MODELLING OF TREE CROSS SECTIONS FROM TERRESTRIAL LASER SCANNING DATA WITH FREE-FORM CURVES

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ABSTRACT
A method to estimate cross sections of tree branches and stems using closed free-form curves is presented. The data is gathered in the form of unstructured point clouds from terrestrial laser scanning of the standing tree in the forest. A method to compute a model for each branch, also applicable to the stem, is presented first. This model describes the branch surfaces with a sequence of overlapping cylinders. In slices orthogonal to these cylinder axes points are selected from the data set and closed B-Splines are fitted to those cross section points. Fitting accuracy of ±1cm can be achieved. The measurement of branch ovality based on these B-Splines is illustrated.

1 INTRODUCTION

Cross sections of tree branches are of interest to forest managers, because economically important measures can be derived from them. The knowledge of e.g. taper (i.e., the decrease in diameter from base to tip) and ovality (ratio between maximum and minimum diameter of a branch) are measures for the shape and quality of wood. Another example are buckles, which are irregularities in the surface, e.g. caused by an earlier injury to the tree, resulting in wood of reduced strength. Additionally, precise knowledge of tree shape enables the study of the growth process as a reaction to environmental factors (e.g. wind).

The traditional way of obtaining cross sections is to cut down the tree, draw slices of it, and measure them. This has the disadvantage, that the quality assessment cannot be performed on the standing tree, but only after cutting it down. Alternatively, direct measurement of quality parameters, e.g. ovality, on standing trees can be performed by means of manual measurement with a calliper. This approach has the disadvantage, that it requires much manual work, measurements can only be performed at a few places, and additionally there is the problem of operator-subjective execution of the measurement process.

In this paper a method is described for reconstructing the cross section of tree stems and branches from terrestrial laser scanner data. A laser scanner is positioned in the forest and captures one or more trees from different sides. This provides a dense set of points in 3D, a point cloud, covering the tree surface and the ground. The principal requirement for cross section determination is that the branches or the stem are covered with points along the entire circumference. A section can then be determined at any position. As the cross sections cannot be described by simple curves like circles and ellipses due to their irregularity, free-form curves (e.g. B-Splines) have to be used.

The organization of the paper is as follows. In Sec. 2 a method will be described which provides estimations of branch axes. This is a requirement for virtually cutting out a section of the tree. In Sec. 3 the reconstruction of the cross sections with adjusting B-Splines will be explained. Examples and applications are presented in Sec. 4. The Appendix gives a brief overview on B-Splines.

2 TREE MODELS BASED ON CYLINDERS

To reconstruct branch cross sections it is necessary to know where the branches are and in which direction their axes are pointing. While this can also be prescribed manually, it is more efficient to determine these measures automatically from the given points. Especially the axis direction, or equivalently the definition of the cross section plane orthogonal to the axis, is difficult to specify manually. First a method to compute a sequence of overlapping cylinders on the branch surface is described. Based on this, the computation of a smooth branch axis and surface is presented.

2.1 Cylinder following

Assuming that we are given a set of points $\mathcal{P} = \{\mathbf{p_i} \mid i = 1, \ldots m\}$ on *one* branch and possibly some outliers, it is possible to determine a rough model of the branch by fitting of multiple, overlapping cylinders. The procedure runs as follows.

1. A subset of points is selected around a start point $\mathbf{p_s}$, which is e.g. the point closest to the barycenter of all the given points. All points within a sphere centered on $\mathbf{p_s}$ and a radius r_s not smaller than the expected branch diameter are selected: $\mathcal{S} = \{\mathbf{p_j} \mid \|\mathbf{p_j} - \mathbf{p_s}\| \leq r_s\}$.
2. An approximation for an adjusting cylinder through the points in $\mathcal{S}$ is determined by first estimating the axis direction $\mathbf{a_0}$. In each point $\mathbf{p_j}$ its normal vector $\mathbf{n_j}$ is estimated by fitting a plane than minimizes the orthogonal distances to $\mathbf{p_j}$ and its k nearest neighbors.[1] The vectors $\mathbf{n_j}$ lie approximately in the same plane, which goes through the origin and is orthogonal to the cylinder axis. This plane is determined by adjustment, and its normal is denoted as $\mathbf{a_0}$. For the position and the radius the choice of approximations is less critical, the barycenter of $\mathcal{S}$, $\mathbf{p_0} = \sum \mathbf{p_j}/|\mathcal{S}|$, and a radius $r_0 = r_s/2$ are sufficient.
3. With the current approximation values for the parameters $(\mathbf{a_0}, \mathbf{p_0}, r_0)$ least square fitting of the cylinder is performed. To avoid divergence or convergence to wrong solutions (e.g. cylinders with very big radii, which are very similar to the adjusting plane in the region of the given points) regularization is applied, e.g. by the Levenberg-Marquardt method (Press et al., 2002). The result is checked (see below) in order to assure that the fitted cylinder is a good approximation for the branch at the given points. The points $\mathbf{p_j}$ are projected to the adjusted axis, and the outermost points define top and bottom of the cylinder.
4. The cylinder is shifted forward along its axis. This yields firstly an area to select a new set of points: proximity to the cylinder skin is the deciding criterion (e.g. with a threshold of three times the measurement accuracy). The skin does not stretch to infinity, but is bounded by top and bottom point on the axis. Secondly the forward shifted cylinder provides approximation values for the parameters of the next cylinder to be fitted. After point selection and cylinder adjustment, the obtained cylinder

[1] The value k depends on the measurement noise and the sampling density. For dense laser data with 5–10 points per cm^2 on the tree surface and a measurement accuracy of a few millimeter k=20 gives good results.

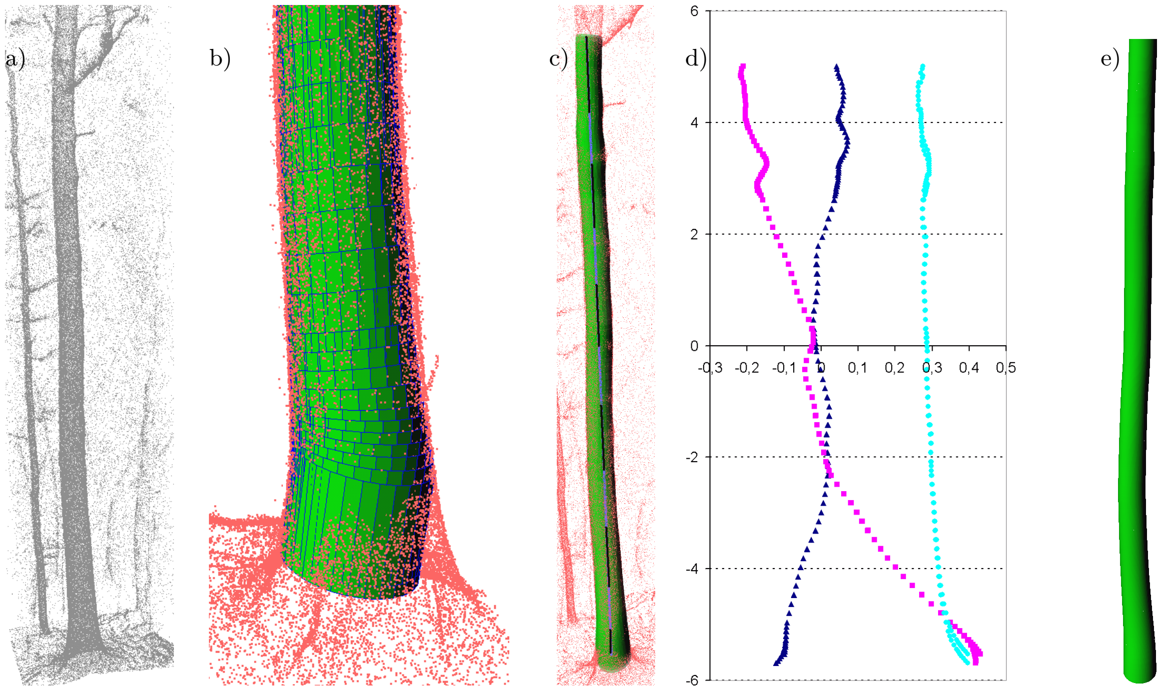

Figure 1: Branch modelling by a sequence of cylinders. In a) the original point cloud is shown (thinned out by a factor 10), in b) a detail of the sequence of cylinders as wireframe models is shown. The average axis for the entire branch together with the fitted cylinders is shown in c). In d) the x-coordinates (squares) and the y-coordinates (triangles) of the cylinder axis, and the radii (circles) are shown as function values, parameterized over the average axis. In e) the branch model interpolated with moving least squares for the x-, y-, and z-coordinate and the radius is shown.

is checked. If fitting was un-successful, another forward shift can be tried, or the length of the cylinder can be reduced, resulting in a reduced set of selected points. If all these measures fail, cylinder following is stopped, otherwise forward shifting and fitting continues.

The forward shift, i.e. the overlap between two consecutive cylinders, maximum length of the cylinder, and criteria for accepting a cylinder (maximum r.m.s.e. of the fit, maximum angle between subsequent axes, ...) can be derived from the measurement accuracy and the sampling density. Further details can be found in (Pfeifer et al., 2004).

For a given point set (Fig. 1.a) cylinder following yields a sequence of overlapping cylinders (Figs. 1.b and 1.c), each describing the tree surface in a certain region.

2.2 Smooth axis and radius function estimation

The cylinders describe the branch surface only piecewise, whereas a smooth (i.e. continuous and continuously derivable) branch model is required for further analysis. This model has a smooth axis $\mathbf{c}(t)$, and a corresponding smooth radius function $r(t)$.

First an average axis in the form of a line $\mathbf{c_p} + t\mathbf{c_d}$ is computed (Fig. 1.c). Its constant point $\mathbf{c_p}$ is the barycenter of the cylinder axes midpoints and its direction $\mathbf{c_d}$ is the average of the cylinder axes directions. For straight branches this line lies within the branch, but for strongly curved branches this is not necessarily the case. As this average axis is only used in an intermediate step for computing the smooth model, its precise location is of minor importance.

For each cylinder the axis start point $\mathbf{s}$ and the end point $\mathbf{e}$ are projected onto the line $\mathbf{c_p} + t\mathbf{c_d}$ and the parameters of the projected points t_s and t_e are computed. With these two parameters, two observation tuples of a 4D curve can be generated: $t_s \mapsto (s_x, s_y, s_z, r)$ and $t_e \mapsto (e_x, e_y, e_z, r)$, where r is the cylinder radius. This leads to observations of the branch axis and its radius function parameterized over the line as shown in Fig. 1.d. With moving least squares (Lancaster and Salkauskas, 1986) these points are approximated to determine a smooth axis $\mathbf{c}(t)$ and a smoothly varying radius $r(t)$. A polynomial model (e.g. a linear function) is fitted independently to the observations of each co-ordinate direction and the radius observations with a weighting scheme, that depends on the parameter t. For a given parameter t_0 the weight functions assigns the highest weight 1 to observations at location t_0 and decreasing weights to observations further away. The smoothness of the weighting function determines the smoothness of the final model. For a bell shaped weight function the model is shown in Fig. 1.e.

The data to be approximated is well distributed in the parameter domain and not noisy. Therefore the simple method of moving least squares gives satisfying results. While this branch model adapts to the curvature of a branch axis, it does not consider the non-circular cross section of real-world branches.

3 ACCURATE CROSS SECTIONS

With the algorithm explained in the previous section a reconstruction of a branch axis is provided, generated from a set of points on the outer branch surface. Branch cross sections orthogonal to this axis have to be determined now.

3.1 Point selection

Given a point $\mathbf{c_0} = \mathbf{c}(t_0)$ on the branch axis $\mathbf{c}(t)$, points have to be selected for reconstructing the cross section at this point. The plane α orthogonal to $\dot{\mathbf{c}}(t_0)$ and going through $\mathbf{c_0}$ is the carrier for the cross section. For having, on the one hand, enough points to determine the cross section reliably, and avoiding, on the other hand, influences of the branch curvature and the changing cross section along the axis, a slice of points from the original point set $\mathcal{P}$ is projected onto α. Let $\mathbf{p}' = \left(\mathbf{I} - \dot{\mathbf{c}}(t_0)\dot{\mathbf{c}}(t_0)^\top\right)(\mathbf{p} - \mathbf{c_0})$ denote this orthogonal projection of $\mathbf{p}$. Then the set of cross section points is

$$\mathcal{C} = \{\mathbf{p_k} \mid \|\mathbf{p'_k} - \mathbf{p_k}\| \leq h \wedge \|\mathbf{p'_k} - \mathbf{c'_0}\| < d\}, \quad (1)$$

the set of points which have a height less than h above or below the plane α and a distance less than d to the axis. The value d is the estimated radius from the cylinder increased by a certain distance in order to compensate for the non-circularity of the branch. With this upper bound it is prevented that other points of the entire point set – lying close to the plane ($\pm h$), but not close to the branch axis – are used for the determination of the cross section curve. The value h is chosen according to the straightness of the axis and point density as discussed above.

The points $\mathbf{p'_k}$ lie in a plane embedded in 3D. With a local coordinate system with $\mathbf{c'_0}$ as origin and any two orthogonal vectors in this plane $\mathbf{p'_k}$ is mapped from 3D to 2D.

This procedure is straight forward, but any error in the inclination of the plane will result in an overestimation of the branch diameter d in one direction. As shown in Fig. 2 the points are projected onto an area (shown in grey) in place of a curve. The angle ε between the actual projection direction (dash-dotted line with arrow) and the correct one influences on the wrongly estimated diameter d' through $d' = d/\cos(\varepsilon)$. If the diameter must not be overestimated by more then 0.3% for example, the maximum value for ε is 4.4°.

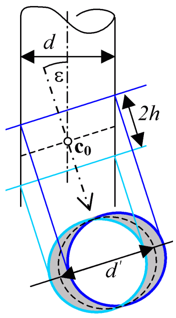

Figure 2:

3.2 B-Spline fitting

In the Appx. a short overview on B-Spline curves is given. For a comprehensive introduction see e.g. (Farin, 2002).

A B-Spline curve of degree n is defined as

$$\mathbf{x}(u) = \sum_{j=0}^{L} \mathbf{d_j} N_j^n(u) \quad (2)$$

with control points $\mathbf{d_j} \in \mathbf{R}^2$ and basis functions $N_j^n(u)$. The basis functions are defined via a recursion (see Appx.) over the knot vector $(u_0, \ldots u_K)$, and the curve is defined in the interval $[u_n, \ldots u_{L+1}]$. The knot vector prescribes the intervals of the individual polynomial curve segments which form the B-Spline curve.

Fitting a B-Spline curve to a set of points $\mathbf{p_i}, i = 1, \ldots m$ means to determine the control points $\mathbf{d_j}$ in such a way, that the sum of squared distances between the curve $\mathbf{x}(u)$ and the points $\mathbf{p_i}$ is minimized (least squares adjustment). The distance between a point and the curve is equivalent to the distance between the point $\mathbf{p_i}$ and its foot point on the curve $\hat{\mathbf{p}}_\mathbf{i}$, i.e. its orthogonal projection onto the curve. Therefore the curve parameter w_i (the running argument u in Eq. 2) of the foot point has to be determined for every point, too, so that $\hat{\mathbf{p}}_\mathbf{i} = \mathbf{x}(w_i)$ with $\mathbf{p_i} - \mathbf{x}(w_i)$ orthogonal to $\dot{\mathbf{x}}(w_i)$. The minimization can be written as:

$$\Delta = \sum_{i=1}^{m} \|\mathbf{p_i} - \sum_{j=0}^{L} \mathbf{d_j} N_j^n(w_i)\|^2 \rightarrow \text{Minimum} \quad (3)$$

This kind of fitting problems are usually solved with two alternating steps. First the foot point parameters w_i are determined, which is an independent operation for each point $\mathbf{p_i}$ (so-called parameter correction (Farin, 2002)). In the second step the control points $\mathbf{d_j}$ are found by setting the derivatives $\partial\Delta/\partial\mathbf{d_j}$ of Eq. 3 to zero and solving the overdetermined linear system of equations by least squares adjustment (Mikhail, 1976). Then iteration commences with the process of determining new foot point parameters for the new curve and subsequent control point determination. If the root mean square error of the adjustment

$$\sigma_0 = \sqrt{\Delta/(m - (L+1))} \quad (4)$$

does not become smaller anymore, the system is end-iterated. A criterion is imposed on the relative change $(\sigma_0^{s-1} - \sigma_0^s)/\sigma_0^{s-1}$, with s denoting the iteration number.

For the first step of determining foot points an initial curve is required and the correct orthogonal projections of all $\mathbf{p_i}$ onto the curve have to found. For closed curves there are at least two orthogonal projections for each point.

For describing closed B-Spline curves the last n control points must be identical to the first n control points, and the knot vector has to be cyclical, too (see Appx.). Taking e.g. four knots and spreading them equally over the parameter interval $[0°, 360°]$, the knot vector for a degree $n = 3$ B-Spline is $[-270, -180, -90, 0, 90, \ldots, 360, 450, 540, 630]$, and $\mathbf{d_4} = \mathbf{d_0}, \ldots \mathbf{d_6} = \mathbf{d_2}$.

3.3 Initial curve and knot insertion

The first approximating curve can be obtained from the radius of the cylinder which was fitted to the point cloud as described in Sec. 2. By using Eq. 3 with four "observed" points $(r\cos(w_i), r\sin(w_i))^\top$, spread equally along the circle with parameters $w_i = 360/4 * i, i = 0, \ldots 3$ a unique solution for four control points is found. Initial values for the foot point parameters w_i of the points $\mathbf{p_i}$ can then be derived from their azimuth to the circle center point.

The number of knots has a large impact on the B-Spline curve. The more knots, the more flexible the curve is, and smaller details of the tree surface can be approximated better. A too high number of knots on the other hand can lead to strongly oscillating curves, self-intersecting curves, and curves that follow the random distribution of noisy points instead of averaging out the random measurement errors. A solution suggested in (Dierckx, 1993) is to start with a low number of knots and incrementally increase the number of knots in intervals of poor approximation quality. For each interval a test value is determined, and in the midpoint of the interval with the highest test value a new knot is inserted (see Appx.). This increases the number of control points and the number of curve segments by one. Adjustment is continued and this process is repeated, until all test values are below a certain threshold.

The test value suggested in (Dierckx, 1993) is based on the mean square distances from the curve to the points in each segment. This quantity is not suited for the application of determining approximating free-form curves for tree cross sections. The m.s.e. (mean square error) is a measure of noise, but it remains unclear, if this noise comes from a poor approximation, i.e. the curve is far away from the points, or from the points themselves. Both situations can occur in our case: the first case indicates indeed a poor approximation, if e.g. one curve segment lies only on one side of the points, but the second case indicates noise, e.g. due to random measurement errors, a locally very irregular tree surface, or small remaining registration errors between the different scans. In the first case a new knot has to be inserted in this interval in order to increase the approximation quality, but in the second case, the segment must not be subdivided into two segments, because this could lead to oscillating or self-intersecting curves (see above). Additionally, a new knot in this segment cannot improve the approximation of the point set, because the adjusting curve is already running in an optimal way (in the least square sense) between the points on either side. What is needed is a test value, that measures *systematic* differences between the given points and the curve for each segment.

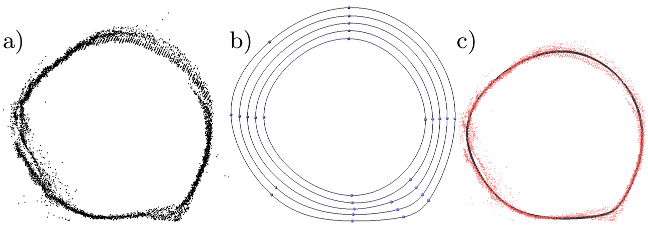

Figure 3: A set of points is shown in a), and a sequence of B-Spline curves with increasing knot number is fitted to it. Scaled versions of the curve are shown in b) with the knots indicated as circles. The final curve with 8 knots is shown in c) with the point set.

To measure a systematic difference between a curve segment and its points, the residuals, i.e. the signed distances between the given point and their curve foot points, have to be analyzed. If the residuals are plotted over their corresponding foot point parameters and a trend can be observed in this graph, i.e. if the expectancy of the residuals is not zero everywhere, a systematic error exists.

The test value we use is computed in two stages. First each curve segment is split into k parts of equal size in the parameter domain. This shall be illustrated by an example. If the initial number of knots is $L = 4$, the parameter bounds of the segments u_s are $0, 90, 180, 270, 360$ and if $k = 5$, then the first curve segment is split into the intervals $[0, 18), [18, 36), [36, 54), [54, 72), [72, 90)$. With $I_{s,j}$ denoting the j^{th} interval in segment s, for each interval the mean residual is determined:

$$r_{s,j} = \sum_{w_i \epsilon I_{s,j}} |\mathbf{p_i} - \mathbf{x}(w_i)| / m_{s,j},$$

with $m_{s,j}$ the number of points in the interval $I_{s,j}$. Here $|\cdot|$ denotes the signed distance from the foot point to the given point[2]. The average number of points in each segment is $\overline{m} = \sum_{l=0}^{L-1} m_l / L$, and the average length of each segment in the parameter domain shall be denoted as $\overline{\Delta u}$. The final test value for each interval is:

$$t_s = \sum_{j=1}^{k} \|r_{s,j}\| \frac{m_{s,j}}{m_s} \times \frac{m_s}{\overline{m}} \times \frac{u_{n+s+1} - u_{n+s}}{\overline{\Delta u}} \qquad (5)$$

The first term is the average of the absolute mean residuals in the intervals of one curve segment, with weighting by the number of points in each interval. Segments in which one or more intervals have a large average residual obtain a large test value. The second term is a weighting according to the relative number of points in the segment. A segments with a few points only shall not be subdivided further and its test value becomes smaller. The third term favors subdivision of larger segments in the parameter domain. The last two terms are both relative measures, yielding factors of 1 for equally distributed data and knots. Eq. 5 measures the systematic difference in meters.

The number of intervals in a segment, k, should be bigger than two to avoid overlooking symmetric systematic deviations. A curve segment lying completely inside the points in the first half of the segment, and completely outside in the second half, can lead to $t_s = 0$ for $k = 1$. We achieved good results with $k = 5$.

The process of B-Spline fitting and sequentially inserting knots is demonstrated with a cross section containing 6826 points (Fig. 3.a), demonstrating the robustness of the method. The curve was started with four knots, and a new knot was inserted in the interval with the largest test value t_s (Eq. 5), as long as $t_s > 0.006$, which is larger than the random measurement accuracy of 3mm, but other influences (poor registration between different data sets) had to be considered in this example, too. The r.m.s.e. σ_0 (Eq. 4) of the spline curves went from ±1.23cm with four knots to ±1.05cm with eight knots. Choosing eight equally spaced knots from the beginning leads to a σ_0 which is only 3% larger, but with 15% larger systematic deviations.

If too few points are given, e.g. only along one third of the circumference, the B-Spline determination can become impossible. Each interval $[u_j, u_{j+n+1}]$ within the curve domain must contain at least one point. Fitting can also fail, if the curves starts oscillating strongly. This can be detected e.g. by looking at the curve's total length and comparing it to the circle circumference, or by analyzing the angles in the control polygon $\mathbf{d_0}, \ldots \mathbf{d_L}$.

[2]Signed distance means, that the distance is positive, if the point lies outside of the curve, and negative if it is on the inner side of the curve.

4 APPLICATIONS AND EXAMPLES

The computation of free-form curves for cross sections is demonstrated for the stem and one branch of an oak tree, including the determination of cross section ovality as a measure of wood quality.

First a definition of ovality, which is suitable for closed free-form curves will be given. Also a method for the estimation of the direction accuracy (remember Fig. 2) will be given. Axis accuracy is important to avoid overestimating the cross section diameter. The presented example is based on these topics and the algorithms of the previous Sections.

4.1 Ovality Measurement

Ovality is a measure for the deviation of a stem cross section from a circle. The instruction for manual measurement is to measure the largest and the smallest diameter. The largest diameter is the length of the largest straight line segment connecting two points on the outer surface of the tree in one slice. Smallest and largest diameter are, by definition, orthogonal. The measurement must be taken once in a height of least 1m. The ovality in percent is (Deutsches Institut für Normung e.V., 2000):

$$(1 - \frac{d_{min}}{d_{max}}) \cdot 100$$

This measurement instruction was mapped to an algorithm in order to calculate the ovality of the B-Spline curves. This could be done analytically, too, but for ease of implementation we decided to discretize the spline as polygon. In order to get a more accurate result and eliminate outliers due to buckles or other irregularities on the stem surface, the ovality is calculated for multiple layers. For each spline curve the largest and the 'smallest' diameter are determined. Afterwards, the ovality values for the layers starting at 1m up to 90% of the stem length are averaged. The upper end of the tree stem may not be used for ovality calculation, because the stem cross section deforms due to the transition to the crown branches.

4.2 Directional accuracy testing

In order to get an estimate if the chosen projection directions are the growing directions, a simple test has been performed for each slice. After projection onto the plane, the points are sorted into a raster with an edge length a, e.g. 3mm, which is the accuracy of the laser scanner used. The number of raster cells containing one or more points is determined. This is repeated for other projection directions deviating from the original direction by a few degrees, e.g. up to 10°. Each projection direction is associated with its number of filled raster cells. Assuming

	Stem	Branch
Number of points	1,669,368	34,363
Cylinder length [cm]	50	20
No. of fitted cylinders	90	60
Axis length [m]	11.05	4.25
Min./Max. σ_0 [m]	0.011, 0.029	0.009, 0.022
Average σ_0 [m]	0.0175	0.0147

Table 1: Results of cylinder following for tree stem and branch. σ_0 refers to the r.m.s.e. of the cylinder fitting.

	Stem (1)	Stem (2)	Branch
Number of knots	6	4	4
Avg. no. pts./slice	6638	6638	613
Number of slices	112	112	59
Not accepted slices	2	0	23
Minimum σ_0 [m]	0.0103	0.0106	0.0093
Maximum σ_0 [m]	0.0266	0.0335	0.0234
Average σ_0 [m]	0.0154	0.0159	0.0139
Avg. angle deviat.	2.88°	2.95°	4.95°
Min./Max. iterations	2, 49	6, 41	2, 18
Avg. no. iterations	11	14	5
Ovality [%]	7.8	7.1	–

Table 2: B-Spline fitting results for the tree stem with different number of knots and the branch.

a homogeneous point distribution along the entire circumference of the tree, and assuming that the tree grows cylindrically (i.e., it has zero gaussian curvature), the direction which leads to the lowest number of filled raster cells is the growing direction. The angular difference between the original projection direction and the new direction is a measure for the correctness of the cylinder axis as treated in Sec. 3.1. As the assumptions are not fulfilled exactly, this angular deviation can only be used as an indication on the correctness of the projection direction.

4.3 Example

The B-Spline fit algorithm can be applied to point clouds of tree stems and of the larger crown branches. The steps for cross section determination are as follows:

1. Execute cylinder following for the relevant point cloud (Sec. 2.1). The result is a list of cylinders which represent the stem or a branch.
2. Determine a smooth axis from the individual cylinders (Sec. 2.2).
3. Separate the point cloud of the stem or branch into cross sections with a defined thickness along the curved axis. (Sec. 3.1)
4. Fit the B-Spline curves to the cross section points and calculate the ovality for each slice. Average the ovality values.

For the example a point cloud of an oak tree was taken. The tree was scanned from four positions around it, with a maximum registration error for the combined point cloud below the measurement accuracy. The whole stem was covered with points, but due to occlusions it is impossible to get points from all around the crown branches. Besides, the number of points on branches is also lower because of their smaller size and larger distance to the scanner.

The 1st step of the cylinder following is illustrated in Table 1. Because the branch axis is curved stronger, the maximum length of the cylinder for point selection was chosen smaller. In the 2nd step a moving linear polynomial was used for the computation of the smooth axis from the individual cylinder "observations". Along the average axis, and with a spacing of 10cm, the axis position and its first derivative were determined. For weighting the observations a bell curve was used, that assigned the weight 1 to observations at the interpolation position and the weight 1/2 to observations at a distance of 0.5m. This yields a smooth axis which eliminates some of the small details but follows the branch axis well (see Figs. 1.d and 1.e).

The selection of points (3rd step) was performed in slices of $h = \pm 10$cm thickness. This ensured a sufficient high number of points for the spline calculation. Within a 20cm thick slice the curvature of the branches are negligible (see e.g. Fig. 4). All points in this height slice with a distance from the axis less than the cylinder radius plus 20cm (Eq. 1) were used for the spline calculation in the next step.

The 4th step of spline calculation was performed for the tree stem with four and with six knots, the latter being a compromise between calculation speed and accuracy. Calculation speed is also the reason for not choosing the time-consuming knot insertion strategy presented in Sec. 3.3. The two different knot numbers were chosen to demonstrate the effect on the accuracies and for comparison to the splines of the branch. As described above, fewer points are available for the branch cross section reconstruction. Therefore the spline calculation must be executed with four knots, in order to avoid having curve intervals without point support. Table 2 shows the results of the spline calculation for the stem and one branch of the oak tree. With six knots the fitting accuracy is ±1.5cm, and the direction accuracy testing value is, on average, below 3°.

The ovality which results of the calculation with a reduced number of knots is a little smaller. This is comprehensible because with a smaller number of knots the spline curves fits the real cross section with less accuracy, it stays closer to an averaging circular shape (see also Fig. 3.b).

Fig. 4 shows the point cloud of the oak tree and the tree stem model consisting of 112 spline curves which have a distance of 10cm to each other. The spline curves and point cloud of the branch can be seen in Fig. 5. Only about half of the branch surface is covered with scan points. The darker point cloud slices are those where no spline curve could be fitted. As can be seen, the spline determination is only reliable where points are measured, too (i.e., the lower side of the branch). This underlines the necessity of having the points covering the entire circumference.

5 CONCLUSIONS

In this paper a method for the determination of cross sections with B-Spline curves was presented. Point clouds from laser scanning are the only data source required. The B-Splines are faithful models of the cross sections with an accuracy between 1cm and 2cm. This accuracy is a product of measurement noise, surface roughness and un-modelled small features of the cross section. All these components are in the order of a few millimeter.

Processing begins first with determining a rough model of the tree, consisting of overlapping cylinders. The next finer model is obtained by approximating these cylinders with a model consisting of a smooth axis and a smoothly varying radius. Slices of points, orthogonal to this axis are drawn from the original point cloud, projected onto a plane, and a closed B-Spline curve of degree three is fitted to these points. For the processing only a few threshold values have to be set, but all these values can be related to general tree parameters (e.g. curvature of a branch axis) and can therefore be determined easily. As expected, cross section reconstruction works only satisfyingly, if the branch is covered with points from all sides.

Minimum and maximum diameter of the tree can be determined in different heights from the cross section curves. In order not to overestimate the diameter, it is necessary

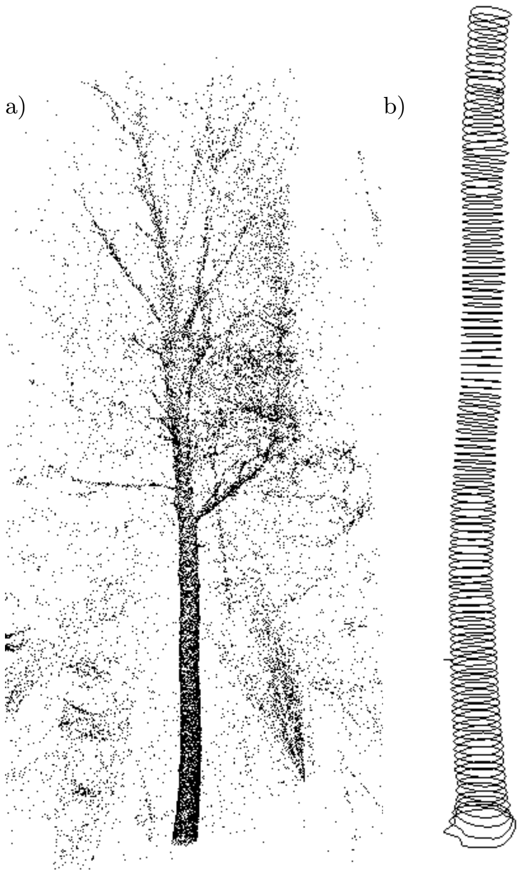

Figure 4: The point cloud of an oak tree is shown in a), and the stem reconstruction with B-Spline curves (six knots) from ground up to crown base in b).

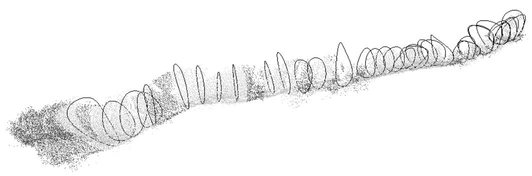

Figure 5: Point cloud and spline curves for a sample branch.

that the cross section plane is orthogonal to the growing direction of the tree. A method has been described, that tests if another projection direction provides a more compact point set in the cross section plane. With this method it has been demonstrated that an error bound of 3% overestimation in diameter was not exceeded in the example.

These different models of branches and branch cross sections allow measurement of the ovality of the stem or a branch, as demonstrated in the paper. Further parameters (e.g., taper) can be determined as well, and the reconstruction of realistic tree models for visualization is possible. One of the next aims is to recognize buckles automatically.

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APPENDIX

B-Splines (Farin, 2002) are so-called free-form curves, allowing the approximation or interpolation of a given set of observed points by a parametric curve $\mathbf{x}(u)$. The curve has a representation of the form

$$\mathbf{x}(u) = \sum_{j=0}^{L} \mathbf{d_j} N_j^n(u),$$

with $\mathbf{d_j}$ as control points in 1-, 2-, or 3D space, and basis functions $N_j^n(u)$, the so-called normalized B-Spline functions, where n is the degree of the curve. The basis functions are defined over the knot sequence $(u_0, \ldots u_K), u_i \leq u_{i+1}$ (a.k.a. knot vector) with the recursion:

$$\begin{aligned} N_l^n(u) &= \frac{u - u_l}{u_{l+n} - u_l} N_l^{n-1}(u) + \frac{u_{l+n+1} - u}{u_{l+n+1} - u_{l+1}} N_{l+1}^{n-1}(u) \\ N_i^0(u) &= \begin{cases} 1 & u_i \leq u < u_{i+1} \\ 0 & \text{otherwise} \end{cases} \end{aligned} \quad (6)$$

The relation between the number of control points $L+1$ and the highest index in the knot vector is $K = L + n + 1$, and the curve is defined over the parameter interval $[u_n, \ldots u_{K-n}]$. Starting the knot vector with $u_0 = u_1 = \ldots = u_n$ leads to the interpolation of the first control point $\mathbf{d_0}$, analogy applies to $\mathbf{d_L}$. The first derivative is

$$\frac{\mathrm{d}\mathbf{x}(u)}{\mathrm{d}u} = n \sum_{j=0}^{L-1} \frac{\Delta \mathbf{d_j}}{u_{n+j} - u_j} N_j^{n-1}(u),$$

where $\Delta \mathbf{d_j} = \mathbf{d_{j+1}} - \mathbf{d_j}$. The B-Spline curve consists of $L - n + 1$ polynomial curve segments of degree n.

Within the curve interval it holds $\sum_{j=0}^{L} N_j^n(u) \equiv 1$, which leads to invariance under affine transformations, and therefore independence of the origin and the rotation of the coordinate system. Additionally, the recursion Eq. 6 is computationally stable, as each of the basis functions remains in the interval $[0, 1]$ and so do the fractions. As $N_j^n(u) \neq 0$ in the interval $[u_j, u_{j+n+1}]$, the curve has local support, which means that changing a control point $\mathbf{d_j}$ only affects the curve in this interval.

A B-Spline curve with $L + 1$ control points can also be described as a B-Spline curve with $L + 2$ control points, if an additional knot $t \epsilon [u_l, u_{l+1})$ is inserted into the knot vector. The new control points $\mathbf{d_j^*}$ are obtained from the old ones by $\mathbf{d_j^*} = a_j \mathbf{d_j} + (1 - a_j)\mathbf{d_{j-1}^*}$ with:

$$a_j = \begin{cases} 1 & : \quad j \leq l - n \\ \frac{t - u_j}{u_{j+n} - u_j} & : \quad l - n < j \leq l \\ 0 & : \quad l < j \end{cases}$$

A closed B-Spline curve of degree n with as many curve segments as control points L can be obtained over the knot vector $\{u_0, \ldots u_{L+2n}\}$ by setting $u_{L+n+r+1} - u_{L+n+r} = u_{r+1} - u_r$ for $r = 0, \ldots n-1$, and cyclic continuation of the control points $\mathbf{d_{L+r}} = \mathbf{d_r}$. This cyclic continuation has be be applied also in the formulae for derivation and knot insertion.