

## NEW ALGORITHM FOR SUB-PIXEL BOUNDARY MAPPING

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### ABSTRACT:

Remotely sensed images often contain a combination of both pure and mixed pixels. Analysis and classification of remotely sensed imagery used to provide information on the spatial pattern of land cover feature suffer from the problem of class mixing within pixels. Therefore, how to get spatial pattern and boundaries information of endmembers in sub-pixel scale has been receiving increasing attention over recent years. A new algorithm for sub-pixel boundary mapping has been proposed in this paper to map the spatial arrangement of land cover targets within pixels. The validity of the technique is demonstrated by applying it to controlled simulated artificial images.

### 1. INTRODUCTION

The existence of mixed pixel, consisting of different features smaller than the resolution of the sensor, is one of the main sources affecting the accuracy of classification. Soft classification techniques can estimate the class composition of image pixels. Their outputs, however, provide no indication of spatial distribution of such classes within a pixel. Mapping subpixel scale land cover features has been developed over recent years. Atkinson (1997a) started this issue, based on the assumption of spatial dependence within and between pixels, by determining where the relative proportions of each class occur within each pixel. Later, Aplin et al. (1999 and 2001) proposed a set of techniques on a per-parcel (herein termed per-field) basis by integrating fine spatial resolution simulated satellite sensor imagery with digital vector data to classify land cover. Tatem et al. (2000, 2001a, 2001b, 2002, 2003) proposed an algorithm to predict the spatial pattern of objects smaller than the ground resolution of the sensor by incorporating prior information on the typical spatial arrangement of the particular land cover types into the energy function of Hopfield neural network as a semivariance constraint. Mertens et al. (2003) proposed a method of combining genetic algorithm with the assumption of spatial dependence to assign a location to every subpixel by evaluating all possible configurations of the subpixels inside a pixel according to the parameter, the neighbouring value. These techniques enable the utilization of providing spatial distribution of classes within pixels. However, high resolution images (Atkinson, 1997b), prior information (Tatem et al., 2002, 2003) or vector data (Aplin et al., 1999 and 2001) as auxiliary data need to be collected prior to implementing these methods. Consequently, some of them were time-consuming. For instance, the running time of the approach proposed by Tatem et al. (2003) used on the task of mapping from real Landsat TM agriculture imagery to derive accurate estimates of land cover was approximately 210 minutes and 510 minutes on a PII-350 computer respectively. Therefore, the technique presented in this paper attempts to overcome these problems and to present a novel and effective solution to mapping the spatial distribution classes within pixels. It utilizes the proportions of every endmember component within central pixel and its 8 neighboring pixels, which are from a soft classification assigning pixel fractions to the land cover classes corresponding to the represented area inside a pixel, to achieve

the location of every endmember component within central pixel. The validity is demonstrated by applying it to a controlled simulated artificial image.

### 2. MAPPING BOUNDARIES OF ENDMEMBERS WITHIN MIXED PIXEL

Assume that a mixed pixel  $P_C$ , depicted in Figure 1, contains two endmember components A and B. The area proportion vector of endmembers  $\{a_c, b_c\}$  and that of neighboring pixel  $P_i$  denoted  $\{a_i, b_i\}$  ( $i=0,1,\dots,7$ ) can be estimated by soft classification techniques implemented some commercial software for remote sensing image processing such as RSI ENVI, and ERDAS IMAGINE, etc.

In Figure 1,  $P_C$  composes of four vertexes  $L$ ,  $C$ ,  $F$  and  $I$ , respectively. The whole boundary of  $P_C$  is divided equally into eight parts in length. The divided line segments are  $AB$ ,  $BCD$ ,  $DE$ ,  $EFG$ ,  $GH$ ,  $HIJ$ ,  $JK$  and  $KLA$ , respectively, where  $AB=2BC$ ,  $LC=2AB$ , etc. Each of surrounding pixels corresponds to a one-eighth line segment. For example,  $P_0$  corresponds to  $AB$  and  $P_1$  corresponds to  $BCD$ . Similarly, the rest surrounding pixels correspond to the rest line segments successively.  $MON$  and  $POQ$  described in Figure 1, are two line segments intersecting perpendicularly at the center  $O$  and are equal in length which is equal to  $AB$ . The basic idea of this method, based on the assumption of spatial dependence within and between pixels, is to determine the boundary of endmember A (B) within  $P_C$  by use of the values of  $a_c$  ( $b_c$ ),  $a_i$  ( $b_i$ ), eight line segments and  $MON$  and  $POQ$ . In next subsection, we will explain how it works step by step. Without loss of generality, we discuss the cases:  $AB$ ,  $MON$  and  $POQ$ .

#### (A) Determining the length of $V_{AB}$ in $AB$

Defined a line segment  $V_{AB}$  in  $AB$ , its length and location in  $AB$  is used to account for the contribution of the endmember A of neighboring pixels  $P_0$ ,  $P_1$ ,  $P_7$  to the boundary of the endmember A within  $P_C$ . The length of  $V_{AB}$  is less than or

equal to the length of  $AB$ . Consequently,  $AB$  is also divided equally into eight parts, and the dividing points are denoted as  $T_3, T_2, T_1, T, T_4, T_5, T_6$ , as shown in Figure 2. There is three situations below to determine the length of  $V_{AB}$ : (I) If  $a_0 = 1$ , then  $V_{AB}$  equals to the length of  $AB$ . (II) If  $a_0 = 0$ , then  $V_{AB} = 0$ . (III) If  $0 < a_0 < 1$ , then  $V_{AB} = \frac{\lceil a_0 / 0.125 \rceil}{8} \cdot |AB|$ , where  $\lceil x \rceil$  is the rounded number of real number  $x$ .

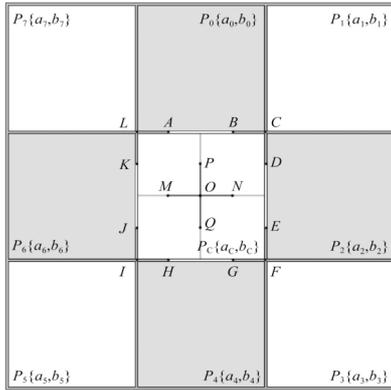


Figure 1. Central pixel  $P_C$ , neighboring pixel  $P_i$  and the inside and boundary of the pixel  $P_C$

**(B) Determining the location of  $V_{AB}$  in  $AB$**

After obtaining the length value  $V_{AB}$  in  $AB$ , we then determine the location of  $V_{AB}$  in  $AB$ . The determining process includes two steps.

**Step 1: Initial position  $V_{AB}^{(0)}$ .**

Starting with the mid-point  $T$  of the line segment  $AB$ , one can take dividing points from  $AB$  on the both sides of  $T$  till the length of all divided parts equals to  $V_{AB}$ . As illustrations in figure 2,  $T_1T_4$  was marked as 0.25 on the line segment  $AB$ , i.e., when  $a_0 = 0.25$ ,  $V_{AB}^{(0)} = |T_1T_4|$ . Similarly, if  $a_0 = 0.50$ ,  $V_{AB}^{(0)} = |T_2T_5|$  and if  $a_0 = 0.75$ ,  $V_{AB}^{(0)} = |T_3T_6|$ .

**Step 2: Calculating position offset  $\Delta_{AB}$ .**

If the numbers of dividing points on the both sides of  $T$  are not equal, an offset arises, that is we need to adjust the position of  $V_{AB}$  according to the comparison of values of  $a_1$  and  $a_7$ . For instance, when  $a_0 = 0.375$ ,  $V_{AB}^{(0)} = |T_1T_5|$  if  $a_1$  is greater than  $a_7$ , otherwise  $V_{AB}^{(0)} = |T_2T_4|$ . The offset can be calculated below

$$\Delta_{AB} = \begin{cases} [|a_1 - a_7| \cdot (1 - a_0) / 0.25], & 0 < a_0 \leq 1 \\ 0, & a_0 = 0 \end{cases} \quad (1)$$

In equation (1), the unit of offset  $\Delta_{AB}$  is  $\frac{|AB|}{8}$ , which refers to a one-eighth length of the line segment  $AB$ . Adding the offset  $\Delta_{AB}$  to  $V_{AB}^{(0)}$ , the final position of  $V_{AB}$  can be obtained.

**(C) Determining the lengths and locations of  $V_{MON}$  in  $MON$  and  $V_{POQ}$  in  $POQ$**

The point  $O$  is the center of pixel  $P_C$ . The line segments  $MON$  and  $POQ$  are perpendicular and their lengths are equal to the half of the edge length of pixel  $P_C$ , and  $OP = OQ$ ,  $OM = ON$ . Dividing the line segments  $PQ$  and  $MN$  into eight equal parts respectively, one can get the equally divided points which are  $V_3, V_2, V_1, O, V_4, V_5, V_6$ , and  $U_3, U_2, U_1, O, U_4, U_5, U_6$  described in Figure 2. In horizontal direction, the length and location of  $V_{MON}$  can be determined by use of  $a_c, a_2$  and  $a_6$ . While in the vertical direction, the length and location of  $V_{POQ}$  can be determined by use of  $a_c, a_0$  and  $a_4$ .

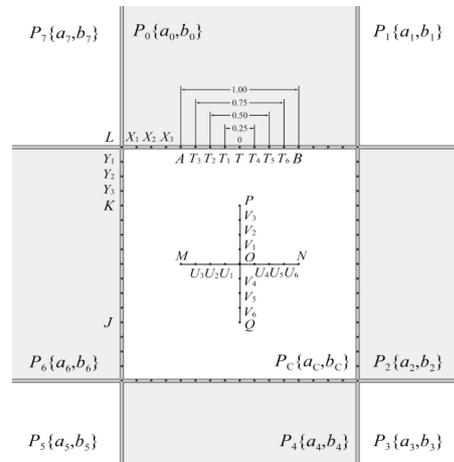


Figure 2. The neighboring pixels and center pixel in boundary line segments of  $P_C$  with area pattern

**(D) Determining the boundary of endmember A in  $P_C$**

After achieving the length and location of line segments in four sides of  $P_C$ ,  $MON$  and  $POQ$ , one connects the vertexes identified by these line segments to form a polygon. The polygon is the boundary of endmember A within  $P_C$ . Repeatedly, we can simulate the spatial distribution of the background endmember B within the pixel  $P_C$  using the values of  $b_i$  and  $b_c$ .

**3. EMPIRICAL ANALYSIS OF ARTIFICIAL IMAGES**

As shown in Figure 3(a), curve  $Z_1Z_2Z_3$  was the real boundary of the endmember A and the background member B between the central pixel  $P_C$  and its neighbouring pixel  $P_i$ . The area of the endmember component A and the background component B in each pixel is:  $P_C\{0.60, 0.40\}$ ,  $P_0\{0.15, 0.85\}$ ,  $P_1\{0.0, 1.0\}$ ,  $P_2\{0.0, 1.0\}$ ,  $P_3\{0.11, 0.89\}$ ,  $P_4\{0.94, 0.06\}$ ,  $P_5\{1.0, 0.0\}$ ,  $P_6\{1.0, 0.0\}$ ,  $P_7\{0.65, 0.35\}$ , respectively.

**Step 1:** First computing the length and location of neighboring pixel  $P_0$  on AB which is the boundary line of  $P_C$ . Given that the area which is belong to A of  $P_0, P_1, P_7$  separately is  $a_0 = 0.15, a_1 = 0.0, a_7 = 0.65$ , so the length of the AB which is the boundary line of A on  $P_C$  is:

$$V_{AB} = \frac{[0.15/0.125]}{8} \cdot |AB| = \frac{[1.2]}{8} \cdot |AB| = \frac{1}{8} \cdot |AB| \quad (2)$$

The initial position of  $V_{AB}$  is  $V_{AB}^{(0)} = |TT_1|$ . The displacement of position is:

$$\Delta_{AB} = [ |0.65 - 0.0| \times (1 - 0.15) / 0.25 ] = [2.21] = 2 \quad (3)$$

The direction of the displacement is the side of the pixel  $P_7$ , then the final position of  $V_{AB}$  is  $V_{AB}^{(1)} = |T_2T_3|$ .

**Step 2:** calculating the length and position of  $P_7$  on the boundary ALK of  $P_C$ . Given the area which is belong to A of  $P_0, P_7, P_6$  separately is  $a_0 = 0.15, a_7 = 0.65, a_6 = 1.0$ , then the length of ALK which is the boundary line of the endmember A on  $P_C$  is:

$$V_{ALK} = \frac{[0.65/0.125]}{8} \cdot |ALK| = \frac{[5.2]}{8} \cdot |ALK| = \frac{5}{8} \cdot |ALK| \quad (4)$$

the initial position of  $V_{ALK}$  is  $V_{ALK}^{(0)} = |Y_3LX_2|$ . The displacement of position is:

$$\Delta_{ALK} = [ |1.0 - 0.15| \times (1 - 0.65) / 0.25 ] = [1.19] = 1 \quad (5)$$

the direction of the displacement is the side of the pixel  $P_6$ , then the final position of  $V_{ALK}$  is  $V_{ALK}^{(1)} = |KLX_1|$ .

**Step 3:** Reckoning the length and position of the central pixel  $P_C$  at the cross line of intersection  $MON, POQ$ . Suppose  $a_c = 0.60, a_6 = 1.0, a_2 = 0.0$ , then the length of  $P_C$  on the line segment  $MON$  is:

$$V_{MON} = \frac{[0.60/0.125]}{8} \cdot |MON| = \frac{[4.8]}{8} \cdot |MON| = \frac{5}{8} \cdot |MON| \quad (6)$$

The initial position of  $V_{MON}$  is:  $V_{MON}^{(0)} = |U_3OU_5|$ . The displacement of position is:

$$\Delta_{MON} = [ |1.0 - 0.0| \times (1 - 0.60) / 0.25 ] = [1.0] = 1 \quad (7)$$

The direction of the displacement is the side of the pixel  $P_6$ , then the final position of  $V_{MON}$  is  $V_{MON}^{(1)} = |MOU_4|$ .

For the same principle, if  $a_c = 0.60, a_0 = 0.15, a_4 = 0.94$ , then we can calculate the length of  $P_C$  on the line segment  $POQ$ :

$$V_{POQ} = \frac{[0.60/0.125]}{8} \cdot |POQ| = \frac{[4.8]}{8} \cdot |POQ| = \frac{5}{8} \cdot |POQ| \quad (8)$$

the initial position of  $V_{POQ}$  is  $V_{POQ}^{(0)} = |V_2OV_6|$ . The displacement of position is:

$$\Delta_{POQ} = [ |0.15 - 0.94| \times (1 - 0.60) / 0.25 ] = [1.264] = 1 \quad (9)$$

The direction of the displacement is the side of the pixel  $P_4$ , then the final position of  $V_{POQ}$  is  $V_{POQ}^{(1)} = |V_1OQ|$ .

**Step 4:** Using the pixel's internal geographical object boundary value rule iteratively, finally we can obtain the spatial distribution's simulated result of the endmember A's pixel  $P_C$ , as is shown in Figure 3(b). The pixel  $P_C$ 's boundary and internal value of the neighboring pixel  $P_i$  and central pixel  $P_C$  is shown as white line segment in Figure 3(b). The value of  $P_0$  at the boundary of pixel  $P_C$  is  $T_3T_2$ , the value of  $P_1, P_2$  is null,  $P_3$ 's value is line segment  $G_1G$ , the value of  $P_4$  is line segment  $HG$ ,  $P_5$ 's value is the curve  $HIJ$ ,  $P_6$ 's value is line segment  $JK$ ,  $P_7$ 's value is curve  $KLX_1$ . Pixel  $P_C$ 's value at the horizontal direction is line segment  $MOU_4$ , at the vertical direction is line segment  $V_1OQ$ . The boundary point of the polygon which is determined by the value of these line segment is  $L, T_2, V_1, U_4, G_1, I$ , then we connect these point to form a polygon. A simulated distribution of endmember on pixel  $P_C$  can be obtained.  $P_C$  was divided into  $16 \times 16$  units, the number of the gray unit is 146, which is the simulated distribution of endmember A on the pixel  $P_C$ . From the simulated distribution we can obtain the area of endmember A at the pixel  $P_C$  is  $a_c = 146/256 = 0.57$ , which is very similar to the real value of  $a_c = 0.60$ .

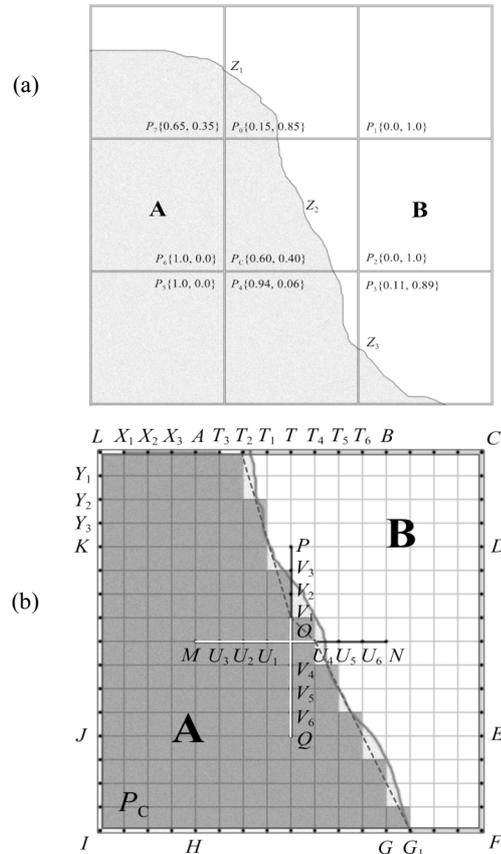


Figure 3. (a) Artificial image with two endmembers A and B; (b) Spatial distribution of endmember A within mixed pixel  $P_C$  by the technique proposed in this paper

#### 4. CONCLUSION AND FUTURE WORKS

In this paper, a new algorithm for sub-pixel boundary mapping is proposed. Two simulated images were used for validating the method. It has been demonstrated that the technique is a simple, robust, and efficient tool for an existing super-resolution target identification technique.

Topics for further investigation include band pattern and point pattern. Furthermore, we have only investigated the spatial distribution of endmember components in simply connected domain of area pattern. It will be essential to investigate the spatial distribution of endmember components in complex connected domain of area pattern.

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