

# STOCHASTIC MODELING AND TRIANGULATION FOR AN AIRBORNE THREE LINE SCANNER

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## ABSTRACT:

From the last decade, commercial vendors have introduced airborne three line scanners; for example ADS40, TLS, JAS, DAS-1, 3-OC and etc. While ADS40, TLS and JAS placed CCD arrays on the focal plane in a single optical system, the 3-DAS-1 and 3-OC use three optical apertures but rigidly fixed to each other. For this reason, we need to develop a photogrammetric model for three different cameras but moving together along a single flight trajectory. The main objective of the research is developing an algorithm to recover internal and external orientation parameters of an airborne three line scanner. In this paper, a general photogrammetric model (including self-calibration process) for a particular three line scanner from Wehrli and Assoc., and triangulation results will be presented.

## 1. INTRODUCTION

The main objective of the research is developing an algorithm to recover orientation parameters of an airborne three line scanner. The phrase, “three line scanner”, refers to a scanner having three pushbroom type line sensors. From the last decade, commercial vendors have introduced airborne three line scanners, instead of more traditional single line scanners. For example, the ADS40 has been jointly developed by Leica and the German Aerospace Center, DLR in 1998 (Sandau et al., 2000). In 2000, STARLABO Corporation designed a three line scanner named TLS jointly with the Institute of Industrial Science, University of Tokyo (Gruen and Zhang, 2003). In 2004, the 3-DAS-1, a digital photogrammetric three line scanner, was jointly developed by Wehrli & Associates and Geosystem (Wehrli et al., 2004). In addition to the 3-DAS-1, Wehrli & Associates and Geosystem also introduced the 3-OC digital oblique camera in 2006. One of the latest three line scanners in the market is JAS (Jena Airborne Scanner) from Jena-Optronik, Germany. These cameras, digital three line scanners, provide a stereo capability within a single flight and increase redundancy in aerial triangulation. In addition to this, real-time direct geo-referencing is possible in theory with the aid of GPS and an inertial sensor.

Airborne multi-line cameras have proven to be an effective tool in the modernization of photogrammetric techniques. But there still remain some technical issues to be addressed. Unlike frame photography, where all pixels in the image are exposed simultaneously, each line of a pushbroom image is collected at a different instant of time (Lee et al., 2000). Therefore, users need to recover exterior orientation parameters for every line to exploit 3D information from a pushbroom type of imagery. For three line scanners, parameters in addition to those needed for a single line scanner should be considered. While ADS40, TLS and JAS placed CCD arrays on the focal plane in a single optical system, the 3-DAS-1 and 3-OC use three optical apertures but rigidly fixed to each other. For this reason, we need to develop a photogrammetric model for three different

cameras but moving together along a single flight trajectory. In this paper, the location of the perspective center for each line sensor is represented by the translations and the rotations with respect to the gimbal center; the perspective centers can be constrained to be the same for a three line scanner with a single optical system.

Since the number of exterior orientation parameters for an image strip increases in proportion to the number of image lines, the total number of unknowns for one image strip becomes significantly larger than that of a traditional frame photograph. Due to the large number of unknown orientation parameters, providing enough observations using ground control points and pass points is not practical. To make this problem feasible, there have been two kinds of approaches. One is reducing the number of unknown parameters and another is providing fictitious observations in addition to the real observations. One of the well known methods of reducing the number of unknown parameters is a piecewise polynomial trajectory. While a piecewise polynomial trajectory model can efficiently reduce the number of unknown parameters, this model can only accommodate smooth trajectories, which airborne platforms do not always provide. Therefore, we propose to add fictitious observations in addition to the real observations to improve the modeling of this problem. For the fictitious observations, we used a first order Gauss Markov model for the exterior orientation parameters of two adjacent lines. Even though we expect this approach can represent well a rapidly changing trajectory, its computational cost is much heavier than that of a piecewise polynomial trajectory model. If we expand this approach to the block triangulation of multiple strips, number of parameters to be adjusted will be drastically increased. We are also investigating a hybrid strategy with fictitious observations between non-contiguous lines and some kind of interpolation strategy between them.

In this paper, a general photogrammetric model for a particular three line scanner from Wehrli and Assoc., and triangulation results will be presented.

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## 2. DATASET AND MODEL

### 2.1 Dataset

#### 2.1.1 Image Acquisition system

3-OC is an airborne three line scanner equipped with a GPS receiver and an inertial measurement unit (IMU). Three cameras are mounted on a stabilizer that is fixed at the bottom of the airplane. The IMU is firmly attached right above the camera system, and the lever-arm from the GPS antenna to the gimbal center is fixed in distance but not in attitude. Each camera has three linear arrays for color imaging, but this paper presents only geometric properties of 3-OC and radiometric characteristics are will not be considered. In this experiment, only the green band image of each camera is used for the triangulation. The nadir camera is looking down, the forward and backward cameras are tilted by 45 degrees in the flight direction with respect to the nadir camera. The size of the CCD array is 8002 pixels, and the physical pixel size is  $9\mu\text{m} \times 9\mu\text{m}$ . To have an identical ground swath for all three cameras, nominal focal lengths are designed as 88mm for nadir, and 110mm for backward and forward cameras. This system is currently operated by Woolpert, Inc. with the service named SmartView™. By the aid of the POS-AV system from Applanix Co. Ltd., the trajectory and the attitudes of the gimbal rotation center, SBET (Smoothed Best Estimated Trajectory), is acquired. SBET data provides locations, attitude and several more information of the gimbal center.

#### 2.1.2 Flight plans

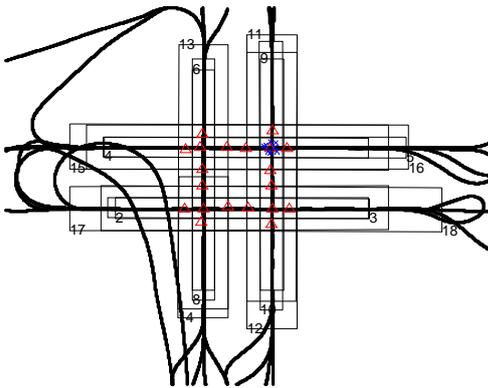


Figure 1 Flight trajectory and approximate footprints of the nadir camera

Total 18 strips of imagery have been acquired through the flight over the Atlanta, Georgia, USA and 16 flights shown in Figure 1 are used for an aerial triangulation. Total 33 ground coordinates are surveyed and 20 points are used as ground controls. Reference coordinate system is Georgia-West state plane coordinate system with NAD88 horizontal datum and NAVD80 vertical datum.

### 2.2 Sensor Model for a three line scanner

#### 2.2.1 Sensor Coordinate System(SCS)

For line scanners, each line has its instantaneous perspective center. Then a pixel coordinate, line  $u$  and column  $v$ , can be transformed into the image coordinate system by Equation 1. Then we can build a collinearity equation between an object

coordinate and the corresponding image coordinate in SCS using equation 2. In equation 4,  $(X, Y, Z)$  is three dimensional ground coordinate in the reference coordinate system and  $(X_L, Y_L, Z_L)$  is the location of an instantaneous perspective center in the reference coordinate system.  $M$  is a 3 by 3 rotation matrix which relates an instantaneous SCS and the reference coordinate system and  $\lambda$  is a scale factor.  $(x_p, y_p)$  is the principal point and  $f$  is the focal length of the camera. Also, the radial distortion terms  $(\Delta r_x, \Delta r_y)$  are added in Equation 2.

$$\begin{aligned} x &= (u - \text{int}(u)) \times 9\mu\text{m} \\ y &= (v - 4000.5) \times 9\mu\text{m} \end{aligned} \quad (1)$$

$$\begin{bmatrix} x - x_p + \Delta r_x \\ y - y_p + \Delta r_y \\ -f \end{bmatrix} = \lambda M \begin{bmatrix} X - X_L \\ Y - Y_L \\ Z - Z_L \end{bmatrix} \quad (2)$$

$$\text{where } \Delta r = k_1 r + k_2 r^3 + k_3 r^5$$

#### 2.2.2 Collinearity equations for a three line scanner

Based on the model for a single line scanner, we need to merge three camera models into a single, integrated model. Since the centers of projection of three cameras are fixed in the navigation coordinate system with respect to the origin of the instantaneous gimbal rotation center, we could estimate the location of the instantaneous perspective center of each camera using a 6 parameter transformation (three translations and three rotations) as described in Equation 3. Since the forward and backward cameras are tilted, a rotation matrix,  $R$ , is introduced to build proper forward and backward sensor coordinate systems in Equation 4. Then  $M$  matrix (transformation matrix from the reference coordinate system to sensor coordinate system) can be used for all three cameras. Finally, three collinearity conditions of one ground point for all the three cameras are introduced in Equation 4; parameters are described in Figure 2.

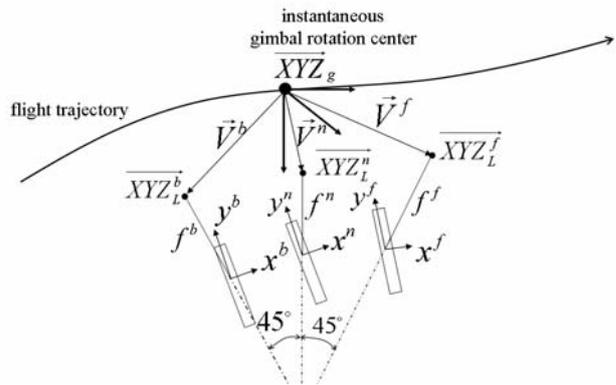


Figure 2 Imaging geometry of a three line scanner

$$\begin{aligned} \overline{XYZ_L^f} &= R(\phi, \theta, \psi) \overline{V^f} + \overline{XYZ_g} \\ \overline{XYZ_L^n} &= R(\phi, \theta, \psi) \overline{V^n} + \overline{XYZ_g} \\ \overline{XYZ_L^b} &= R(\phi, \theta, \psi) \overline{V^b} + \overline{XYZ_g} \end{aligned} \quad (3)$$

where,

$R(\phi, \theta, \psi)$  is 3D rotation matrix  
 roll= $\phi$ , pitch= $\theta$ , heading= $\psi$   
 $\overline{XYZ_g}$  : gimball center  
 $\overline{V^f}, \overline{V^n}, \overline{V^b}$  : from gimbal to perspective centers  
 $\overline{XYZ_L^f}, \overline{XYZ_L^n}, \overline{XYZ_L^b}$  : location of perspective centers

$$R \begin{bmatrix} x^f - x_p^f + \Delta r_x^f \\ y^f - y_p^f + \Delta r_y^f \\ -f^f \end{bmatrix} = \lambda M \begin{bmatrix} X - X_L^f \\ Y - Y_L^f \\ Z - Z_L^f \end{bmatrix}, \text{ where } \begin{bmatrix} X_L^f \\ Y_L^f \\ Z_L^f \end{bmatrix} = \overline{XYZ_L^f} \quad (4)$$

$$R \begin{bmatrix} x^n - x_p^n + \Delta r_x^n \\ y^n - y_p^n + \Delta r_y^n \\ -f^n \end{bmatrix} = \lambda M \begin{bmatrix} X - X_L^n \\ Y - Y_L^n \\ Z - Z_L^n \end{bmatrix}, \begin{bmatrix} X_L^n \\ Y_L^n \\ Z_L^n \end{bmatrix} = \overline{XYZ_L^n}$$

$$R \begin{bmatrix} x^b - x_p^b + \Delta r_x^b \\ y^b - y_p^b + \Delta r_y^b \\ -f^b \end{bmatrix} = \lambda M \begin{bmatrix} X - X_L^b \\ Y - Y_L^b \\ Z - Z_L^b \end{bmatrix}, \begin{bmatrix} X_L^b \\ Y_L^b \\ Z_L^b \end{bmatrix} = \overline{XYZ_L^b}$$

### 2.3 Trajectory Model

In this experiment, stochastic trajectory model is applied. Stochastic trajectory model is based on a first-order Gauss Markov process. The criterion for a first order Markov process is that the conditional probability distribution of a random process be dependent only on the one most recent point in the past (Lee, 1999). The associated differential equation is

$$\frac{dx}{dt} + \beta(t)x = w \quad (5)$$

where,  $w$  is white noise

If we assume that the probability density function of  $x$  and  $w$  to be Gaussian, this process  $x$  is called a first order Gauss-Markov process and its autocorrelation function is represented by Equation 6.

$$\Phi_{xx}(\tau) = \sigma^2 e^{-\beta|\tau|} \quad (6)$$

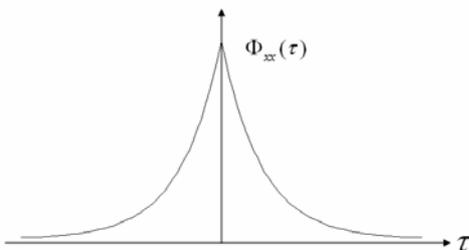


Figure 3 Autocorrelation function of 1st-order Gauss-Markov process

Previously, a first-order Gauss-Markov process is applied to the exterior orientation parameters of the two adjacent lines of image. If a first-order Gauss-Markov process is directly applied to the dataset described in this paper, the number of unknown exterior orientation parameters is larger than 25 million and computational cost is too expensive compared to the more traditional piecewise polynomial approach. To reduce computational cost, a first-order Gauss-Markov process applied only to the lines having observations. From this model, we could have a pseudo observation equation and its weight expressed in Equation 7.

$$\begin{aligned} G_{X_L} &= \Delta X_L^{i-1} - \Delta X_L^i = 0 \\ G_{Y_L} &= \Delta Y_L^{i-1} - \Delta Y_L^i = 0 \\ G_{Z_L} &= \Delta Z_L^{i-1} - \Delta Z_L^i = 0 \\ G_\phi &= \Delta \phi^{i-1} - \Delta \phi^i = 0 \\ G_\theta &= \Delta \theta^{i-1} - \Delta \theta^i = 0 \\ G_\psi &= \Delta \psi^{i-1} - \Delta \psi^i = 0 \\ W_G^{(i-1,i)} &= \sigma^2 e^{-\beta|\tau_i - \tau_{i-1}|} \end{aligned} \quad (7)$$

## 3. TRIANGULATION AND THE RESULT

### 3.1 Aerial triangulation

The condition equations for an image observation is described in Equation 8 and the pseudo observation equation is expressed in Equation 7.

$$\begin{aligned} F_x &= x - x_p + \Delta r_x - f \frac{U}{W} = 0 \\ F_y &= y - y_p + \Delta r_y - f \frac{V}{W} = 0 \end{aligned} \quad (8)$$

where,  $R^{-1}M \begin{bmatrix} X - X_L \\ Y - Y_L \\ Z - Z_L \end{bmatrix} = \begin{bmatrix} U \\ V \\ W \end{bmatrix}$

For an aerial triangulation, we applied the generalized least squares method (Mikhail and Ackermann, 1976). In this experiment, exterior orientation parameters of the lines having image observations, interior orientation parameters and radial distortion coefficients are set to the free parameters.

$$\begin{aligned} v + B\Delta &= f \\ v_G + G\Delta &= 0 \\ v_x - \Delta &= f_x \end{aligned} \quad (9)$$

Then, normal equation can be described as Equation 10.

$$\begin{aligned} N\Delta &= t \\ \text{where, } N &= B^T W B + G^T W_G G \\ t &= B^T W f - W_{xx} f_x \end{aligned} \quad (10)$$

Since  $N$ , a normal matrix, is very sparse (contains numerous zeros terms), a sparse matrix solver named IMSL (Version 6.0) is used. Each non-zero element of  $N$  is stored in the sparse matrix storage. To invert a normal matrix, the LU decomposition is used.

### 3.2 Triangulation result

In this experiment, total 20 ground control points are used for a triangulation. After 7 iterations, the model converged in 0.58 pixels and RMSE of image misclosures for 8 check points is 1.06 pixels. Total processing time is 49 seconds on the 16 core x86\_64 Linux machine with 64 gigabytes of memory; multi-core arithmetic operations are not implemented. After adjustment, the estimated radial distortion is compared to the laboratory calibration result and shown in Figure 4.

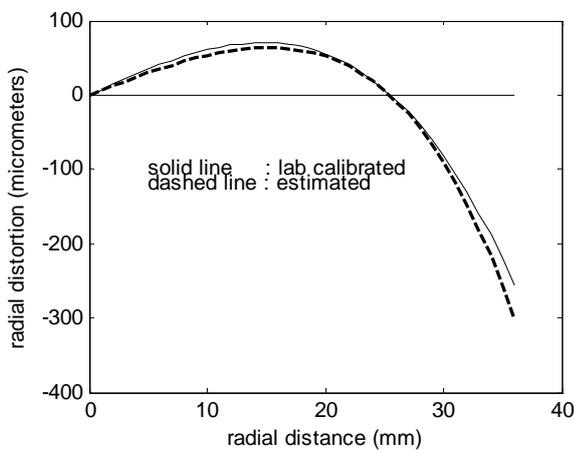


Figure 4 Estimated and lab calibrated radial distortions for forward camera

## 4. CONCLUSION

In this research, a stochastic trajectory model is implemented for the 3-OC three line scanner. Exterior orientation parameters and interior orientation parameters are estimated through the

aerial triangulation and RMSE of image misclosures for check points is 1.06 pixels. To be fully benefitted by a stochastic trajectory model, we are currently working on the pass points manipulation.

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