

ADVANCES IN THE APPLICATION OF MATHEMATICAL MORPHOLOGY IN SPATIAL DATA PROCESSING AND ANALYSIS

HU Zhuowei ^{a,b,*}, GONG Huili ^{a,b}, LI Xiaojuan ^{a,c}, ZHAO Wenji ^{a,c}

^a College of Resources Environment and Tourism, Capital Normal University, #105 West Sanhuan North RD., Beijing, 100037, China – forevergis@hotmail.com, gonghl@263.net, xiaojuanli@vip.sina.com, zhwenj1215@163.com

^b Beijing Key Lab of Resources Environment and GIS, #105 West Sanhuan North RD., Beijing, 100037, China

^c Key Lab of 3D Information Acquisition and Application, MOE, #105 West Sanhuan North RD., Beijing, 100037, China

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ABSTRACT:

Mathematical morphology originated in 1960s. It uses a structuring element to measure and extract corresponding objects in images. The purpose is to analyze and identify these images. Mathematical morphology can be widely applied in geoscience because not only the research object but also the data format in geoscience research is so consistent with mathematical morphology. This article sets out from the theory of mathematical morphology and probes the cardinal application in geoscience. Aiming at the complexity of earth observation data, we discussed the suitability of mathematical morphology in geoscience research. The prospect of application of mathematical morphology in geoscience was expected.

1. INTRODUCTION

Mathematical morphology comes from the research on the geometry of penetrative material, such as sandstones. This type of material consists of two phases: the holes embedded in main body. It is binary. In 1967, Matheron and Serra introduced a set theory that can be used to analysis binary images. In fact, we can define the main body as 1 in binary set and the hole as 0. They are complement of each other. Matheron and Serra defined a set having special shape and size and then moved this set on the binary image. The special set likes a probe and can be used to extract objects having special structure. The name of this special set is Structuring Element (SE). The idea is of checking if the object making up of pixels overlapped by SE is fit in with special structure. The object fitting in with special structure can be extracted. In this point of view, morphological operations are based on the set operation, such as intersection, union, complementation and translation and so on (Soille, 2002; Matheron, 1967; Serra, 1982).

The basic theory of mathematical morphology can be found in references (Soille, 2002; Matheron, 1967; Serra, 1982; TANG, 1990; Serra, 1986; Sternberg, 1986; GONG, 1997).

Because mathematical morphology has characteristics of parallel computing, easily implementation, much attentions have been put on it since its emergence. During these decades, mathematical morphology developed vigorously. Firstly, mathematical morphology was extended to the field of gray image processing after set operations were applied to gray images. For example, in set operations of gray images, the intersection is defines as choosing minimum value pixel by pixel. Based on these basic set operations, special objects extracting can be implemented by translating SE on gray images.

Except for binary mathematical morphology and grayscale mathematical morphology, mathematical morphology still has lots of branches, such as soft mathematical morphology, fuzzy

mathematical morphology and soft fuzzy mathematical morphology (DAI, 2001). Mathematical morphology is mainly applied to digital images and the processing and analysis of computer raster data. It has had a widely and efficiently application in computer digital image processing, medical image processing and analysis, geoscience earth-observation data processing and analysis and so on.

Geoscience takes spatial objects as research objects. It acquires the position, shape and state of spatial objects, sums up their rule and predicts their development trends. Raster data has a big proportion in geoscience research, such as remote sensing images, digital terrain models (DTMs) and other field data stored in raster form. All the data or computer images can be expressed in the form of set. As a result, whether considering the data format or considering the research objects, mathematical morphology should have a beautiful future in the research field of geoscience. Especially, with the development of spatial data mining and knowledge discovery, mathematical morphology will play a more and more important role in it (DI, 2000).

However, because the requirement in geoscience research is diverse, we must develop and improve continually the theory of mathematical morphology in order to making it more powerful and robust in the field of geoscience data processing and analysis. This article introduces the theory and method of mathematical morphology aiming existing problems in geoscience data processing and analysis by mathematical morphology. The application and evolvement of mathematical morphology in geoscience are studied. In the following part, the future development and application trend of mathematical morphology in geoscience data processing and analysis are discussed.

* Corresponding author.

2. BASIC APPLICATION OF MATHEMATICAL MORPHOLOGY IN SPATIAL DATA PROCESSING

2.1 Edge Detection

Morphological edge detection uses erosion, dilation, open and close operation in mathematical morphology to detect object edge in image according to basic theory of morphology. In this process, one or more appropriate structuring elements will be selected and a compound morphological algorithm will be implemented. Edge detection by erosion uses this equation:

$$A - A \ominus B \quad (1)$$

Edge detection by dilation uses this equation:

$$A \oplus B - A \quad (2)$$

where: A is raw image and B is structuring element.

In this process, the compound mode and structuring element will determine the result. Especially, the choice of structuring element will affect the effect and precision of result (LIN, 2003; RAO, 2004). Having lots of speckle in raw images, speckle-remove and smooth operations must be performed before the execution of any other operations. RAO (2004) proposed to remove speckle by using close-open sequential operation and then smooth image by close operation. After image smoothing, dilation operation was executed. The difference between the image after dilation and image before dilation is image edges. The equation was:

$$E_{A,B} = (M \bullet B) \oplus B - M \oplus B \quad (3)$$

where: $M = (A \bullet B) \circ B$

A is raw image

B is structuring element

Figure 1 is the result of edge detection implemented by this method.

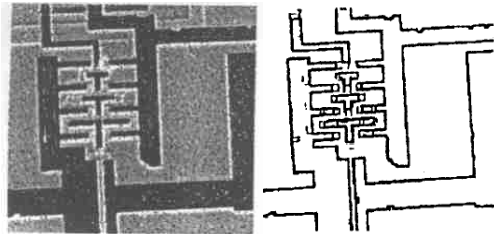


Figure 1. Morphological Edge-detection. Equation (3) was used. A 5*5 structuring element was used.

In spatial data processing, edge-detection can be used to extract object edges and have a great value of application to automatic digital map vectorization.

2.2 Image Segmentation

Image segmentation distinguishes areas with similar properties from raw images and plots them out. Watershed segmentation based on mathematical morphology can overcome shortcomings of conventional image segmentation methods, such as low precision, bad effect and hard treatment. ZHANG (2003) introduced the theory of watershed segmentation. From the theory of watershed segmentation we can deduce the idea of watershed segmentation algorithm: (1) Find out areas with extreme values. (2) Dilate these areas to get catchment basin. (3) Delineate the boundary of catchment basin (dividing lines of watershed). Areas with extreme values can be obtained by transformation from raw images to binary images. However, being speckle exist, over-segmentation will take place. We should use morphological transformation to remove speckle. The method can be used is morphological open reconstruction or morphological close reconstruction. The definitions are respectively:

$$\tilde{\bullet}_n(A) = \bullet^{(rec)}(\bullet_n(A), A) \quad (4)$$

$$\tilde{\circ}_n(A) = \circ^{(rec)}(\circ_n(A), A) \quad (5)$$

where: A is raw image

(rec) is reconstruction operation

n is the size of structuring element in operation

In contrast with simple open (close) operation, open (close) reconstruction can be used not only to remove speckles but also to recover the boundary that doesn't be entirely filtered by open (close) operation. That is to say open or close reconstruction will only cut or fill those peaks or valleys not being entirely covered by constructing element. So, with the open or close reconstruction operation, the geometric information of main objects are kept as well as the images are simplified. Please refer to (VINCENT, 1993; LIU, 2002, LU, 2000) to get more information about the definition and method of morphological reconstruction.

Also, morphological watershed transformation can be implemented on the base of morphological gradient images. Morphological gradient is defined as:

$$MG_{A,B} = A \oplus B - A \ominus B \quad (6)$$

where: A is raw image

B is structuring element

The size of gradient structure (B) has some influence on gradient image result. For fuzzy edges, using structuring element big enough will generate edges as high as slope. But it will result in too wide trapezoid output also. As a result, the edges will be confused. On the contrary, although little structuring element will promise higher result precise, it only generate a very little output. To overcome those two problems, structuring elements with different sizes can be combined together. This method was named multi-scale method (Luo, 2004; Paul, 1996; Salembier, 1994).

2.3 Object Detection

In object detection operation, one or a series of structuring elements was defined according to the shape and size of objects will be detected. Then, special morphological transformation will be implemented on the raw image by using this (these) structuring element(s) to extract objects. For example, in order to detect linear objects whose length is n and having darker background, a series of linear structuring elements with n length and various directions can be defined. Then, morphological close operation can be implemented on the raw image by using those structuring elements. The difference between union of result and raw image are objects we want to get.

Destival (1986) recognized the usability of application mathematical morphology to extract objects with special shapes for the first time, such as roads, rivers and villages. Martel (1989) recognized that morphological methods are selectable consistent analysis methods that can be used to detect typical features on the earth surface. In this paper, the directional morphological filter method was used to detect gravity abnormal. Pesarsei held that resampled morphological gradient had equivalent accuracy with manpower image interpretation method if they are used to extract building covered areas from SPOT-P images (Pesare, 1994). Lea (1993) reported a method used to extract areas of vortices in infrared ocean images. Morphological Top-Hat operation was used to extract domes and deflations in desert panchromatic SPOT images. Chanussot researched the method of extracting roads network from SAR images by using directional filters. Arrighi (1999) studied how to extract contour lines from digital relief maps.

In fact, whether definitions or methods of all those spatial data processing methods based on mathematical morphology intersects and integrates with each other. They are all take morphology as foundation and take functions defined in the earth space as research objects. The complex computation and analysis are achieved by extracting spatial features with special geometries. More basic application and information about mathematical morphology in the field of geoscience research can be found in reference (Soille, 2002).

3. THE NEW APPLICATION OF MATHEMATICAL MORPHOLOGY IN GEOSCIENCE SPATIAL DATA PROCESSING AND ANALYSIS

Raster (such as remote sensing image and digital elevation model) is important data format in geoscience research. It has been widely applied in resources survey, ground objects classification, topographical analysis, objects detection and survey and so on. Because the data related to geoscience is huge and complex, there are often some difficult problems in the processing of geoscience data (Pan, 2004; YANG & HU, 2003). Complex morphological information must be treated before the application of mathematical morphology in spatial data processing and analysis. Following sections will analysis and discuss the feasibility of using mathematical morphology to process complex spatial data from some aspects.

3.1 The Influence of Contrast Between Objects and Background on Morphological Operations

According to the introduction of basic mathematical morphology operations and geometrical meaning, we often consider that research objects are brighter than background. Brightness reflects on pixel value in raster images. Objects with high brightness reflect on high value pixels and backgrounds with low brightness reflect on low value pixels. That is to say

the definition of erosion and dilation are relative. Dilation can be understood as erosion of low brightness pixel set taking high brightness pixel set as background and vice versa. This relativity is not very obvious in ordinary image processing because we often hold that the contrast is coincident in a single image. However, it will become obvious and troublesome if we process complex spatial data (for example, when we apply remote sensing images to perform automatic classification of land use). Taking remote sensing image processing as example, research objects distributing on the whole image have various local contrast with their backgrounds according to the reflectivity of their backgrounds. The result of morphological close operation may replace the result of morphological open operation. It makes the difficulty of data processing and analysis increase greatly.

Duality in morphology explains this phenomenon. In fact, all the basic morphological operations introduced above appear in the dual form. Assuming Ψ and Φ are two operations in mathematical morphology and C is set complementation operation, Ψ is a dual-operation of Φ with respect to complementation C if applying complementation to Ψ is equivalent to applying Φ to the complementation of raw image. The mathematical form is:

$$\Psi(A) = C\Phi C(A) \quad (7)$$

where: A is raw image

In mathematical morphology, dilation and erosion, open and close are all dual operations with respect to complementation. In figure 2 we can find the dependency of morphological open operation on the local contrast between objects and backgrounds (Soille, 2002).

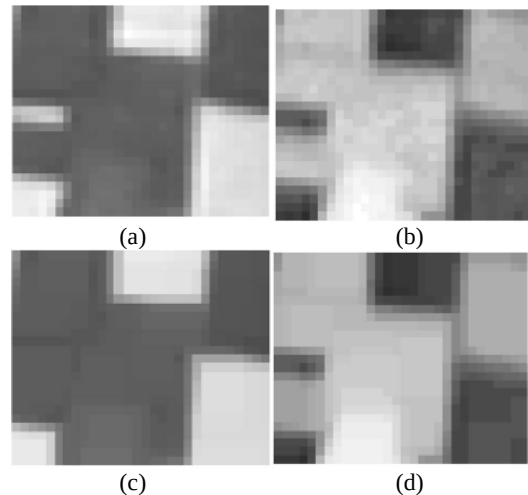


Figure 2. The result of open operation depends on the local contrast. (a) raw image, (b) complementation of (a), (c) open operation of (a), the white strips were removed, (d) open operation of (b), the black strips were not removed.

To resolve the problem of dependence of single operation on image local contrast, its dual operation can be performed on the mid result image after a morphological operation. For example, a close operation can be performed after an open operation. Also, according to the shape and size of processed objects, a

series operation with different structuring elements can be performed. It is morphological alternative filter (Sternberg, 1986; Serra, 1998), which has almost equal processing ability to high brightness objects and low brightness objects. But itself is not self-dual operation yet.

Self-dual operation is a series of operations those don't depend on image local contrast. In mathematical morphology, assuming Ψ is a self-dual operation with respect to complementation C , the equation is:

$$\Psi(A) = C\Psi C(A) \quad (8)$$

where: A is raw image

Self-dual operation can be constructed by applying morphological center operation to a series of dual operations (Meyer, 1989). Assuming $\{\Psi_1, \Psi_2, \dots, \Psi_n\}$ is a set of transformations. The morphological center of this set of operation is defined as:

$$\beta = (id \vee (\bigwedge_i \Psi_i)) \wedge (\bigvee_i \Psi_i) \quad (9)$$

where: id denotes identity transformation,
 $\vee$ is the pointwise maximum operator,
 $\wedge$ is the pointwise minimum operator

Morphological center operation will become middle value calculation if n equals to 2. If these two operations are monotonously increased and monotonously decreased respectively, the morphological center may become identity transformation. As a result, in the process of constructing morphological enter operation, we must ensure that the pair of dual operations cannot be monotonous or identity transformation of raw image. For example, we can select following pairs of dual morphological operations:

$$\{\Phi\gamma, \gamma\Phi\}, \{\gamma\Phi\gamma, \Phi\gamma\Phi\}$$

where: γ is open operation,

Φ is close operation.

Figure 3 is an example of morphological center (Soille, 2002).

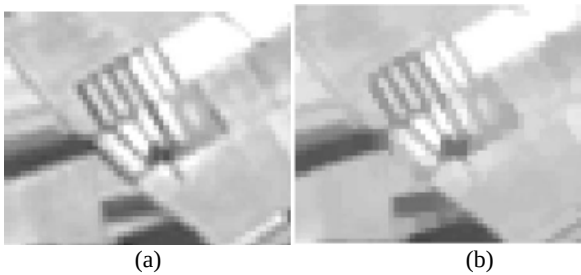


Figure 3. Self-dual morphological operation. (a) is raw image, (b) is the result image

In practical application, self-dual morphological operations can be constructed by other methods. Soille (1999) introduced a self-dual morphological operation consisting of a series of thinning filters and thickening filters. Heijmans (1994) reported a general method of constructing self-dual morphological filters. Another method of resolving problem that mathematical morphology depends on local contrast is transformation having

self-complementary property. Assuming Φ is self-complementary filter, the equation is:

$$\Phi(A) = \Phi C(A) \quad (10)$$

where: A is raw image.

Morphological gradient is a type of self-complementary filter. As a result, morphological gradient watershed transformation based on morphological gradient watershed transformation can ensure that the segmentation result is independent of local contrast (Vincent, 1991; Meyer, 1990).

3.2 The Influence of Speckles on Complex Objects

Spatial data acquired by various methods always has speckles. So, speckle remove operation must be performed before morphological transformations. However, because the research object is complex and the number of speckles is large, many structures divided by speckles become incomplete. Moreover, some small structures may be erased. The result is greatly influenced (in figure 4.)

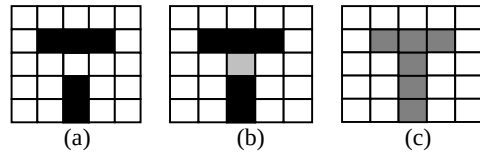


Figure 4. Influence of speckles on image processing. (a) image structure influenced by speckle, image (b) was derived from raw image (a) by speckle remove, (c) is structuring element. Obviously, the 'T' shape structure in the image can not be detected.

In order to enhance the ability of resolving this type of problem of mathematical morphology, ranked morphological filter was proposed. In ranked filters, a parameter value can be assigned. This parameter denotes the flexibility of structuring elements. It allows in image detection one or more pixels not being coincident with structuring element to exist in images. This method often applied to processing of image having speckles or detection of uncertain image structures.

There is no fixed structuring element in ranked filters. A series of structuring elements (B_i) will be derived from an appointed main structuring element (B). Taking set into considering, this series of structuring elements is a sub-set of the main structuring element. In the ranked opening operation, a series of open operations using this set of structuring elements will be performed on the raw image. The union set of those mid results is the final result. The mathematical expression is:

$$\gamma_{A,B,k} = \bigvee_i \{\gamma_{A,B_i} \mid B_i \subseteq B, \text{card}(B_i) = k\} \quad (11)$$

where: A is raw image,

B is main structuring element,

K is rank amount,

B_i is sub-structuring element derived from main structuring element,

Card is the cardinal number of the sub-structuring element, $1 \leq k \leq n$,

The cardinal number of main structuring element is n , which is the number of pixels constituting main structuring element.

It is obvious that the flexibility of ranked open operation decreases along with the decreasing of k . However, the increasing of flexibility will bring on the increasing of operation spending. From equation (11), we know that the maximum number of sub structuring elements is $n!/(k!(n-k)!)$. To almost all the application, it cannot be endured. Heijman (1994) provided a resolving scheme for this problem. The ranked open filter equation can be changed to this one:

$$\gamma_{A,B,k} = id \wedge (\delta \hat{B}, \zeta_{B,n-k+1}) \quad (12)$$

Ranked morphological open filter proposed by Ronse for the first time (Ronse, 1986). Although its anti-speckle ability is good, it is not well applied yet. Soille (2001) proposed a speedy algorithm applying ranked morphological filter in line segmentation.

3.3 Multi-band Spatial Data Processing

With the development of earth observation technology, spectral resolution as well as spatial resolution was enhanced greatly. The number of bands of an image may be often greater than 100. It brings forward new question to mathematical morphology. That is how to deal with multi-value images. According to the morphological theory has been introduced previously, mathematical morphology is based on including relation. In order to implement various operations in mathematical morphology, the standard of being high or low, the including relation between sets and the including relation between operations must be defined in advance. That is to say that the theory foundation of mathematical morphology is complete lattices. As a result, various operations in mathematical morphology cannot be applied to multi-value images directly. If we regard one pixel in multi-value image as a vector, the relation between vectors must be defined. Although some research work has been carried through, such as reduced ordering in (Corner, 1999; Barnett, 1976), there isn't a definite definition of vector ordering. Soille (2001) proposed a vector ordering method considering pixel positions, which was applied in the vector middle value filter.

Another resolving scheme is that every band can be regarded as a single image and be processed respectively. Because the method will change the spectral contents of images, it is difficult to use it. Goutsias (1995) proposed to perform a component transformation (such as principal component analysis) on the raw image and then take the first component into account only. The two methods are all based on the morphological operation on gray images. Talbot (2000) and Chanussot (1998) provided a method of defining full ordering in vector space, the shortcoming of which is that it did not consider the physical meaning of raw data.

Those new methods introduced above are all proposed aiming at resolving problems appeared in the process of application of mathematical morphology in the field of geoscience research. Furthermore, in the process of spatial data processing and

analysis, many new theories and new methods are put forward. Soille (2002, 1996) proposed a speedy algorithm to extract linear objects having special length and arbitrary direction. It can be used to detect automobiles on roads. Soille (2002), Mathern (1997) and Meyer (1997) introduced a new leveling and compressing method. Soille (2002) introduced a method of using morphological profiles to extract complex objects. This method performed a set of morphological operations on raw image with a series of structuring elements having various sizes and directions. Profiles are constructed according results. Profiles took the parameter (size and direction) of structuring element as independent variable and pixel value as dependent variable. The final pixel values were calculated according to a special rule. Ranwez (1999) introduced an order independent skeletonization method. This method was applied to extract crest line (Vincent, 1991).

4. CONCLUSIONS

Being compared with other digital image processing method, mathematical morphology has the most important property that we can study in the object-oriented mode. It put emphasis not only on the exhibition of image but also on the spatial relation between objects. As a result, mathematical morphology has great significance to the geoscience, which takes spatial objects as principal research objects. With the development and progress of the theory and method of mathematical morphology, it will be applied more and more widely to the field of geoscience research. At the same time, if those problems having been encountered in the process of application are resolved, mathematical morphology will receive more powerful life-force in the future.

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