DERIVING SPATIOTEMPORAL RELATIONS FROM SIMPLE DATA STRUCTURE

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Commission II, WG II/1

KEY WORDS: Data Structure, Operation, Query, Object, Spatial, Temporal

ABSTRACT:
A spatiotemporal data model is incomplete without three components: classes, consistency constraints, and operators. Classes define the structure of the model, constraints enforce consistency in the model, and operators operate on the structure of the model. These operators can be static or dynamic (Raza, 2004). Dynamic operators change the state of the system, for example, create, kill, delete, or reincarnate operators. Static operators are query operators. Past research on spatiotemporal models (STM) mainly focused on the classes of the model. Research on operators of STM is not abundant. Static operators are the focus of this paper. These operators are utilized to query spatial, temporal, and spatiotemporal relations. Past studies mainly focused on purely spatial or temporal relations. Spatial relations that are valid for a certain time period are called spatiotemporal relations. Past studies mainly focused on purely spatial or temporal relations. Spatial relations that are valid for a certain time period are called spatiotemporal relations. Spatial relations that are valid for a certain time period are called spatiotemporal relations. Spatial relations that are valid for a certain time period are called spatiotemporal relations. Spatial relations that are valid for a certain time period are called spatiotemporal relations.

This paper discusses the spatiotemporal relations based on temporal cell-tuple structure of an object-oriented, cell-tuple-based spatiotemporal data model (Raza, 2001; Raza and Kainz, 1999). The object-oriented cell-tuple-based spatiotemporal data model (CTSTDM) consists of three main classes: spatial, attribute, and temporal. A spatiotemporal class is the aggregation of spatial and temporal classes (Figure 1).

1. INTRODUCTION
A spatiotemporal data model has three components: the classes, consistency constraints, and operators. Classes define the structure of the model, constraints enforce consistency in the model, and operators operate on the structure of the model. These operators can be static or dynamic (Raza, 2004). Dynamic operators change the state of the system, for example, create, kill, delete, or reincarnate operators. Static operators are query operators. Past research on spatiotemporal models (STM) mainly focused on the classes of the model. Research on operators of STM is not abundant. Static operators are the focus of this paper. These operators are utilized to query spatial, temporal, and spatiotemporal relations. Relations in spatiotemporal databases can be categorized into three groups (i.e., spatial, temporal, and spatiotemporal relations). Past studies mainly focused on purely spatial or temporal relations. Spatial relations that are valid for a certain time period are called spatiotemporal relations.

This spatiotemporal class is also a super class of three classes—ZeroTCellClass (ZeroTCell), OneTCellClass (OneTCell), and TwoTCellClass (TwoTCell). TemporalCellTuple class is the aggregation of three classes: ZeroTCellClass, OneTCellClass, and TwoTCellClass. Operations pertaining to the TemporalCellTuple class are the focus of this paper. This paper elaborates the topological relations (spatiotemporal topology) derived from the simple temporal cell-tuple structure of...
TemporalCellTuple class. The details of this structure can be found in Raza and Kainz (1999).

First, the spatial and temporal relations are briefly discussed in §2. The temporal cell-tuple class is explained in §3. Section 4 elaborates the spatiotemporal relations. The paper concludes in §5.

2. SPATIAL AND TEMPORAL RELATIONS

Static operators are query operators. These operators are used to query spatial, temporal, and spatiotemporal relations. Relations in spatiotemporal databases can be categorized into three classes:

- Spatial relations
- Temporal relations
- Spatiotemporal relations

These spatial relations are grouped into four classes: set-oriented, metric, topological, and Euclidean relations (Worboys, 1992). Spatial order relations were also introduced (Kainz, 1989). These spatial relations can be grouped into five categories:

- Spatial metric relations
- Spatial topological relations
- Spatial order relations
- Set-oriented spatial relations
- Euclidean spatial relations

### Table 1. Adapted temporal relations for bounded interval (Allen, 1984)

<table>
<thead>
<tr>
<th>Temporal Relations</th>
<th>Illustration</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Before</td>
<td>![Before Illustration]</td>
</tr>
<tr>
<td>2 After</td>
<td>![After Illustration]</td>
</tr>
<tr>
<td>3 Equal</td>
<td>![Equal Illustration]</td>
</tr>
<tr>
<td>4 Meets</td>
<td>![Meets Illustration]</td>
</tr>
<tr>
<td>5 Met</td>
<td>![Met Illustration]</td>
</tr>
<tr>
<td>6 Overlaps</td>
<td>![Overlaps Illustration]</td>
</tr>
<tr>
<td>7 Overlapped</td>
<td>![Overlapped Illustration]</td>
</tr>
<tr>
<td>8 Covers</td>
<td>![Covers Illustration]</td>
</tr>
<tr>
<td>9 During</td>
<td>![During Illustration]</td>
</tr>
<tr>
<td>10 Started</td>
<td>![Started Illustration]</td>
</tr>
<tr>
<td>11 Finishes</td>
<td>![Finishes Illustration]</td>
</tr>
<tr>
<td>12 Starts</td>
<td>![Starts Illustration]</td>
</tr>
<tr>
<td>13 Finished</td>
<td>![Finished Illustration]</td>
</tr>
</tbody>
</table>

Temporal operations refer to temporal relations. Temporal operations are isomorphic to the spatial relations. These relations could be defined as using metric, topological, and order-theory concepts. For example, "two hours" is a metric relationship, "one hour later" is a topological temporal relationship, and "four weeks in a month" is an order relationship.

Therefore, the temporal relations can be subclassified into five categories:

- Temporal metric relations
- Temporal topological relations
- Temporal order relations
- Set-oriented temporal relations
- Euclidean temporal relations

As mentioned earlier, this paper will focus on temporal topological relations. These relations are associated with TemporalCellTuple class.

3. TEMPORALCELLETTUPLE CLASS

The object of a spatiotemporal class is called an \( n \)-tcell. The boundaries (\( \partial \)) of an \( n \)-tcell are its \( (n-1) \) faces at time \( t \). The coboundary (\( \Phi \)) of an \( n \)-tcell produces the \( (n+1) \) cells incident with \( n \)-tcell at time \( t \). In the temporal cell complex, intracell complex relations (i.e., relations between cells in the cell complex) can be described using boundary and coboundary relations. The boundary and coboundary relations capture two types of topological relationships: adjacency and containment. Relations between spatial objects can be found based on boundary/coboundary relations between cells. The boundary and coboundary relations are encapsulated in a simple temporal cell-tuple structure, which is an extension of the cell-tuple structure of Brisson (1990). A cell-tuple \( T \) is an \( (n+1) \) tuple of cells \( \{c_0, c_1, c_2, \ldots, c_n\} \), where any \( i \)-cell is incident with a \( (i+1) \)-cell.

TemporalCellTupleClass preserves the temporal cell-tuple structure (Figure 2) and is the aggregation of ZeroTCellClass (ZTC), OneTCellClass (OTC), and TwoTCellClass (TTC) (Figure 1). The object of TemporalCellTupleClass has a unique tuple ID and a unique combination of ZTC, OTC, and TTC. Each tuple must have a ZTC, zero or one OTC, and zero or one TTC. Therefore, a temporal cell-tuple structure encapsulates the spatiotemporal topology of each spatiotemporal object. A temporal cell tuple (TCT) is a set of \( C \) and \( T \), as follows:

\[
TCT = \{C, T\}
\]

where \( C \) is a set of cells
\[
C = \{c_0, c_1, c_2, \ldots, c_n \mid c_i \in TCC\}
\]

and \( T \) is a time interval (1-T)
\[
T = \{T_{From}, T_{Until} \mid (T_{From} < T_{Until}) \land (T_{From} \in ST)\}
\]

Therefore,
\[
TCT = \{c_0, c_1, c_2, \ldots, c_n, T_{From}, T_{Until}\}
\]
Any \( i \)-tcell \((c_i)\) is incident with an \((i+1)\)-tcell \((c_{i+1})\). Every \( c_i \) cell is a boundary of a \( c_{i+1} \) cell, where \( 0 \leq i \leq n \) and \( n+1 \) is the maximum number of cells in each tuple. For \( n = m \), the first cell \( c_0 \) is a ZTC, the second cell \( c_1 \) is an OTC, the third cell \( c_2 \) is a TTC, and \( m \)-cell \( c_m \) is an \( n \)-TCT. In \( \{c_0, c_1, c_2, \ldots, c_n, T_{\text{From}}, T_{\text{Until}}\} \) at time \( t \), any \( i \)-tcell \((c_i)\) is either a boundary of an \((i+1)\)-tcell \((c_{i+1})\) or coboundary of an \((i-1)\)-tcell \((c_{i-1})\). The advantage of TCT is that it stores topology implicitly. It is dimension-independent, that is, it can accommodate objects of dimension \( k \) \((k \geq 1)\), and it encapsulates boundary and coboundary and order relations over time. We can formulize the spatiotemporal structure. These spatiotemporal relations are preserved in the TCT structure. Spatiotemporal relations derived from the TCT structure are based on two primary relations: boundary and coboundary. In the following subsections, each operation, operands, results, and syntax in unified modeling language (UML) is presented.

### 4.1 Boundary \((\partial)\) and Coboundary \((\Phi)\)

The boundary \((\partial)\) of an \( n \)-tcell is its \((n-1)\) faces at time \( t \). The coboundary \((\Phi)\) of an \( n \)-tcell produces the \((n+1)\) cells incident with \( n \)-tcell at time \( t \).

The \( \Phi \) and \( \partial \) history of \( k \)-tcell at time \( T_i \) can be formalized as

\[
\Phi(k-tcell)_{T_i} = \{\forall (k+1)tcell \mid T_{\text{From}} \leq T_i\}
\]

\[
\partial(k-tcell)_{T_i} = \{\forall (k-1)tcell \mid T_{\text{From}} \leq T_i\}
\]

Whereas, the boundary of \( k \)-tcell at time \( T_i \) is

\[
\partial(k-tcell)_{T_i} = \{\forall (k+1)tcell \mid T_{\text{From}} = T_i\}
\]

The coboundary \( k \)-tcell at time \( T_i \) can be defined as

\[
\Phi(k-tcell)_{T_i} = \{\forall (k+1)tcell \mid T_{\text{From}} = T_i\}
\]

In Figure 3, the boundary of \( A_1 \) at time \( T_2 \) can be calculated as

\[
\partial(A_1)_{T_2} = \{a_1, a_2, a_3\}
\]

\[
\Phi(a_1) = \{[A_1, A_1]\}
\]

\[
\Phi(a_2) = \{[A_2, A_1]\}
\]

\[
\Phi(a_3) = \{A_1, A_1\}
\]

where symbol \([\_]\) represents null.

Therefore, \( a_3 \) is excluded from the boundary of \( A_1 \) because the coboundary of \( a_3 \) is the same.

\[
\partial(A_1)_{T_2} = \{a_1, a_2\}
\]
The two \(n\)-cells (\(n = 1, 2\)) are disjoint if the intersection of their faces is empty. Disjoint relations of point and ZTC are straightforward. The \(\Omega\) relations of OTC and TTC can be expressed as
\[\text{Disjoint}(P: \text{TTC}_1, P: \text{TTC}_2); \text{Boolean}\]
\[\{2\text{-tcell}_1 \ \Omega \ 2\text{-tcell}_2 \ \text{true} \ | \ (2\text{-tcell}_1) \ \cap \ (2\text{-tcell}_2) = \emptyset\]\n
Disjoint \((\text{P: OTCT}_1, \text{P: OTCT}_2)\); Boolean
\[\{1\text{-tcell}_1 \ \Omega \ 1\text{-tcell}_2 \ \text{true} \ | \ (1\text{-tcell}_1) \ \cap \ (1\text{-tcell}_2) = \emptyset\]

For example, consider Figure 4[a],
\[\Omega(\text{TTC}_2, \text{TTC}_3) = \text{FASLE because}\]
\[\Omega(\text{TTC}_2, \text{TTC}_3) \ = \ (\text{TTCC}_2)_{\text{TTC}_1} \ \cap \ (\text{TTCC}_3)_{\text{TTC}_2} \ = \ \{ (a_2, a_3) \ \cap \ (a_3, a_4) \} \ = \ \{ a_3 \} \]

### 4.3 Contains \((\chi)\)

The containment relations can be between spatiotemporal objects of the same spatial dimension or different spatial dimensions. For example, a TTC can contain a TTC, an OTC, or a ZTC; these relations are depicted in Figure 5, Figure 3, and Figure 6, respectively.

**Figure 6. ZTC interior of TTC**

At time \(T_2\), 2-tcell contains 2-tcell;\nContains(P:TTCC, P:TTCT): Boolean
\[\{2\text{-tcell} \ \alpha \ 2\text{-tcell} \ \text{true} \ | \ T_{\text{From}} = T_i \ \land \ \hat{\partial}(2\text{-tcell}) \ \cap \ \hat{\partial}(2\text{-tcell}) = \emptyset\}\]

At time \(T_2\), 2-tcell contains 1-tcell;\nContains(P:TTCC, P:OTC): Boolean
\[\{2\text{-tcell} \ \alpha \ 1\text{-tcell} \ \text{true} \ | \ T_{\text{From}} = T_i \ \land \ \hat{\partial}(2\text{-tcell}) \ \cap \ \hat{\partial}(1\text{-tcell}) = \emptyset\}\]

At time \(T_2\), 2-tcell contains 0-tcell;\nContains(P:TTCC, P:ZTC): Boolean
\[\{2\text{-tcell} \ \alpha \ 0\text{-tcell} \ \text{true} \ | \ T_{\text{From}} = T_i \ \land \ \hat{\partial}(2\text{-tcell}) \ \cap \ \hat{\partial}(0\text{-tcell}) = \emptyset\}\]

At time \(T_2\), 2-tcell contains 0-tcell;\nContains(P:TTCC, P:ZTC): Boolean
\[\{2\text{-tcell} \ \alpha \ 0\text{-tcell} \ \text{true} \ | \ T_{\text{From}} = T_i \ \land \ \hat{\partial}(2\text{-tcell}) \ \cap \ \hat{\partial}(0\text{-tcell}) = \emptyset\}\]

For example, to check whether TTC contains a TTC or not, consider Figure 5, where at time \(T_2\), TTC(3) \(\alpha\) TTC(2).
\[\{\chi(3) \ \cap \ \hat{\partial}(2) \} = \{\chi(2)\}\]
\[\{(a_1, a_2) \ \cap \ (a_2)\} = \{(a_2)\}\]
\[a_2 = \{(a_2)\}\]

### 4.4 Inside \((\chi)\)

At time \(T_2\), a ZTC, OTC, or TTC can be inside a TTC. The same logic is employed to discern the \(\chi\) relations between two \(n\)-cells. For example:

At time \(T_2\), 2-tcell is inside 2-tcell;
Inside(P:TTC, P:TTC): Boolean

\{ 2-tcell \n\n\chi \neg \exists \text{true} | T_{from} = T_t \land \partial(2\text{-tcell}) \cap \partial(2\text{-tcell}) = \partial(2\text{-tcell}) \}

At time \(T_t\), 1-tcell is inside 2-tcell;

Inside(P:OTC, P:TTC): Boolean

\{ 1-tcell \n\n\chi \neg \exists \text{true} | T_{from} = T_t \land \partial(2\text{-tcell}) \cap \partial(1\text{-tcell}) = \partial(1\text{-tcell}) \}

At time \(T_t\), 0-tcell is inside 2-tcell;

Inside(P:ZTC, P:TTC): Boolean

\{ 0-tcell \n\n\chi \neg \exists \text{true} | T_{from} = T_t \land \partial(2\text{-tcell}) \cap \partial(0\text{-tcell}) = \partial(0\text{-tcell}) \}

\[
\begin{align*}
\{ \text{2-tcell}_{T1} = & 2\text{-tcell}_{T2} | \partial(2\text{-tcell})_{T1} = \partial(2\text{-tcell})_{T2} \} \\
\text{4.5 Equal (=)} & \\
\end{align*}
\]

Checking Equal relations between two points or ZTCs is straightforward. TTC at time \(T1\) is in equal relation to TTC at time \(T2\) if the boundaries of both are the same.

\[
\begin{align*}
\{ \text{2-tcell}_{T1} = & 2\text{-tcell}_{T2} | \partial(2\text{-tcell})_{T1} = \partial(2\text{-tcell})_{T2} \} \\
\end{align*}
\]

Although it is a topological relation, the Equal relation between two OTCs may not be checked correctly in the TCT structure (based on boundary/coboundary relations) because these OTCs can be defined by different intermediate points, regardless of the same boundary. A geometric calculation is needed to check this relation.

\[
\text{4.6 Meet (\delta)}
\]

A TTC at time \(T1\) can meet with TTC, OTC, or ZTC at time \(T2\). Similarly, an OTC at time \(T1\) can meet with OTC or ZTC at time \(T2\).

Meet(P:TTC, P:TTC): Boolean

\{ 2\text{-tcell}_{T1} \n\n\delta \exists \text{true} | T_{from} = T_t \land \partial(2\text{-tcell})_{T1} \cap \partial(2\text{-tcell})_{T2} \neq \emptyset \}

Meet(P:OTC, P:TTC): Boolean

\{ 2\text{-tcell}_{T1} \n\n\delta \exists \text{true} | T_{from} = T_t \land \partial(2\text{-tcell})_{T1} \cap \partial(1\text{-tcell})_{T2} \neq \emptyset \}

Meet(P:TTC, P:ZTC): Boolean

\{ 2\text{-tcell}_{T1} \n\n\delta \exists \text{true} | T_{from} = T_t \land \partial(2\text{-tcell})_{T1} \cap (0\text{-tcell})_{T2} \neq \emptyset \}

Meet(P:OTC, P:OTC): Boolean

\{ 1\text{-tcell}_{T1} \n\n\delta \exists \text{true} | T_{from} = T_t \land \partial(1\text{-tcell})_{T1} \cap \partial(1\text{-tcell})_{T2} \neq \emptyset \}

Meet(P:OTC, P:ZTC): Boolean

\{ 1\text{-tcell}_{T1} \n\n\delta \exists \text{true} | T_{from} = T_t \land \partial(1\text{-tcell})_{T1} \cap (0\text{-tcell})_{T2} \neq \emptyset \}

For example, consider Figure 4[b]. At time \(T2\), TTC (2) and TTC (3) have Meet relations.

\[
\begin{align*}
&& \{ \partial(2) \cap \partial(3) \} & \neq \emptyset \\
&& \{ \partial(a2, a3) \cap \partial(a3, a4) \} & \neq \emptyset \\
&& \{ (n2, n2) \cap (n2, n1) \} & \neq \emptyset \\
&& \{ (n2, n1) \} & \neq \emptyset \\
\end{align*}
\]

Similarly, consider Figure 7. At time \(T2\), TTC (A1) and OTC (a3) have Meet relations.
Consider Figure 9. Let TSTO1 = \{2, 3\} and TSTO2 = \{3, 4\} at time T2. These two TSTOs overlap because their intersection with TTC or OTC is nonempty (\cap \neq \emptyset).

Overlap (\cap) is similar to relations and is not discussed further.

4.9 Overlap (\cap)

A TCC is a partition of spaces; therefore, TTC or OTC cannot overlap each other. However, a two spatiotemporal object (TSTO) and one spatiotemporal object (OSTO) at time T1 can overlap with TSTO and OTC at time T2, respectively, if their intersection with TTC or OTC is nonempty (\cap \neq \emptyset).

Consider Figure 9. Let TSTO = \{2, 3\} and TSTO2 = \{3, 4\} at time T2. These two TSTOs overlap because

\[(\{2,3\} \cap \{3,4\} \neq \emptyset) \]

\[(\{3\} \neq \emptyset)\]

Figure 9. Boundary of TTC intersects with boundary-interior of TTC, and interior of TTC intersects with boundary of TTC.

5. CONCLUSION

In this paper, operators pertaining to a simple temporal cell-tuple structure are presented. These operators are formulated by employing relational algebra. Examples are provided to derive these relations (spatiotemporal topology) from temporal cell-tuple structure. It has been proved that almost all spatiotemporal relations between OTC-OTC and TTC-TTC in the spatial domain and some other relations can be derived from temporal cell-tuple structure, which is based on the boundary and coboundary of cells. However, depending on time, some relations cannot be derived because of the inherent nature of temporal cell-tuple structure. For example, Overlap and CoveredBy relations for the same time cannot be derived because temporal cell complex is a partition of spaces. Similarly, for the Equal relation, geometric calculation is still needed. More research is needed to evaluate the performance of operators derived from the TCT structure. The composite B-tree index on the elements of this structure may perform better.

REFERENCES


