MULTISCALE GEOGRAPHIC INFORMATION WITH MULTIGRAPH OF MULTIGRAPHS

José de Jesús Cruz Guzmán[a], Elzbieta Lewandowicz[b], Zbigniew Oziewicz[a]

[a]Universidad Nacional Autónoma de México, Facultad de Estudios Superiores Cuautitlán. Apartado Postal # 25, C.P. 54714 Cuautitlán Izcalli, Estado de México, México - (cruz,oziewicz)@servidor.unam.mx
[b]Uniwersytet Warmia i Mazury, Faculty of Geodesy and Land Management, Department of Surveying. ulica Jana Heweliusza 12, PL-10-957 Olsztyn, Poland - leela@uwm.edu.pl, www.ela.mapa.net.pl

KEY WORDS: Multigraph, Hierarchization, Peculiarity, Graph of Graphs, Preciseness, Multiscale Information

ABSTRACT:

Multiscale resolution database MRDB include not only several levels of details (ie preciseness), but also several layers of *peculiarity*. The data *preciseness* means the variable levels of details for the fixed set of attributes. Less details leads to data aggregation, more details leads to data de-aggregation. The data *peculiarity* means the growing and reduction of the number of variable attributes for the fixed level of details. More attributes leads to higher hierarchizated data (better, finer structural data trees), less attributes leads to less organized data. We propose to consider the concept of a *multiscale* to be derivable from the more primitive above concepts of preciseness and peculiarity. A mathematical model of multiscale database is proposed, in terms of multigraph theory, with idealized mathematical distinction among the concept of preciseness, and the concept of peculiarity. We relate conceptually the grade of the preciseness with the internal structure of the graph of graphs. Instead, the grade of the peculiarity we wish to relate with the external structure of the *multi*graph (or multi-hyper-graph). Hierarchical data information we propose to encode into external structure of the multigraph, and this goes beyond the usual hierarchical tree. The hierarchical information is illustrated by means of a linking the thematic maps of the same family of objects.

1. INTRODUCTION

The increasingly geographic data information analysis for a variety of users, within multiple disciplines and particular concerns (data providers, service providers, final users, etc.), demands open information integrated technology to make geographic information an integral part of information systems (Kim et al. 2001; Percivall 2002; Reed 2005).

Information integration and services in Geographic Information System needs dynamic geo-spatial data store processes, spatial modeling and analysis features, operation on data sets, and visualization at different detail and peculiarity levels.

Database and visualization of spatial data, based on different technologies, becomes a challenge when users requires considerable amount of cartographic information from geographic phenomenon represented as the user-independent data sets. Visualization at different scales and object representation of real world information, represented by models builded from spaced sample points on the surface, requires nobel techniques for transformation of map resolution or map concern to support rapidly expanding spectrum of Geographic Information System user needs.

In Geographic Information System graph theory is useful for data organization, modeling, analysis and visualizations. Graphs provide description of systems and analysis which displays relationships between various operations, processes, organizations, etc. Graphs underline category theory which provide precise mathematical terms, morphism, functor, natural transformation, etc.

This paper concern primarily the two concepts of scale issues of geographic information and spatial data handling. Namely, the data preciseness and the data peculiarity. In the present paper we define these concepts in terms of the variable set of attributes and variable sets of designated values of these attributes. This is how the users see the database DB by means of attributes and their values. It is convenient to visualize schematically this as the data-free fiber bundle, where the base set of a bundle is a set of attributes, and to each attribute is associated a fiber of a designated values, and we do not require that fibers must have the same cardinality. Figure 1 illustrate an example of four attributes with four fibers of their values.

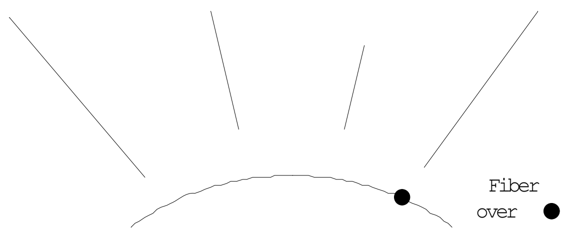

Figure 1. Base set is a set of attributes. Each fiber is a set of values of his attribute

This fiber bundle structure is more fundamental that of cartesian product of two sets. The cartesian product is a pair of fiber bundles, with two projections, and moreover the cardinality of each fiber, in cartesian product, must be exactly the same.

1.1 Definition (Preciseness)

The data *preciseness* means the variable levels of details for the fixed set of attributes. The change of preciseness is identified with the change of the cardinalities of fibers. Less details leads to data aggregation, more details leads to data de-aggregation.

1.2 Definition (Peculiarity)

The data *peculiarity* means the growing and reduction of the number of variable attributes, without the change of the fibers, ie for the fixed level of details. More attributes leads to higher hierarchizated data (better, finer structural data trees), less attributes leads to less organized data.

The multiscale database is derivable from the more primitive concepts of preciseness and peculiarity.

In this paper we propose the mathematical model of multiscale database, in terms of multigraph theory, with idealized mathematical distinction among the concept of preciseness, and the concept of peculiarity. We relate conceptually the grade of the preciseness with the internal structure of the graph of graphs. Instead, the grade of the peculiarity we wish relate with the external structure of the *multi*graph (or multi-hyper-graph).

An element of the hierarchical system is said to be a module. Hierarchical system is usually described by refinement of some modules, adding an extra structure to some modules. This way of thinking about hierarchical system as a disjoint union of its modules leads to consideration of any part of the system, part of the data, on another abstraction level. This would make impossible to consider at the same time, parallel, properties of parts of system being of different levels of abstraction that could be comparable by means of the hierarchical tree order (Cruz et al., 2003).

Graph based models of hierarchical systems seen as 'graph equipped with some refinements', weights etc., do not allow to consider jointly data and phenomena happened on different levels of the system. We propose a *multi*-graph formalism allowing to consider a hierarchical data system as a garland of hierarchical trees, with all levels parallel `at the same time', and all levels of abstractions in exactly the same way.

2. GRAPH

The most primitive concept is a set V, a set of vertices without edges among different vertices. Such graph with the empty set of edges is said to be 0-graph. A graph G that consists of collection of objects V = G, and not empty collection of arrows (among different objects) E = G, is said to be 1-graph. More familiar terminology: an object in V is called a vertex of the graph G and accordingly is visualized geometrically as zero-dimensional point. An arrow (edge) in E is visualized geometrically as one-dimensional directed segment of some line or curve. A *type* consists of two operations: domain/source $s{:}G_1 \to G_0$ and codomain/target $t{:}G_1 \to G_0$,

$$V = G_0 \underset{tarjet}{\overset{source}{\leftleftarrows}} G_1 = E \qquad (1)$$

A directed graph can be seen as the parallel arrows, $V \Leftarrow E$, or as the source and the target maps $s, t : E \to V$, or as a type map

$$V \times V \xleftarrow{type} E \qquad (2)$$

The geometrical terminology, point$\in G_0$, line$\in$G1, surface$\in G_2$, volume$\in G_3$, …is *not* always appropriate when applied for particular geographic data. For example when considering gas and liquid pipeline, or other similar transmissions, it is more natural to consider segment$\in G_0$ and connecting points$\in G_1$. Analogously, a two-dimensional geographic surface need not to be considered necessarily always as an element of G_1 that is connecting some geographic one-dimensional road data in G_0 but could be beneficial to consider every municipality surface, council surface, town surface, as an element of G_0, and each municipal boarder as the connecting morphism in G_1.

These examples shows that not appropriate terminology, like vertex, point, edge, could be obstacle for graph based modeling geographical *data*. More adequate terminology is taken from biology, 0-cell $\in$ V and 1-cell $\in$ E (Burroni, 1981), because the biological `cells' terminology, allows free flexibility to treat any cell as a point, or as an arrow or as a surface, etc, as would be more convenient in many applications. Alternatively, the object of G_0 is said to be 0-morphism, an arrow in G_1 is said to be 1-morphisms.

For a cell $f \in G$, the value sf is said to be a domain or a source of f, and tf is a co-domain or target of f. Arrows/1-cells can be weighted, and/or attribute-dependent. In some situations it is suitable to consider a globular graph, ie a graph consisting of arrows only. We will not consider this in what follows and refer to (Cruz et al., 2003). The directed graph underlie the theory of categories. A category is a directed graph with additional structure. Pre-category and *diagram scheme* or just a *scheme* are just other new names for the old and more familiar *directed graph*.

3. MULTIGRAPH

3.1 Definition

A multigraph G, or an n-graph, is a collection of families $\{G_i\}$ of i-cells for which there exists $n \in N$ such that $\forall\ i \in N$, $G_{i+n} \approx G_n$, and such that $G_i \Leftarrow G_{i+1}$ is a 1-graph with source $s{:}G_{i+1} \to G_i$ and target $t{:}G_{i+1} \to G_i$, (Burroni, 1981),

$$G_0 \underset{t}{\overset{s}{\leftleftarrows}} G_1 \underset{t}{\overset{s}{\leftleftarrows}} G_2 \cdots G_{n-1} \underset{t}{\overset{s}{\leftleftarrows}} G_n \qquad (3)$$

A multi-graph generalize the notion of the directed graph ($\equiv$ one-graph as diagram of arrows). Instead of considering only two levels of abstraction, nodes and links only, vertices and edges, states and transitions, and many geographic information networks, we can treat a hierarchical system (Section 7) as a sequence of nested three or more families of elements, hyper-links, called in these context cells, connected by two operations having the same properties as the well known operations of the source and the target in a directed one-graph. The structure thus obtained, diagram of oriented surfaces, diagram of oriented volumes and beyond, is called directed (or oriented) multi-graph or more precisely a directed n-graph for $n \in N$.

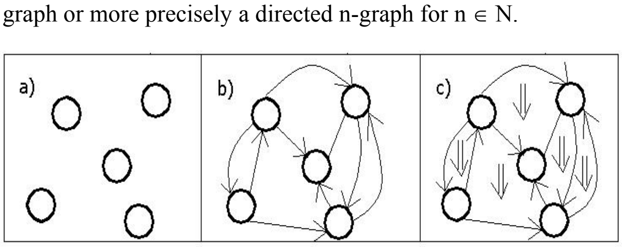

Figure 2. a) 0-graph, b) 1-graph, c) 2-graph

A multigraph is N-graded, or better N-filtered, N-nested, n-graph G, with a sequence of type maps (orientation), {type$\equiv$source,target}, $G_{i+1} \to G_i \times G_i$ for $i \in N$, like, Edges $\to$

Vertices × Vertices. Another generalization of the graph is a hyper-graph. In hyper-graph a type is generalized to assignment,

$$Edges \to 2^{Vertices} \quad and \quad G_{i+1} \to 2^{G_i} \tag{4}$$

The origin of the concept of the higher-dimensional graph must be seen in the enriched category theory. The last Chapter XII on structures in categories of the Mac Lane's monograph (1998) is the elementary survey on higher-dimensional categories. The concept of the directed multi-graph (n-graph) was introduced by Burroni (1981). Our aim is a conceptual applications of multi-graph to multi-scale geographic information.

4. GRAPH OF GRAPHS

Another generalization of the concepts of 0-graph, V, and of 1-graph, $V \Leftarrow E$, is specification of *interior* structure of V (agregation of more details): this leads to the concept of graph of graphs, and, afterwards, to multigraph of multigraphs.

4.1 Definition (Graph of graphs)

A graph $\{V \Leftarrow E\}$ is said to be a *graph of graphs*, if every node of V is an 1-graph. Hence every `edge' in E must be morphism of 1-graphs (see the next Definition). A 1-graph $\{V \Leftarrow E\}$ of 1-graphs is denoted by (1,1)-graph.

4.2 Definition (Graph morphism)

A k-graph morphism $f:E \to F$, $f \in Graph(E,F)$, with $E \equiv (E_0 \Leftarrow E_1 \Leftarrow \ldots \Leftarrow E_k)$ and $F \equiv (F_0 \Leftarrow F_1 \Leftarrow \ldots \Leftarrow F_k)$ be two k-graphs, is a collection of functions $f_i:E_i \to F_i$, (Cruz et al. 2004),

$$\begin{aligned} s \circ f &= f \circ s, \\ t \circ f &= f \circ t \end{aligned} \tag{5}$$

$$\begin{array}{ccccc} E_0 & \overset{\leftarrow}{\underset{\leftarrow}{}} & E_1 & \overset{\leftarrow}{\underset{\leftarrow}{}} & E_2 \cdots \\ \downarrow f_0 & & \downarrow f_1 & & \downarrow f_2 \\ F_1 & \overset{\leftarrow}{\underset{\leftarrow}{}} & F_1 & \overset{\leftarrow}{\underset{\leftarrow}{}} & F_2 \cdots \end{array} \tag{6}$$

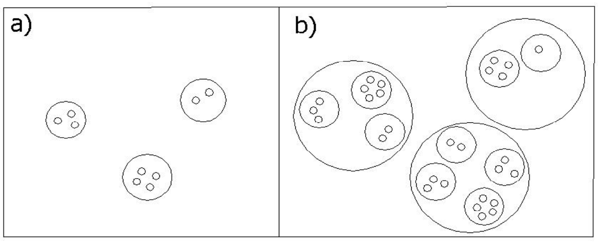

Figure 3. a) (0,0)-graph , b) (0,0,0)-graph

The above concept of the graph of graphs, has the easy generalization for all natural numbers, k, m, n ∈ N.

Consider k-graphs E, F ∈ G_0 to be 0-cells of the graph G Hence graph(E,F) ⊂ G_1. Then $G \equiv (G_0 \Leftarrow G_1)$ is a 1-graph of k-graphs, and this we denote by (1,k)-graph. This means that every 0-cell of G is a k-graph and every 1-cell of (1,k)-graph is a k-graph morphisms, $f \in G_0(E,F) \equiv graph(E,F) \subset G_1$.

Let 1-cell $f \in G_0(E,F) \subset G_1$, be a k-graph morphisms, k-pre-functor. Hence every 2-cell $t \in G_1(f,g) \subset G_2$, where $f, g \in G_0(E,F)$, is k-graph natural transformation,

$$G_0(E,F)(f,g) \subset G_1(f,g) \subset G_2 \tag{7}$$

Analogously, every 3-cell is a k-graph modification, every 4-cell is a k-graph modifications of modifications, etc. Then an m-graph G, where every 0-cell is an k-graph, is said to be a multigraph of multigraphs, and is denoted by (m,k)-graph.

Let G be an m-graph such that every 0-cell is a n-graph of k-graphs. Then G is said to be a (m,n,k)-graph. Every 1-cell in G_1 must be a morphism (≈(n,k)-pre-functor) of (n,k)-graphs. By iteration we define more and more subtle structure, namely an $(n_1,\ldots,n_k)$-graph.

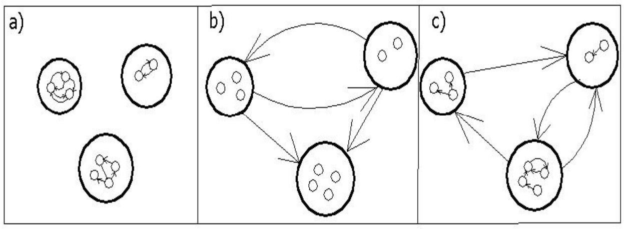

Figure 4. a) (01)-graph, b) (1,0)-graph, c) (1,1)-graph

5. DATABASE AS THE SECTIONS OF FIBRE BUNDLE

The users of the databases DB see them in terms of attributes and designated values of these attributes. It is convenient to visualize this as the data-free `fiber bundle', where the base set of a bundle is a set of attributes, and to each attribute is associated a fiber of chosen values (identified with level of details), as it is illustrated on Figure 1,

$$Bundle = \{Atributes \leftarrow Values \quad of \quad Atributes\}. \tag{8}$$

Hence, every one concrete data is a selection of one value for each attribute, ie a section of this fiber bundle,

$$Attributes \xrightarrow{one \quad section} \begin{matrix} values \quad off \\ Attributes \end{matrix} \tag{9}$$

DB ⊂ section of bundle

The number of attributes is not fixed, and each user is interested in another variable set of attributes. Similarly, the cardinality of each fiber is variable because different users are interested in different data preciseness.

5.1 Multiscale databases

Multiple users view the information system from their own perspective. Geographic Information System provide general and specialized services that can be used via client application that needs application processes on spatial data of geographic information.

Multiscale databases (DB), as it is presently understood, it is a database with several levels of resolution, multiresolution MRDB, multi-representation (Anders and Bobrich 2004; Hampe, Anders and Sester, 2003; Hampe, Sester and Harrie, 2004; Bobzien, Burghardt and Petzold, 2005; Gotlib et al., 2004-2005). Such multiscale database MRDB include not only several levels of details (ie preciseness), but jointly also several layers of peculiarity.

We propose a mathematical model of multiscale database in terms of graph theory with mathematical idealized distinction among the concept preciseness and the concept peculiarity. We derive the concept of a multi-scale from elementary *preciseness* (levels of details that can be related to aggregation and de-aggregation processes), and from elementary *peculiarity*, that we wish to relate with the hierarchization of information and de-hierarchization processes.

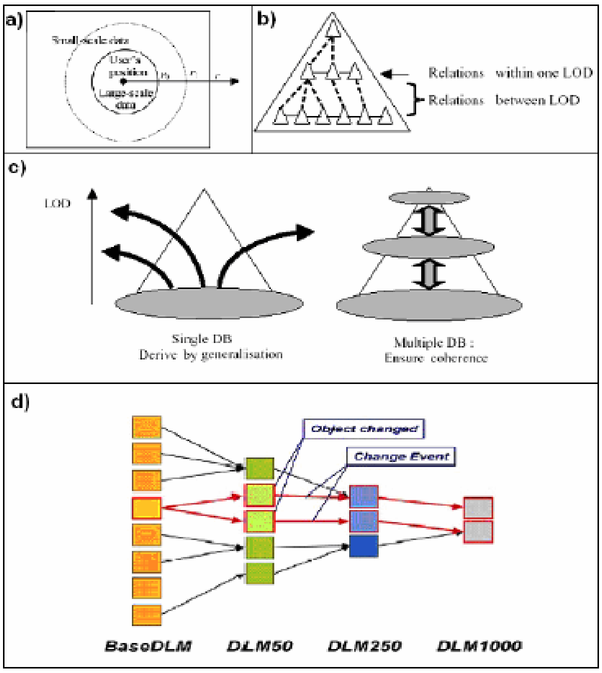

Figure 4. Models for variable-scale maps a) Harrie et al., 2002, b) Ruas, 2002, c) Ruas, 2002 d) Anders and Bobrich 2004

6. DETAILIZATION: SCALE MAPS AS GRAPH OF GRAPHS

Spatial representation of geographic information at different resolution levels can be approached in terms of an internal structure of (multi)graph of multigraphs.

One can ask how the internal concept of the graph of graph could be related with many different geographic operations of data visualization (spatial data transformations), like simplification, collapse, amalgamation, refinement, smoothing, aggregation, merge (Longley et al., 2001). Is our hypothesis that most of these spacial data transformations can be understand in terms of the preciseness, ie in terms of the levels of details, without changing the number of attributes. This means that we guess that the source of most of these data transformations, are variable fibers of attribute values, see Figure 1. In contrast, the following data transformations, enhancement, exaggeration and displacement, can not be related to the concept of preciseness.

Developing GIS application satisfying the various user requirements needs collections of the essential and optional functions as in the Multiple Representation/Resolution Database MRDB. Such model store the structural layer of multiple representations and links for corresponding objects between levels. Based on definition of map union/products and their associated map scales, the objects in an hierarchical layers can be considered to be in some exterior i-graph, and the objects layer in a scale change can be in interior j-graph. Applying integrity rules and spatial relationships to the process going from the crude small scale i to the larger scale (i,j) can be described like going from i-graph to (i,j)-graph, where (i,j)-graph is short for i-graph of j-graphs.

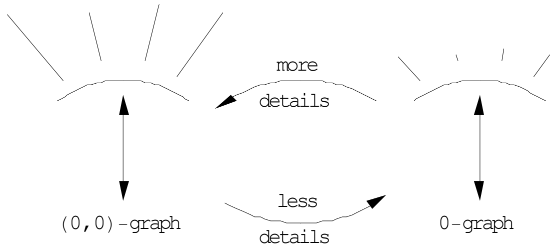

Figure 5. Precisness = graph of graphs

7. HIERARCHY TREE FROM GRAPH

What is a process of hierarchization of information? Not hierarchical data can be seen as a 0-graph. A family of objects, data, V, is said to be organized hierarchically, if each objects is a vertex of a directed *tree* diagram, expressing the dependencies among members of this family. Clearly not every graph is a tree.

The user with his given set of attributes (keeping the fixed level of details), associate to the DB as the set of sections, hierarchical tree,

$$DB \xrightarrow[attributes]{user} \left\{ \begin{matrix} Hierarchical \\ tree \end{matrix} \right\} \xleftarrow[number]{in\ \&\ out} Graph \qquad (10)$$

We will explain first how every 1-graph determine hierarchy on the set of 0-cells. The association of the hierarchical tree-diagram is according to the joint # of in-arrows, incoming to the given vertex, and # of out-arrows, out-going from that vertex. More relations have the given 0-cell, more important it is in hierarchical structure of the data. This means that arrows in G_1 describe the simplest hierarchical organization of objects in G_0. A family of 1-cells G_1 introduce the linking (hierarchy) among data G_0, and a process going from 0-graph G_0 to 1-graph $\{G_0 \Leftarrow G_1\}$ can be thought as the first data hierarchization.

The simplest hierarchical structure provided by 1-graph possess the natural exterior hierarchical extension to multi-graph, where 1-morphisms are organized hierarchically in terms of 2-morphisms etc.

Higher hierarchical data information we propose to encode into external structure of the multigraph, and this goes beyond the usual hierarchical tree. For example 2-graph $G \equiv \{G_0 \Leftarrow G_1 \Leftarrow G_2\}$, gives the hierarchical tree for elements of G_0, and this tree in completely encoded into G_1. However the set of 2-cells, define the hierarchical organization of all 1-cells. Therefore, each 2-graph define a pair of hierarchical trees, and each n-graph gives a garland of (n-1) hierarchies,

$$
\begin{aligned}
0-graph &\Rightarrow \quad empty \quad hierarchy, \\
1-graph &\Rightarrow \quad hierarchy \quad for \quad 0-cells \\
2-graph &\Rightarrow \begin{cases} hierarchy \quad for \quad 0-cells \\ \qquad and \\ hierarchy \quad for \quad 1-cells \end{cases}
\end{aligned} \tag{11}
$$

We will refer to garland of n hierarchical trees, as to n-hierarchy.

Generalizing, we say that an exterior extension from n-graph to (n+1)-graph, is said to be hierarchization, and the inverse forgetful process from (n+1)-graph to n-graph is said to be de-hierarchization (Cruz et al. 2003 and 2006). Therefore the concurrence in hierarchical system is modeled in terms of multi-graph.
Every 0-cell can be visualized as a section of a bundle over the set of attributes. We think of hierarchization as adding more independent (essential) attributes,

$$
\begin{aligned}
&DB \xrightarrow[number\ of\ attributes]{user\ with\ less} \{\text{n-hierarchy}\} \Leftarrow \text{n-graph} \\
&DB \xrightarrow[attributes]{user\ with\ more} \{\text{(n+1)-hierarchy}\} \Leftarrow \text{(n+1)-graph}
\end{aligned} \tag{12}
$$

Architectural concepts can be related in a hierarchy.

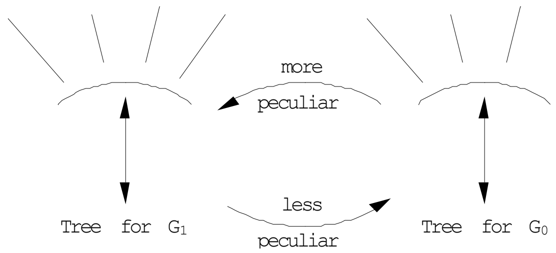

Figure 6. Change of number of attributes

8. THEMATIC MAPS

Graph theory provide mathematical approach for description and analysis of geographic service. Here we focus on visualization of thematic maps and different scale maps.

The hierarchization could be illustrated by means of a linking the thematic maps of the same family of objects. Consider two thematic maps $\{E_0 \Leftarrow E_1\}$ and $\{F_0 \Leftarrow F_1\}$, as 1-graph $\{(E_0 \approx F_0) \Leftarrow (E_1 \cup F_1)\}$. Then, every linking f among these themes must be considered as the graph morphism (Cruz et al., 2004), ie as a pre-functor $f \in graph(E,F) \approx E_2$. Therefore the linking among thematic maps can be thought as a hierarchization from 1-graph to 2-graph, $1 \mapsto 2$, or alternatively, $(0,1) \rightarrow (1,1)$.

Numerous custom applications have been developed for managing and analyzing the geographic data. Geographic Information Systems share geographic layers of information out of which more complex components are built. Such systems share many components for inter-operability implementation as in (Denzer et al., 2005). The efforts of standardization are motivating architectural definition whether for specification or for design (Turner, 1997).

Pipeline companies organize features, or grouping features, according to a hierarchy. Typically hierarchy is based on where particular station series features are located. There is no standard hierarchy structure that pipeline companies adhere to other than some kind of hierarchy present in most pipeline systems. For example APDM is a geodatabase model that was developed for implementing transmission (gas and liquid) pipelines. In the ArcGis Pipeline Data Model (ESRI, 2004),

"... a typical hierarchy will place a station series feature belonging to a single line. Each line will belong to a single pipeline system. Many pipeline systems will belong to a pipeline company. Even a simple hierarchy can be broken down into more complex organizational structures such as discharge subsystems, valve sections, branches, and main lines."

Above hierarchy can be thought from 0-cells in G_0, to 1-cells in $\{G_0 \Leftarrow G_1\}$. A linking process between thematic layers is a graph morphism, and it is a 2-cell in G_2. The new higher objects/cells can be linked via new hierarchization process in terms of higher cells, and then a new cells enter into the game.
Generalizing, a thematic map looks like $\{G_0 \Leftarrow G_1 \Leftarrow G_2 \ \dots \Leftarrow G_n\}$

REFERENCES

Anders Karl-Heinrich and J. Bobrich, 2004. MRDB approach for automatic incremental update. http://ica.ign.fr/Leicester/ paper/ Anders-v1-ICAWorkshop.pdf.

Burroni Albert, 1981. Algèbres graphiques. Cahier.

Benenson Itzhak, Michael Sofer and Izhak Schnell, 1998. Geographical information systems as a visualization tool, Applied Geography, 18 No. 2 pages 117–-135.

Bobzien Matthias, Dirk Burghardt and Ingo Petzold, 2005. Re-generation and construction - Two alternative approaches to automated incremental updating in MRDB, XXII International Cartographic Conference (ICC2005), A Coruña, Spain, ISBN: 0-958-46093-0.

Cruz Guzmán, José de Jesús, Waldemar Korczynski and Zbigniew Oziewicz, 2003. Multilevel system as multigraph. Lecture Notes in Computer Science (Springer-Verlag, Berlin Heidelberg New York) 2658 pp. 832-840.

Cruz Guzmán, José de Jesús and Zbigniew Oziewicz, Morphism of Network of Networks, 2004. Lecture Notes in Computer Science 3037 pp. 602-605.

Cruz Guzmán, José de Jesús, El.zbieta Lewandowicz and Zbigniew Oziewicz, 2006. Multigraphs of multigraphs model for structural data, American Mathematical Society 2006-Spring-Southeastern Meeting # 1015, Florida International University. Session on Mathematical Models in Image and High-Dimensional Data Analysis, abstract number 1015-18-249.

Denzer Ralf, Carlo Riparbelli, Matteo Villa, and Reiner Güttler, 2005. GIMMI: Geographic Information and Mathematical Models Inter-operability, Environmental Modelling & Software, Volume 20 Issue 12, December 2005, Pages 1478-1485, Environmental Knowledge and Information Systems.

ESRI, 2004. ArcGIS Pipeline Data Model, An ESRI Technical Paper J9137, September 2004.

Gotlib Dariusz, M. Lebiecki and R. Olszewski, 2004. Investigating possibilities to develop the BDT in Poland as MRDB type database, ICA Workshop on Generalization and Multiple Representation, Leicester 2004.

Gotlib Dariusz, A. Iwaniak and R. Olszewski, 2005. Jedna referencyjna baza danych topograficznych, czy to mo.zliwe?, Geodeta 1 (January 2005) p. 116.

Gotlib Dariusz, A. Iwaniak and R. Olszewski, 2005. SDI in Poland - concept of topographic reference system for thematic, harmonized databases, XXII International Cartographic Conference (ICC2005), A Coruña, Spain, ISBN: 0-958-46093-0.

Hampe Mark, Karl-Heinrich Anders and Monika Sester, 2003. MRDB applications for data revision and real-time generalization.

Hampe Mark, Monika Sester, L. Harrie, 2004. Multiscale Databases to Support Visualisation on Mobile Devices, http://www.lantm.lth.se/personal/lars/,ISPRS_2004.pdf.

Harrie, L., Sarjakoski, L., T., Lehto, L., 2002. A Mapping Function for Variable of Geospatial Engineering, Vol. 2 No. 3, pp. 111-123.

Kim Do-Hyun, Kwang-Soo Kim, Haeock Choi, and Jong-Hun Lee, 2001. The Design and Implementation of Open GIS Service Component, IEEE, pp. 1922-1924.

Longley P., M. Goodchild, D. Maguire, D. Rhind, 2001. Geographic Information Systems and Science, John Wiley & Sons.

Mac Lane Saunders, 1971, 1998. Categories for the Working Mathematician. Springer-Verlag.

Percivall George, Editor, 2002. The Open GIS Abstract Specification Topic 12: OpenGIS Service Architecture, Version 4.3, OpenGIS Project Document Number 02-112.

Reed Carl, Editor, 2005. The OGC, Abstract Specification Topic 0: Abstract Specification Overview, Version 5, Open Geospatial Consortium, Project Document Number 04-084.

Ruas, A., 2002. MRDB and Generalization. http://www.ikg.uni-hannover.de/isprs/ppt-files/MRDB-gene.pdf

Turner K. J., 1997. Specification architecture illustrated in a communications context, Computer Networks and ISDN Systems 29 347-411.

ACKNOWLEDGEMENTS

The International Archives of the Photogrammetry, Remote Sensing and Spatial Information Sciences. The 12th International Symposium on Spatial Data Handling, ISPRS TC II Symposium, July 10-12, 2006, University of Vienna, Austria.

Authors are indebted to both anonymous Referees for many very valuable questions and suggestions to the first draft. Both Referee correctly ask, among other, for the *practical* use of multi-scale databases in the framework of multi-graph theory, whereas the present Authors still have a lot of conceptual fundamental problems to resolve. Evidently, the present paper does not close this subject, and Authors expect further advices and help from practicing specialist in spatial data handling.

Supported by el Consejo Nacional de Ciencia y Tecnología (CONACyT) de México, grant # 41214-F.

Supported by Tecnologías para la Universidad de la Información y la Computación. Universidad Nacional Autónoma de México (UNAM).

Zbigniew Oziewicz is a member of Sistema Nacional de Investigadores in México, Expediente # 15337.