partial b_p} = x_i \qquad \frac{\partial y_i}{\partial c_p} = 1 \qquad (6)$$

The observation equations for the constraints can be written as:

$$0 - (a'_p x'^2_a + b'_p x'_a + c'_p - a_1 x'_a - b_1) = \frac{\partial f_1}{\partial a_p}\Delta a_p + \frac{\partial f_1}{\partial b_p}\Delta b_p + \frac{\partial f_1}{\partial c_p}\Delta c_p + \frac{\partial f_1}{\partial x_a}\Delta x_a$$

$$0 - (a'_p x'^2_b + b'_p x'_b + c'_p - a_1 x'_b - b_2) = \frac{\partial f_2}{\partial a_p}\Delta a_p + \frac{\partial f_2}{\partial b_p}\Delta b_p + \frac{\partial f_2}{\partial c_p}\Delta c_p + \frac{\partial f_2}{\partial x_b}\Delta x_b$$

$$0 - (2a'_p x'_a + b'_p - a_1) = \frac{\partial f_3}{\partial a_p}\Delta a_p + \frac{\partial f_3}{\partial b_p}\Delta b_p + \frac{\partial f_3}{\partial x_a}\Delta x_a$$

$$0 - (2a'_p x'_b + b'_p - a_2) = \frac{\partial f_4}{\partial a_p}\Delta a_p + \frac{\partial f_4}{\partial b_p}\Delta b_p + \frac{\partial f_4}{\partial x_b}\Delta x_b$$

where, (7)

$$\frac{\partial f_1}{\partial a_p} = x'^2_a \qquad \frac{\partial f_1}{\partial b_p} = x'_a \qquad \frac{\partial f_1}{\partial c_p} = 1 \qquad \frac{\partial f_1}{\partial x_a} = (b'_p - a_1)$$

$$\frac{\partial f_2}{\partial a_p} = x'^2_b \qquad \frac{\partial f_2}{\partial b_p} = x'_b \qquad \frac{\partial f_2}{\partial c_p} = 1 \qquad \frac{\partial f_2}{\partial x_b} = (b'_p - a_2)$$

$$\frac{\partial f_3}{\partial a_p} = 2x'_a \qquad \frac{\partial f_3}{\partial b_p} = 1 \qquad \frac{\partial f_3}{\partial x_a} = 2a'_p$$

$$\frac{\partial f_4}{\partial a_p} = 2x'_b \qquad \frac{\partial f_4}{\partial b_p} = 1 \qquad \frac{\partial f_4}{\partial x_b} = 2a'_p$$

3.3 Example of vertical curve fitting

A set of data to illustrate this aspect is shown in Table-1. The constants a_1, b_1, a_2 and b_2 have been selected to represent the coefficients of lines that have been fitted to the coordinates acquired in the mapping of the entering and exiting straights. The “curve data” has been generated with all coordinates having a standard deviation of 0.3m.

Straights Data		Curve Data	
constants		X	Y
a_1	4.00	4.00	68.01
b_1	56.00	5.00	75.81
a_2	-4.00	6.00	80.35
b_2	120.00	7.00	83.57
		8.00	84.02
		9.00	83.17
		10.00	80.46
		11.00	75.60
		12.00	68.31

Table 1. Data for vertical curve fitting.

The force with which the constraint measurements are applied is controlled by the assigned weights of these measurements. Table-2 shows the effect of varying the standard deviation of the constraint measurements from 1.0m to 0.001m. It illustrates that applying a standard deviation of 0.001m to the constraint measurements effectively enforces the required condition of continuity of the curves. This is evident as the residuals for the constraint measurement approach zero for all practical purposes.

standard deviations (metres)	1.000	0.100	.010	.001
Constraints	residuals			
f_1	-0.348	-0.103	-0.001	0.000
f_2	0.000	0.000	0.000	0.000
f_3	-0.371	-0.114	-0.002	0.000
f_4	0.000	0.000	0.000	0.000

Table 2. Effect of varying standard deviations of constraint measurements.

4. FITTING OF HORIZONTAL CURVES

4.1 An algorithm for fitting transition curves

The fitting of horizontal curves is, on the surface at least, a more complex matter due to the need to fit three curves. That is, when moving from a straight into a circular curve a transition curve is required to smoothly effect the change in curvature. The transition curve selected is one that has a constant “rate of change of curvature with length” as it moves from the straight (zero curvature) to a circular curve of radius R (curvature of 1/R). The curve with this property is the Euler spiral. In earlier times, the cubic parabola, a close approximation, was used to avoid difficulties in computing and setting out.

For a transition curve that moves to the left from a straight running due east, its coordinates are given by Clark (1968) as

$$e = l - \frac{l^5}{40(RL)^2} + \cdots \qquad n = \frac{l^3}{6RL} - \frac{l^7}{336(RL)^3} + \cdots \quad (8)$$

Here, e and n are the east and north coordinates, l the length from the tangent point, L the total length of the transition curve and R the final radius of the transition curve. For transition curves on railways, the second terms in these series are very small and the others negligible.

The fact that a transition curve is needed provides an opportunity for an elegant solution to the problem. In brief, the process is as follows:

1. The locations of the tangent points, the points which mark the start and end of the transition and circular curves, are determined from a plot of the running curvature of the track (the methodology is discussed in Section 4.2).
2. Lines are fit by least squares to the set of coordinates acquired in the mapping between the tangent points that define the straights.
3. Circles are fit to the set of coordinates between the tangent points that define the circular curves.
4. The two parameters that define a transition curve are its length and final radius. As the latter is known (the radius of the circular curve), it remains only to determine the length of the transition curve that runs, for example, from the tangent point on a straight to meet another on the circular curve.

It is noteworthy that in this process the location of the straight and the location and radius of the circle, determined in steps (2) and (3), are not varied. Neither is the transition curve fitted to the coordinates mapped along its length. Rather, the length of transition is chosen so that it will leave the tangent point on the straight and meet the circular curve tangentially.

4.2 Determining tangent points and rate of change of curvature of transition curve

There is a difficulty in estimating the location of tangent points from a plot of the raw coordinates acquired in the mapping of the track. This can be seen in Figure-5, which has been generated from typical data. The panel on the right shows the plot of the track coordinates. As can be seen, the locations of the tangent points of a straight to transition curve, and a transition curve to a circular curve, cannot be discerned with any confidence.

The panel on the left of Figure-5, in which the locations of the tangent points are more clear, shows the plot of the running curvature. The running value of the curvature is computed by continually fitting a circle to the last point mapped and the two preceding points. The raw values of curvature display significant noise and they are smoothed before plotting, as illustrated in Figure-5. In the initial section the curvature is zero and is associated with a straight. The second section shows a constant increase in curvature along a transition. The third section shows positive constant curvature as the mapping platform moves through a circular curve.

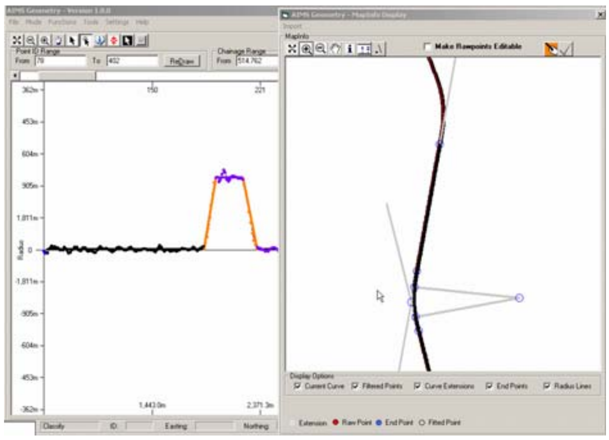


Figure 5. Plot of curvature and track centreline

Figure-6 shows in more detail an idealised plot of curvature over a 3km section. The track enters a curve on the right at 400m and the curvature varies until entering, at 700m, a circular curve of 1000m radius. The curvature remains at 1/1000 until 1100m when it returns to a straight at 1400m. Later the track enters a curve on the left at 1700m and a similar pattern of curvature variation follows.

In practice, determination of the locations of the tangent points is aided by least squares fitting of straight lines to the various sections of the plot of curvature. In determining the tangent point that marks the end of a transition curve, for example, an initial estimate of the location is taken manually from the plot. The tangent point at the start of the transition curve will have been previously computed in a similar manner while fitting the line to the curvature of the straight. This determines which of the mapped points are to be used in fitting the line that represents the constant rate of change of curvature. The process is iterated by moving the selection of the tangent point back and forwards until an optimum is reached as indicated by the sum of squares of the residuals.

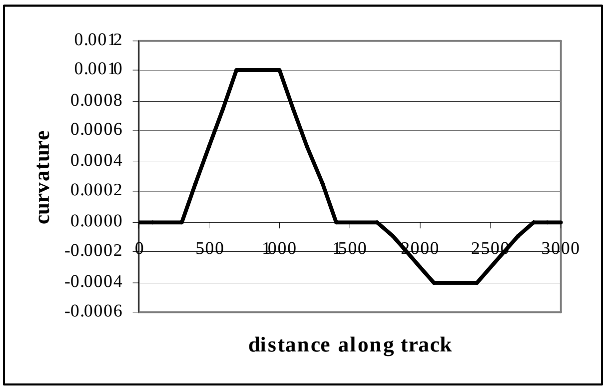


Figure 6. Idealised plot of track curvature.

At the conclusion of this process the mass of mapped coordinates is reduced to a set of data where:

» the location of the tangent points which encompass the straights have been determined,

» the locations of the centres and the radii of the circular curves have been determined, and

» estimates of the lengths of the transitions have been made.

4.3 Refining the length of the transition

Due to noise in the mapped coordinates, the set of reduced data is unlikely to be coherent. That is, if a straight, transition curve and section of circular arc are plotted there is likely to be a discontinuity where the transition curve should meet the circular curve. This is illustrated in Figure-7 where, ideally, the transition curve leaving the straight at tangent point T_1 would terminate with the point P being coincident with the tangent point T_2 on the circular curve.

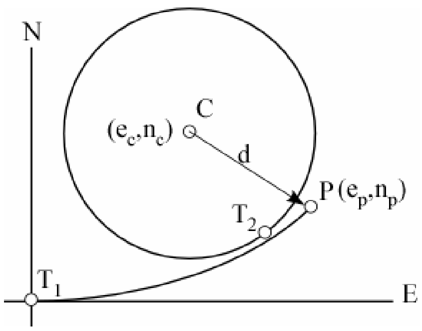


Figure 7. Plot of transition curve with inaccurate, estimated length.

The cause of this discontinuity could be errors in all or any of the components of data: the position of the straight, the position and radius of the circular curve or the estimated length of the transition. As mentioned earlier, the logic followed here is to hold both the location of the straight and the centre, along with the radius of the circular curve, and vary the length of the transition curve so that it will meet the circular curve tangentially. Under this logic, the cause of the discontinuity shown in Figure-7 is that the length of the transition is too large causing the transition to pass below the tangent point T_2. This can be deduced by considering the first term of the formulae for the north coordinate:

$$n \approx \frac{l^3}{6RL} \tag{9}$$

Figure-8 shows that the error in the transition length causes an error of $\partial n = \partial r \cos(180 - \theta)$ in the northing of point P where ∂r is the radial error in the position of P.

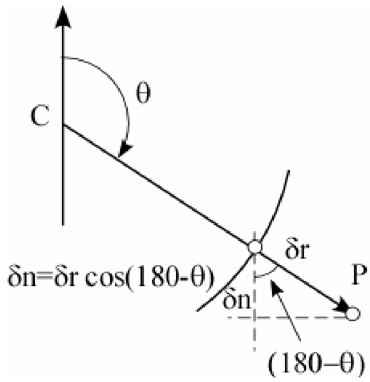


Figure 8. Radial error in the terminal of transition curve due to error in length.

The following differential correction for the length L applies at the terminal of the transition curve:

$$\delta L \approx \frac{\partial L}{\partial n}\delta n \approx \frac{-6R}{L}\delta n \tag{10}$$

An improved estimate of the transition length L' follows as

$$L' = L - \frac{6R}{L}\delta n \tag{11}$$

Upon iteration of Eqn.11, the point P moves to the tangent point T_2 as shown in Figure-9.

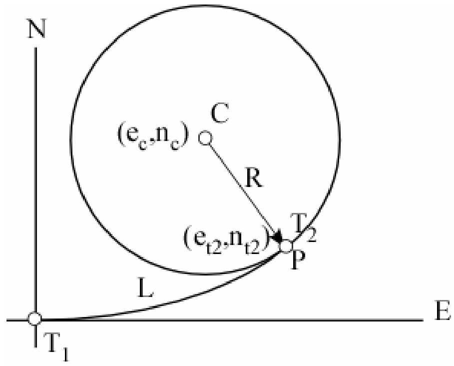


Figure 9. Length of transition selected so as to meet the circular curve tangentially.

4.4 Example of fitting transition curve

Figure-9, although not at all to scale, can also be taken to represent the design of a transition curve that moves from a straight running due east to a circular curve on the left.

The transition curve has been designed with the following parameters:

» the tangent point on the straight (T_1) is at the origin of an east/north axes system,

» the circular curve has a radius of 1000m,

» the tangent point on the circular curve (T_2) is at coordinates 99.975 east and 1.666 north, and

» the length of transition curve is 100m.

As a test of the algorithm represented by Eqn. 11, the length of the transition was assumed to be 110m. Table-3 shows the variation of the length of the transition L and the radial error δr in the position of its terminal as the algorithm is iterated.

	Coordinates of terminal point P			
Iteration	east	north	δr	L
1	109.967	2.016	0.199	110.000
2	99.114	1.638	-0.014	99.138
3	99.964	1.666	0.000	99.989
4	99.975	1.666	0.000	100.000

Table 3. Variation in length of transition L and radial error δr with iteration of Eqn. 11.

Upon completion of the computation:

» the terminal of the transition curve has moved onto the circular curve at the designed tangent point,

» the length of the transition has converged to the design length of 100m, and

» the radius of curvature of the transition at the tangent point is 1000m, as expected since this is a fundamental property of the Euler spiral.

The curvature of the Euler spiral is obtained as (Clarke, 1964):

$$K = \frac{l}{RL} \qquad (12)$$

where l is the length from the tangent point, L the total length of the transition curve and R the final radius of the transition curve.

Thus, at the terminal of the transition curve, where l = L, the curvature is

$$K = \frac{1}{R} \qquad (13)$$

5. CONCLUDING REMARKS

The need to reduce the strings of coordinates acquired in the mobile mapping of railways to a continuous series of straights and curves arises from advantages this format has in its integration into a spatial data model which underpins asset management systems. The challenge to the mapping process is to include algorithms that reduce the mass of coordinates to this format with particular emphasis on the need for continuity.

For the vertical profiles, the continuity is achieved by adding constraint measurements when applying the least squares technique of fitting the parabolas to the previously determined straights. For the horizontal curves, initially, the least squares technique is used to fit straights and circular curves to the associated coordinates. Continuity is then achieved by manipulating the length of the transition curve.

The examples discussed refer to the simple case of moving from a straight to a circular curve. However, the principles can be applied to compound curves where the transition joins a circular curves of differing radii.

6. REFERENCES

References from Journals:

Fraser C.S. and Judd A.M., 1999. The Calibration of Imaging Sensors Integrated into a Rapid Route Mapping System. *International Workshop on Mobile Mapping Technology*, Li R and Murai S (eds), International Archives of Photogrammetry & Remote Sensing, Bangkok, 32: 4-1-1 to 4-1-7..

Fraser, C.S., 1997. Digital Camera Self-Calibration. *ISPRS International Journal of Photogrammetry & Remote Sensing.* Vol. 52: 149-159.

Judd, A. M. and Leahy, F. J., 2005, Route mapping for railway asset management, *Geomatics World*, 14(1): 22-26

Leahy, F.J. and Judd, A.M., 1998. Precision Rapid Route Mapping Incorporating Stereo Imagery, *ISPRS Commission II, W/G II/1*, Cambridge, July 1998: 10p.

References from Books:

Clark, D., 1968. *Plane and Geodetic Surveying for Engineers. 5th Edition*, Constable, London, pp. 446-449.

References from Other Literature:

Leahy, F.J. and Judd, A.M., 1996. A Simple Process of Smoothing Route Mapping by GPS, *Technical Papers, 37th Australian Surveyors Conference:* 411-421.