

# SPATIO-TEMPORAL OBJECT MODELING IN FUZZY TOPOLOGICAL SPACE

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### ABSTRACT:

Fuzzy spatial objects have become more and more important in GIS applications. When spatial phenomena are generalized by the crisp form, a lot of quantitative information is lost. Land cover as a fuzzy spatio-temporal object should be modeled in a fuzzy framework. In addition, topological relations between fuzzy spatial objects are much more complicated than that are defined between crisp spatial objects. The purpose of this paper is to construct topological relationships; as one of the most important characteristics of spatio-temporal objects; based on fuzzy logic with its applications to land cover. An area in Tehran which has lots of land cover types was considered and topological relationships between them were constructed both spatially and temporally.

## 1. INTRODUCTION

Many natural phenomena are not limited to clear boundaries. Fuzzy spatial objects are those with indeterminate boundaries. Fuzzy spatial object's uncertainty should be considered in all aspects of a GIS to have a better understanding of the real world.

In order to fully define and model fuzzy spatial objects such as land covers, it is necessary to investigate their fuzzy topological relationships. Topology is one of the core concepts in Geospatial Information System (GIS). Topological relationships are invariant with respect to homeomorphic transformation. Intuitively speaking, it deals with the structural representation of spatial features and their properties that remain invariant under certain transformations. Topology and topological relationships have a wide range of applications in robotics, artificial intelligence, and GIS. Topological relationships are non-metric properties of geospatial objects that remain constant when the object space is continuously distorted (Tang, 2004).

It is possible to categorize methods of constructing topological relationships into two classes, namely: point based methods such as 9-intersection matrices that can be based on crisp set or fuzzy set theory or intuition fuzzy set theory. Egenhofer and Franzosa (1991) and Egenhofer and Herring (1990) have introduced 4-intersection and 9-intersection approaches in a connected topological space by using the interiors, boundaries and exteriors between two crisp subsets. Eight relations have been identified between two simple regions in the two-dimensional Euclidean space. In addition, thirteen topological relations between two temporal intervals were identified by Allen using temporal operators; Begin and End which provide respectively the beginning and ending instances of a time interval. These approaches do not consider the uncertainty and fuzziness in any aspect of a spatio-temporal object. In order to deal with the fuzziness of the spatial objects, Tang (2004) defined boundary of the boundary region and interior of the boundary. Based on these core concepts 5\*5-intersection matrix is defined. Malek (2004) has modeled spatial relationships using intuitionistic fuzzy topology.

This paper proposes a new method to construct topological relations for land covers as spatio-temporal objects. Land use and

land cover; most of which is obtained from the classification results of satellite images; may be good examples of a fuzzy spatial objects. As a case study a part of Iran which has different parcels and also different land cover types is selected. Fundamental concepts for uncertainty modeling of spatial relationships are analyzed from the view point of fuzzy logic. It is demonstrated that how fuzzy logic can provide a model for fuzzy region; i.e., regions with indeterminate boundaries.

In general, after classification, each pixel in the image is assigned to a particular land cover type; therefore, a pixel belongs to one and only one type (Tang, 2004). In reality, all pixels contain a number of different contributing land use types. In general, land cover is continuously distributed in nature, and there is seldom a clear boundary between different natural phenomena. In this study land cover as a spatial fuzzy object is modeled. Results are scientifically assessed in the paper and showed the superiority of the model over conventional ones.

The remainder of this paper is structured as follows: Section 2 introduces necessary concepts and preliminaries. Section 3 introduces the implementation of the approach. Section 4 gives a summary and an outlook to our future research topics.

## 2. FUZZINESS

Almost all the information that we possess about the real world is uncertain, incomplete and imprecise. Uncertainty or fuzziness may include five aspects: inaccuracy and error, vagueness, incompleteness, inconsistency, and imprecision (Worboys 1998).

In order to deal with fuzziness, Zadeh proposed the famous fuzzy set theory (Zadeh, 1985). The fuzzy set and fuzzy logic are the most powerful tools for solving these fuzzy problems.

Zadeh generalized a fuzzy set from classical set theory by allowing intermediate situations between the whole and nothing. For a fuzzy set, a membership function is defined to describe the degree of membership of an element to a class. The membership value ranges from 0 to 1, where 0 shows that the element does not

belong to a class, 1 means “belong”, and other values indicate the degree of membership to a class.

Fuzzy set theory is the extension of classical set theory by allowing the membership of elements to range from 0 to 1. Let  $X$  be the universe of a classical set of objects. Membership in a classical subset  $A$  of  $X$  is often viewed as a characteristic function  $\mu(x)$  ( $x$  is a generic element of  $X$ ) from  $X$  to  $\{0, 1\}$  (Dubois and Prade, 1980).  $\{0, 1\}$  is called a valuation set. If the valuation set is allowed to be the real interval  $[0, 1]$ ,  $A$  is called a fuzzy set.  $\mu(x)$  is the membership value (or degree of membership) of  $x$  in  $A$ . Clearly,  $A$  is a subset of  $X$  that has no sharp boundary. A fuzzy set  $A$  can be represented by the set of pairs:

$$A = \{(x, \mu(x)), x \in X\}.$$

### 2.1 Fuzzy topology

Fuzzy topology is constructed based on fuzzy sets. It is an extension of general (crisp) topology.

Let  $A$  be a fuzzy subset of an ordinary (crisp) set  $X$ , and  $\wp(X)$  be the fuzzy power set of  $X$ .  $\forall \delta \in \wp(X)$  if

$$\begin{aligned} &\Phi, X \subseteq \delta \\ &\forall A_i \in \delta \cup_i A_i \in \delta \\ &\forall U, V \in \delta; U \cap V \in \delta \end{aligned}$$

Then  $\delta$  is called a fuzzy topology on  $X$  ( $i \in I$  is an index set).  $(X, \delta)$  is called a fuzzy topological space Chang (1968).

Every element of  $\delta$  is called an open (fuzzy) set in  $(X, \delta)$ . A set  $A$  is a closed (fuzzy) set if its complement  $A^c$  is open. The union of all open sets contained in  $A$  is the interior of  $A$ , denoted by  $A^o$ . The intersection of all closed sets containing  $A$  is called the closure of  $A$ , denoted by  $A^-$ . The exterior of  $A$  is the complement of  $A^-$  and is denoted by  $A^e$ . Obviously  $A^e$  is an open set. The boundary  $\partial A$  of a subset  $A$  is the intersection of the closure of  $A$  with the closure of the complement of  $A$  (Figure1).

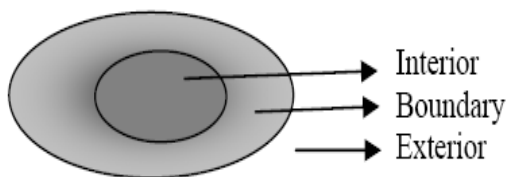


Figure1: The concept of Interior, Exterior, and Boundary of a set

The boundary of a subset may also have its interior and its boundary of the boundary. On the other hand, the interior and the closure of a subset also have their boundaries. For example the boundary of the boundary of a fuzzy set  $A$  is the union of the boundary of the closure and the boundary of the interior of a fuzzy set (Tang, 2004).

Based on this information one can define a maximum of 5 areas for each object: interior, boundary of the boundary, interior of the boundary, boundary of the interior, and exterior.

Consequently, one can develop the traditional 9-intersection matrix into 25-intersection ones.

### 3. FUZZY LAND COVER OBJECTS AND THEIR TOPOLOGICAL RELATIONSHIPS

The importance of land cover needs no more explanation, since it plays a fundamental role in a lot of fields. Land resources pose one of the biggest problems all around the world. Because of the growth in population and the economy, the contradiction between land resources and humans is becoming more and more severe. On the one hand, more arable land is necessary to feed more people. On the other hand, the growth of the economy accelerates urbanization, which always results in a decrease in cultivated lands. In Iran as a developing country Land Use and Land Cover (LULC), is taken into the great consideration by the government because the government may introduce the correct land policies to achieve the dynamic balance of cultivated lands (Tang, 2004). We address the method for forming fuzzy land covers from Spot images.

First of all, these images were classified. Because we needed the membership values so some special programs were developed to achieve our purpose. Consequently, we can assign the pixels into the land cover classes and have their memberships. It seems to have a set whose elements are the pair of pixels and their memberships. So the basis of our study is the fuzzy set. We can define the interior and the boundary of the boundary and boundary of the interior and the exterior based on the pixel’s memberships. The pixels whose memberships are between 1 and 0.8 are categorized into interior of each class. The pixels whose memberships are between 0.8 and 0.6 are categorized into Boundary of the interior of each class. Interior of the boundary of each class includes pixels whose memberships are between 0.4 and 0.6. The Boundary of the Boundary region includes the pixels with membership value between 0.2 and 0.4 and the rest are categorized into exterior of each class. So it is possible to show the relationships between the regions into the 25-intersection matrix.

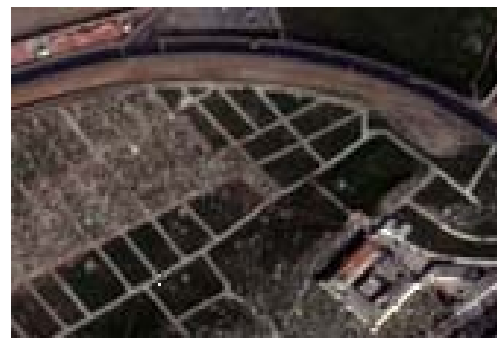


Figure 2: Spot image of the study area

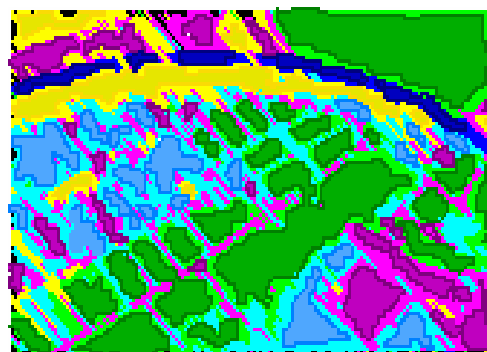


Figure 3: image classification result

Each Region is valid for an interval. The comparison of land cover maps is the basis for many dynamic analysis of land use and land covers. The traditional method usually compares the differences based on a crisp pixel-by-pixel method. But in this way our judgment is much more precise. Considering the update images it is possible to find the valid time interval of the regions. Based on Allen's temporal operators, End and Begin, temporal topological relations can be deduced.

As can be seen in Figure 3, five classes were categorized. For each class, interior, interior of the boundary, boundary of the interior, boundary of the boundary and exterior are determined. Next, 25-intersection matrices for each two classes are calculated. The classified image (Figure 3) includes five classes; Bush, Residential, Grassland, Waste land and Water, respectively. Based on newer images valid temporal interval can be found. And based on Allen's operators temporal topological relations are generated.

#### 4. CONCLUSION

Land cover as a fuzzy spatial object should be modeled in a fuzzy framework. In addition, the relationships between these fuzzy spatial objects are much more complicated than between crisp spatial objects. We can define some area for constructing topological relationships between regions which is essential. In this regard 5 areas can be considered and 25-intersection matrix is achieved.

#### REFERENCES

- [1] Allen J.F., 1983. Maintaining knowledge about temporal intervals. *Communications of the ACM* 26(11)
- [2] Cheng Tao, A process-oriented data model for fuzzy spatial objects, 1999
- [3] Cheng T. Molenaar M. Lin H. 2001. Formalizing fuzzy objects from uncertain classification results. *International Journal of Geographical Information Science*,
- [4] Egenhofer Max J. and Al-Taha Khaled K.-"Reasoning about gradual changes of topological relations"-USA
- [5] Gomert C. and Alkan M.-"The design and development of a temporal GIS for cadastral and land title data of Turkey"-Tachnical university of Turkey geodesy and photogrammetry department-Turkey
- [6] Malek Mohammad R. and Frank Adrew U. and Delavar Mohammad R.-"A logic-based foundation for spatial relationships in mobile GIS environment"-Iran Tehran
- [7] Malek Mohammad R. and Twarochi Florian-"An introduction to intuitionistic fuzzy spatial region"- Wien Austria
- [8] Muller Philippe-"Topological spatio-temporal reasoning and representation"-Paul Sabatier university
- [9] Nadi S. and Delavar M.-"Toward a general spatio-temporal database structure for GIS applications"-Tehran university-Iran
- [10] Schneider Markus-"Uncertainty management for spatial data in databases: Fuzzy spatial data types"- Fern university Hagen-Germany
- [11] Schneider Markus-"Finite resolution crisp and fuzzy spatial object"-Fern university Hagen-Germany
- [12] Tang Xiniming-"Spatial object modeling in fuzzy topological spaces with applications to land cover change"-2004. Jan-China- printed by ITC printing department
- [13] Zadeh L A. Fuzzy sets as a basis for a theory of possibility. *Fuzzy Sets and Systems*,
- [14] Worboys M.F., 1998. Imprecision in finite resolution spatial data. *GeoInformatica*
- [15] William Zhu, 2005. Topological approaches to covering rough sets. *Information Sciences* 177 (2007) 1499-1508

