

# EVALUATION OF CORRELATION CRITERIA FOR SAR IMAGES

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## ABSTRACT

The aim of this paper is to study the use of cross-correlation for radargrammetric applications. Two other criteria derived from Mean Square Error analysis are proposed. These three criteria are studied using two sets of tests on real data (high resolution, one-look image of a semi-urban area). The first test is based on the analysis of the heights of a set of buildings (the heights being obtained using the thresholded disparities computed by one of the criteria). Then the performances are analyzed for each building using the available ground truth. The second test is based on the matching of a set of points. These points are manually associated and the displacement is compared to the one given by the matching criterion. Eventually, the ROC curves are computed to compare the 3 criteria in different situations.

## 1 INTRODUCTION

SAR images are nowadays widely used for Digital Elevation Model (DEM) production. The interferometric potential of the Synthetic Aperture Radar (SAR) data permits to obtain accurate DEM and the recent SRTM mission is a new proof of this capability. Nevertheless, SAR data are also able to produce height information using the classical stereo-vision principle (Rosenfeld, 1968). However, rather few works have been done in this domain (Toutin, 1995). One of the main difficulties in radargrammetry is the matching step which associates the pixels of both images. Due to the speckle phenomenon, this step is particularly difficult with SAR images. The normalized centered cross-correlation coefficient, which is widely used for optical images gives noisy results with SAR data. Among the proposed solutions, pre-processing like edge detection or post-processing like clever filtering of the associated pairs can be used.

The aim of this paper is to analyze correlation criteria and study their behaviors when applied on real SAR images. We first describe 3 criteria which will be studied in the paper. Then we describe the two sets of tests that have been done to do the comparison. The first test is based on the analysis of the heights of a set of buildings (the heights being obtained using the thresholded disparities computed by one of the criteria). Then the performance are analyzed for each building using the available ground truth. The second test is based on the matching of a set of points. These points are manually associated and the displacement is compared to the one given by the matching criterion. Eventually, the ROC curves are computed to compare the 3 criteria in different situations.

## 2 URBAN SAR IMAGES AND RADARGRAMMETRY

SAR image interpretation in urban areas is known as very difficult task because of many phenomena: speckle, distance sampling, lay-over areas, and shadows. As pointed out in (Hardaway et al., 1982) (Tupin et al., 2002) the appearance of an object is strongly related to its geometrical properties in regards to the along track direction and the incidence angle. It is also depending on its roughness compared to the wavelength. Because of multiple bounce scatterings, many very bright features corresponding to dihedral or trihedral configurations are present in the data

(Franceschetti et al., 2002). They correspond to wall / ground corners, balconies, chimneys, posts, street lamps, etc. Therefore SAR images in urban areas are usually composed of very bright features on a darker background with speckle. An example is shown in Figure 1. Since some roofs are quite smooth compared to the wavelength (9cm in S band), their mean radiometry is rather close to ground radiometry.

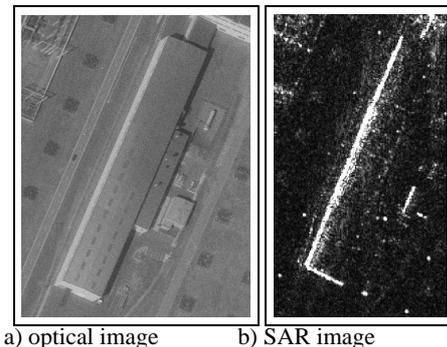


Figure 1: Appearance of a building on the SAR image. Most of the information is given by very bright features (points or lines) corresponding to dihedral or trihedral configurations.

For these reasons, classical stereovision algorithms based on the correlation coefficient between the two acquisitions are not very successful. The phenomenon has been observed for satellite data and therefore many original approaches have been developed for radargrammetric applications (Ansan and Thouvenot, 1995) (Marinelli et al., 1998) (Paillou and Gelautz, 1999). We study here the use of dedicated correlation measures adapted to multiplicative noise (Tupin, 2002).

## 3 CORRELATION CRITERIA

The problem to solve is to test the correspondence between two signals. In radargrammetric applications, the final purpose is to recognize if the first signal is present in the second one, each signal being a small part of the two images.

This first part is dedicated to the presentation of the three criteria that will be tested in the following of the paper. The first

criterion is the classical normalized centered cross-correlation, widely used for optical images. The second one called “variation coefficient criterion” has been proposed in a previous work (Tupin, 2002) and is a mixture of the classical cross-correlation coefficient and the variation coefficients (standard deviation normalized by the mean). The third one is a new criterion called “logarithmic criterion” in the following which is derived using the multiplicative noise model of the SAR images. These three criteria are defined in the following three sub-sections.

### 3.1 Cross-correlation criterion

The cross-correlation coefficient is usually presented using a probabilistic approach. Let  $X_1$  and  $X_2$  be two real random variables, the centered normalized correlation  $\rho$  is defined by:

$$\rho = \frac{\text{cov}(X_1, X_2)}{\sigma_1 \sigma_2} = \frac{\mathbb{E}(X_1 X_2) - \mu_1 \mu_2}{\sigma_1 \sigma_2} \quad (1)$$

with  $\mathbb{E}$  the expectation,  $\mu_i$  and  $\sigma_i$  the mean and standard deviation of  $X_i$ . It measures the linearity of the relationship between  $X_1$  and  $X_2$ . For normalized centered random variables, it reduces to  $\mathbb{E}(X_1 X_2)$ .

Having  $\rho$  defined by eq.1 and  $N$  real<sup>1</sup> samples of  $X_1$  and  $X_2$  denoted by  $x_{1i}$  and  $x_{2i}$  for  $i \in \{1, \dots, N\}$ , there are many estimators  $\hat{\rho}$  of  $\rho$ .

The moment estimator is given by:

$$\hat{\rho} = \frac{\frac{1}{N} \sum_i x_{1i} x_{2i} - \hat{\mu}_1 \hat{\mu}_2}{\hat{\sigma}_1 \hat{\sigma}_2} \quad (2)$$

with  $\hat{\mu}_j = \frac{1}{N} \sum_i x_{ji}$  and  $\hat{\sigma}_j^2 = \frac{1}{N} \sum_i x_{ji}^2 - \hat{\mu}_j^2$ .

Another estimator which can be used to estimate the correlation coefficient is the maximum likelihood (ML) estimator.

For the binormal distribution, the ML estimator is the same as the one obtained by the moment method and is given by eq.2 (Kendall and Stuart, 1969). The bias and variance of this usual sample correlation coefficient can be derived in the case of binormal distribution assumption.

Another way of deriving the correlation coefficient is the use of the Mean Square Error (MSE) to compare the  $X_1$  and  $X_2$  populations:  $\text{MSE} = \mathbb{E}[(X_1 - X_2)^2]$ . This quantity is minimized to search the best correspondence. To allow radiometric variations between the two signals, the MSE is applied to the centered normalized random variables which corresponds to  $\text{MSE} = 2(1 - \rho)$ . Thus the minimization of the MSE for normalized random variables is equivalent to the maximization of the cross-correlation coefficient.

<sup>1</sup>Note that in the case of SAR images, many different data can be used (complex field  $Z$ , intensity measure  $I = |Z|^2$ , amplitude data  $A = |Z|$ ). There have been many works on the coherence estimation, specially in the interferometric framework addressing the interest of using complex or intensity data (Guarnieri and Prati, 1997) and studying the bias and variance of various estimators. But the aim is totally different in the radar-grammetric context, since we are here interested in the correlation of the underlying scene of the 2 images. The speckle is for us de-correlated and thus a disturbing phenomenon, whereas in interferometry speckle is the “real” signal and the scene is the disturbing phenomenon (Guarnieri and Prati, 1997). Therefore, there is no interest in our context of using complex images instead of amplitude or intensity ones.

### 3.2 Variation coefficient criterion

The probabilistic approaches which have been developed for SAR data strongly rely on scene and speckle assumptions. Since they are rarely verified for un-natural areas, we preferred to use the Mean Square Error approach, to derive new criteria adapted to multiplicative noise. To keep the criterion increase in the case of a good match, we use the inverse of the MSE.

Using this idea, the criterion  $v$  has been derived keeping a radiometric difference to define the error, but using a ratio based normalization and integrating the variation coefficient  $\gamma_j$  of the random variables  $X_j$  to favor featured areas (Tupin, 2002):

$$1/v = \frac{\mathbb{E}[(\frac{X_1}{\mu_1} - \frac{X_2}{\mu_2})^2]}{\gamma_1 \gamma_2} \quad (3)$$

In fact, we can see that  $v$  has also the following expression:

$$1/v = \frac{\gamma_1}{\gamma_2} + \frac{\gamma_2}{\gamma_1} - 2\rho$$

It means that this coefficient has a global behavior similar to the one of the classical correlation, but tends to select windows with the same variation coefficients ( $v$  is maximized when  $\rho = 1$  and  $\gamma_1 = \gamma_2$ ). This criterion is referenced as “variation coefficient criterion” in the following.

It has been shown in (Tupin, 2002) using simulated data with theoretical Gamma distributions of different patterns and with different contrasts that:

- the variation coefficient gives better results in terms of detection probability and false alarm rates on the original SAR data; results are only slightly better if a preliminary averaging is applied on the data;
- the localization accuracy of the match is better with the variation coefficient criterion than with cross-correlation.

### 3.3 Logarithmic criterion

Since the speckle is modeled by a multiplicative noise (Goodman, 1975), a natural idea would be to use the logarithmic transformation to define adapted criteria. A first test has been to apply the correlation coefficient on logarithmically transformed images. But this criterion is not adapted to urban areas. Indeed, after logarithmic transformation, the small size bright targets are more difficult to detect. Since they are very important features in real urban scenes, the logarithm should not be used in practice.

Instead of using normalized centered variables  $\frac{X-\mu}{\sigma}$  we propose to introduce the variable  $X' = (\frac{X}{\mu})^\alpha$  and to define the criterion by:

$$1/w = \mathbb{E}[(\log X'_1 - \log X'_2)^2] \quad (4)$$

Using the multiplicative noise model,  $X = RS$  with  $R$  the scene reflectivity, and  $S$  the normalized speckle, with  $\alpha = 1$  we obtain the following relationship (supposing that the two scenes are equal ( $R_1 = R_2$ ):

$$\begin{aligned} 1/w &= \mathbb{E}[(\log \frac{R_1 S_1}{\mathbb{E}(R_1)} - \log \frac{R_2 S_2}{\mathbb{E}(R_2)})^2] \\ &= \mathbb{E}[\log \frac{S_1^2}{S_2^2}] \end{aligned}$$

This expression is not satisfying in a radargrammetric application since there is no way to distinguish homogeneous areas and pattern areas. Therefore, we introduced the scene variability in the  $\alpha$  definition by taking:

$$\alpha = \frac{1}{\sqrt{\sigma^2 - \mu^2 \sigma_S^2}}$$

$\sigma$  and  $\mu$  being the standard deviation and mean of the image ( $X$ ),  $\sigma_S$  being the standard deviation of the speckle given by the look number of the SAR image ( $\sigma_S = \frac{1}{\sqrt{L}}$  for intensity images). In this case, we have:

$$1/w = \frac{1}{\sigma_R^2(\sigma_S^2 + 1)} E[(\log \frac{S_1}{S_2})^2] \quad (5)$$

and the intrinsic variability of the scene is taken into account by the standard deviation of the scene  $\sigma_R$ .

### 3.4 Examples of application

An example to show the different behaviors of the three criteria is presented in figures 2 and 3 for a punctual target. The ‘‘correlation images’’ are displayed which means that the correlation value for each displacement around the true match (in the center of the image) is displayed. It is a way to see the decreasing of the criteria around the good match and to see other maxima.

In figures 4 and 5 another result is presented. This time, in each pixel the correlation value for the best match is displayed. As usual, the higher correlation values are obtained for patterns (specially the bright lines here).

A quantitative analysis is given in the two following sections using two HR SAR images on a semi-urban areas.

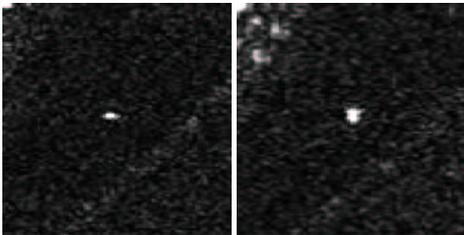


Figure 2: Left and right SAR images used for the correlation

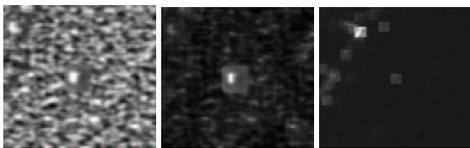


Figure 3: Correlation images for the three criteria: from left to right: values of cross-correlation coefficient, the variation coefficient criterion, and the logarithmic criterion.

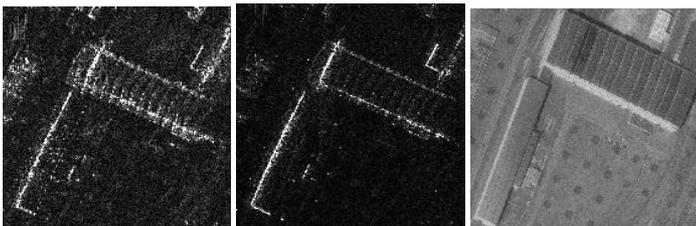


Figure 4: Left and right SAR images used for the correlation

A quantitative analysis is given in the two following sections using two HR SAR images on a semi-urban areas.

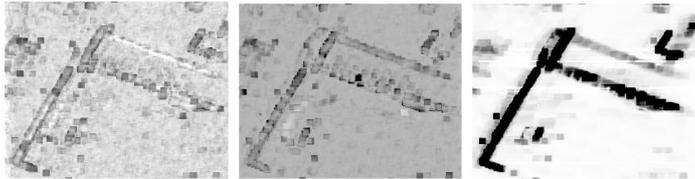


Figure 5: Correlation results for the three criteria: from left to right: cross-correlation coefficient, variation coefficient criterion, the logarithmic criterion (in negative display: the darker the pixel, the best the correlation value)

## 4 TEST ON BUILDING HEIGHTS

In this section, the comparison of the three criteria is done using a building map. For each building, we have a ground truth giving its true height (obtained by stereo-vision with optical images). For the SAR images, they are in epipolar geometry<sup>2</sup>. For each criterion, for each match above a fixed threshold, the associated disparity is converted in an height associated to the point.

The first step of this comparison is the study of the dynamic of each criterion to define an adapted threshold. This value is of course crucial since it influences the match selection. For the three criteria the value distributions have been empirically computed on a real SAR image. Then the mode of the probability density function has been selected, and will be used as a threshold value in the following tests. On the ground truth 10 buildings have been selected and subdivided into two sets: textured buildings and un-textured ones. We have used two criteria to study the performance of a correlator. The first criterion is the percentage of points which have been matched to the right height. The second criterion is the mean square error of the computed height (compared to the real one).

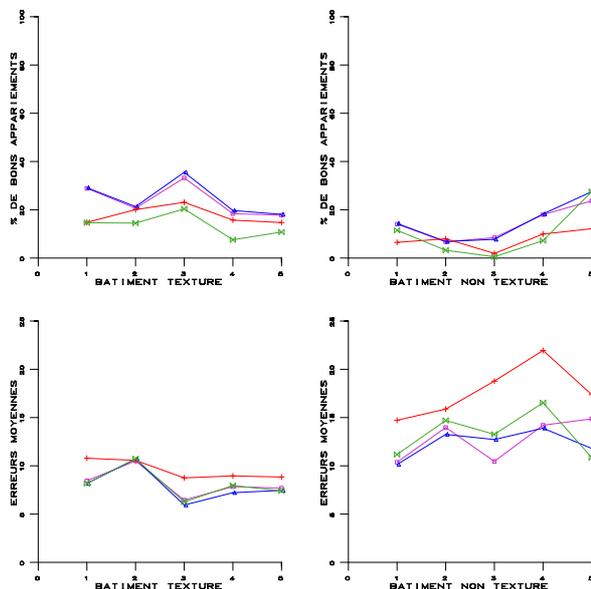


Figure 6: Percentage of good matchings (top) and mean square errors (bottom) for textured buildings (left) and untextured ones (right) for the three correlators (pink: cross-correlation; blue: variation coefficient criterion; red: logarithmic criterion; green: fusion of the three correlators).

The fusion of the three correlators is done by selecting pixels for which the three matches are above the thresholds and giving the same disparity.

<sup>2</sup>Thanks to Thales for the epipolar geometry computation.

Using Figure 6, the following conclusions can be given:

- results are better for both criteria for the textured buildings and this is true whatever the correlator is;
- The two first correlators (cross-correlation and variation coefficient criterion) have similar performances (with a slight advantage to the variation coefficient criterion) whereas the logarithmic criterion is worst than the two other ones.

To complete this analysis, Figure 7 presents a classification of the pixels depending on their accordance to the theoretical height of the building.

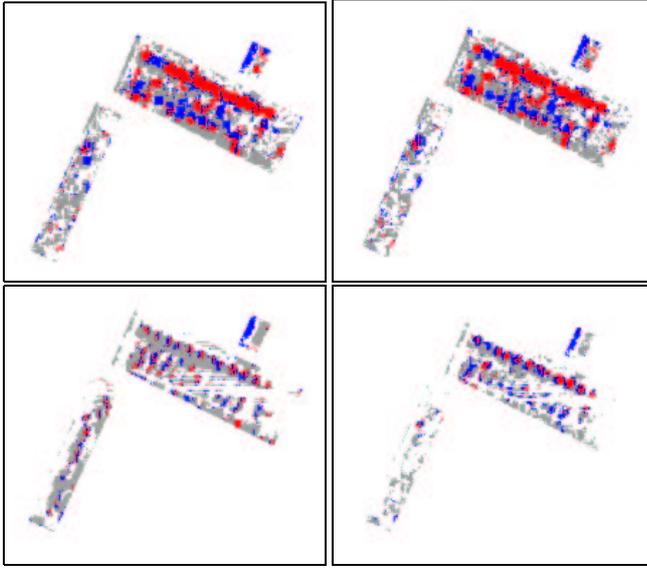


Figure 7: Colored classification of the pixels depending on their error compared to the theoretical height (in red: with errors less than 2m; in blue: with errors less than 4m; in grey: with errors more than 4m -from top left to bottom right: cross-correlation, variation coefficient criterion, logarithmic criterion, fusion of the three correlators-).

On this Figure 7, it seems that the logarithmic correlator gives right matches accurately located on the bright targets, whereas the two other ones give spread responses, thus increasing the percentage of well classified pixels. This correlator is thus more adapted to cross-shaped targets. This point is investigated in the second part.

## 5 TEST ON A SET OF POINTS

In this section, we are interested in the ROC curves computed for a set of manually selected points.

The test is done in the following way. The pair of points  $(P_1, P_2)$  in each image is selected manually. This selection is done in a very accurate way using visual criteria (contextual knowledge, etc.). Two sets of points have been defined. The first one correspond to bright punctual isolated targets and the second one to "L" shape corners (usually corresponding to ground/wall corner reflectors). For each point  $P_1$  the best match in image 2 for a given correlation window and a given search area is computed. This match is localized in  $P_2'$  with value  $v$ . Three situations are considered for a given threshold  $th$  on the correlation value:

- if  $d(P_2, P_2') \leq 2$  pixels and  $v > th$ : good detection;

- if  $d(P_2, P_2') > 2$  pixels and  $v > th$ : false alarm;
- if  $v \leq th$ : point not taken into account.

Using these definitions, for each threshold on the correlation criterion, a probability of detection and a false alarm probability is computed. The ROC curves are then deduced for each criterion.

### 5.1 Results for the punctual bright targets

An example of a manually selected pair of points is shown on Figure 8. The results of the correlation values in the search area for a  $9 \times 9$  window are presented Figure 9.

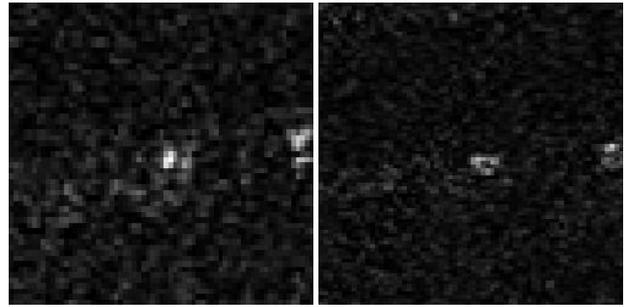


Figure 8: Pair of points manually selected in the two images (bright punctual target).

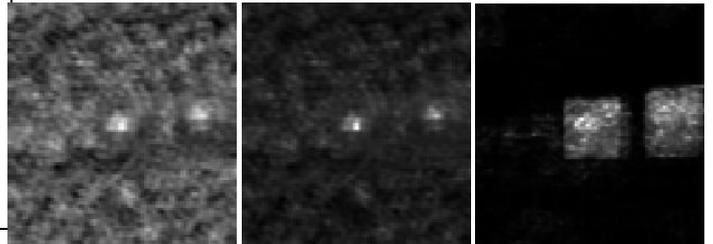


Figure 9: Correlation images for the three criteria (from left to right: cross-correlation, variation coefficient criterion, logarithmic criterion) applied on the pair of Figure 8.

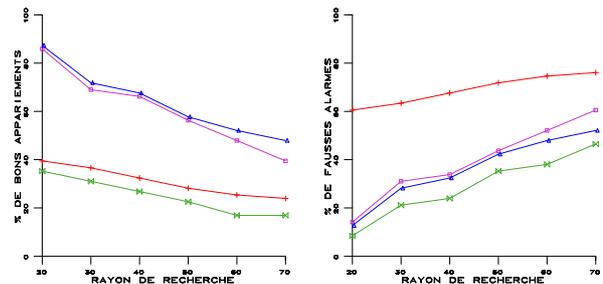


Figure 10: Percentage of good matches (on the left) and false alarms (on the right) versus the size of the search area (in pink: cross-correlation; in blue: variation coefficient criterion; in red: logarithmic criterion; in green: merged criterion).

Figure 10 shows the behavior of the 3 correlators when the search area is increased. As expected, the percentage of good matches decreases when the search area increases, whereas the false alarm percentage increases. A radius of 40 pixels for the search area has been eventually chosen to compute the ROC curves.

On this Figure, it can be observed that the variation coefficient criterion has better performances than the others.

This result is confirmed on the Figure 11 (ROC curves) showing that the best performance is obtained for the variation coefficient criterion.

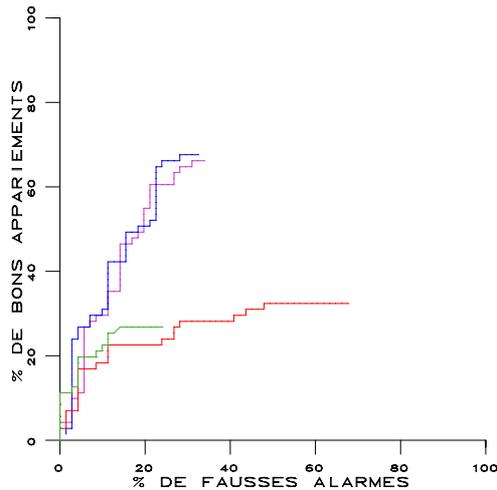


Figure 11: ROC curves for the punctual targets (in pink: cross-correlation; in blue: variation coefficient criterion; in red: logarithmic criterion; in green: merged criterion).

## 5.2 Results for the “L” shape corners

Once again, an example of a manually selected pair of points in the set of “L shape” points is shown on Figure 12. The results of the correlation values in the search area for a  $9 \times 9$  window are presented Figure 13.

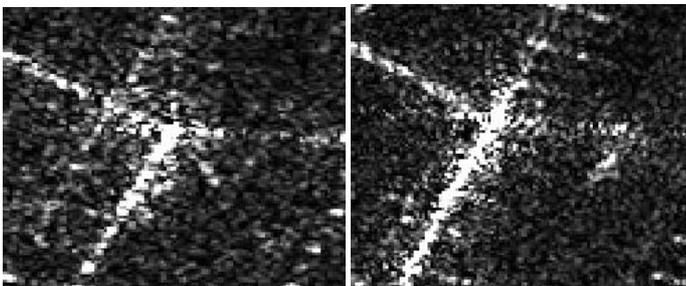


Figure 12: Pair of points manually selected in the two images (“L” shape corner).

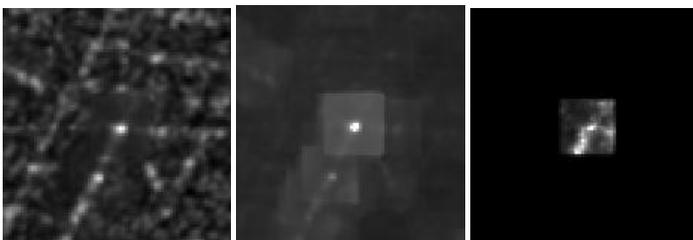


Figure 13: Correlation images for the three criteria (from left to right: cross-correlation, variation coefficient criterion, logarithmic criterion) applied on the pair of Figure 12.

Once again, a radius of 40 pixels for the search area has been chosen.

For the case of “L-shape” pattern, the results are very different, since this time the best performances are given by the logarithmic criterion, whereas the variation coefficient criterion gives the worst performances.

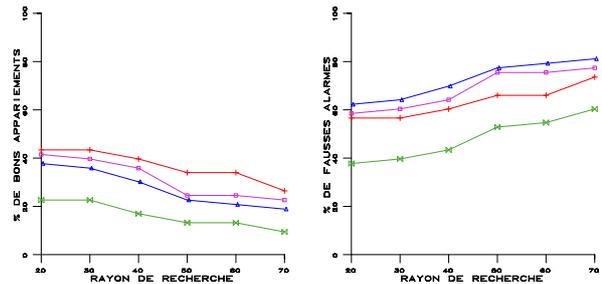


Figure 14: Percentage of good matchings (on the left) and false alarms (on the right) versus the size of the search area (in pink: cross-correlation; in blue: variation coefficient criterion; in red: logarithmic criterion; in green: merged criterion).

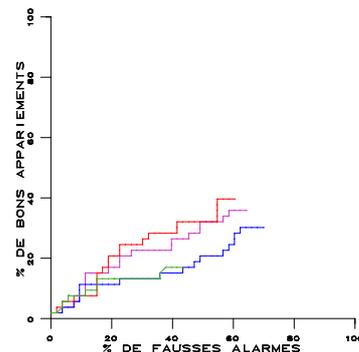


Figure 15: ROC curves for the “L-shape” targets (in pink: cross-correlation; in blue: variation coefficient criterion; in red: logarithmic criterion; in green: merged criterion).

## 6 CONCLUSION

In this paper a new logarithmic based criterion has been proposed. A study of the behaviors of 3 correlators has shown that the performances depend on the kind of buildings on the one hand (first set of tests), and on the kind of considered targets (corners or punctual targets) on the other hand (second set of tests).

The best way to process SAR images would be to use the logarithmic criterion for “L shape” pattern and the variation coefficient criterion for punctual targets, but to the price of a higher computational complexity.

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