

# UNCERTAINTIES IN REMOTE SENSING INFORMATION APPRAISE BASED ON THEORY OF ROUGH SETS

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## ABSTRACT:

In the remote sensing data not only have radiation error and geometry error, need geometry correction and radiometric correction, but most of them also have "same object different spectrum" and "same spectrum different object" phenomena. The report of Digital Orthophoto Map quality is subjective and limited, so it confine the application level of remote sensing imagery. Some of which may seriously distort the reliability of products. Thereby, Uncertainties in Remote Sensing Information problem, receive more and more broad attention. Some important international academic institutions or organizations hold the Remote Sensing Information problem as the 21st century's major research topic. In this paper use the theory of rough sets appraise remote sensing information. At the same time it also compared with other uncertainties remote sensing data method. In the classical fashions, e.g. error matrix and kappa coefficient, the performance of the classification models is estimated directly on the training data. Whereas it is actually not appropriate. The error matrix based on the training data set can not be regarded as the measurement of overall accuracy of classification models, and these models performance need to be evaluated on "out-of-sample-data" data that have not been used in constructing the model. Propose a new method of discretization of continuous attributes based on dynamic-layer-cluster. A unified framework of the rough set theory to deal with discrete and continuous attributes is suggested.

## 1. INTRODUCE

With the development of these techniques in each space observation. The information processing of remote sensing has transformed from a quantitative analysis into a qualitative analysis, and the processing system of remote sensing has translated from functionality into high quality. The classification of land-cover plays an important role in the processing and application of remote sensing. Under the uniform criterion, the issue of verifying the classification algorithm for land-cover and assessing the accuracy of classification has become one of the fundamental problems in the field of remote sensing.

Rough set theory is a new mathematical approach to deal with vagueness and uncertainty. Which is different from probability theory, fuzzy set theory and the theory of evidence is it uncertainty to deal with the any prior information out of the Data collection. For example, the statistical probability distribution, DS evidence in the theory of basic probability assignment, fuzzy set theory of membership; At the same time that have highly complementary with these theories. In the rough set theory a way measurement uncertainty is using Shannon (Shannon, 1948), or its deformation. Basic idea can be briefly described as follows: Use the knowledge divide of data space, Make out a relation to the information derived system discrete probability distribution, this probability distribution's Shannon Can be used to measure knowledge of the rough entropy. When knowledge is composed with information system, At this time Shannon can be considered as the system Shannon.

## 2. BASIC CONCEPT OF ROUGH SET THEORY

Rough sets theory is a relatively new soft computing tool to deal with vagueness and uncertainty. Considering the feature of remote sensing images and the basic theory and applications of rough sets. we first introduce the basic theory and character of rough sets and its applications in recent years are also pointed out.

The basic notions in rough set theory are the lower and upper approximation operators. There are at least two methods to define approximation operators, the constructive and axiomatic approaches. In the constructive approach, binary relations on a universe of discourse, partitions of the universe of discourse, neighborhood systems, and Boolean algebras are all primitive notions. The lower and upper approximation operators are constructed by means of these notions. On the other hand, the axiomatic approach takes the lower and upper approximation operators as primitive notions and a set of axioms is used to characterize approximation operators that are the same as the ones produced by using the constructive approach. Under this point of view, a rough set algebra is a set algebra with two additional approximation operators and rough set theory may be interpreted as an extension theory with two additional unary operators. The lower and upper approximation operators are related to the necessity (box) and possibility (diamond) operators in modal logic, the interior and closure operators in topological space, the belief and plausibility functions in the Dempster-Shafer theory of evidence. Thus the axiomatic approach helps us to gain much more insights into the mathematical structures of rough set approximation operators.

### 2.1 Information system framework

An information system in different category have many different meaning. In the theory of Rough Sets information to be of the opinion that classification capability. People's behavior on the basis of the ability to distinguish reality or abstract object.

Let be  $I = (U, A)$  an information system (attribute-value system), where  $U$  is a non-empty set of finite objects (the universe) and  $A$  is a non-empty finite set of attributes such that  $a:U \rightarrow V_a$  for every  $a \in A$ .  $V_a$  is the set of values that attribute  $a$  may take. In words, the information table simply assigns a value in  $V_a$  to each attribute  $a$  of each object in universe  $U$ .

With any  $P \in A$  there is an associated equivalence relation

$IND(P)$ :

$$IND(P) = \{(X, Y) \in U^2 \mid \forall a \in P, a(x) = a(y)\} \quad (1)$$

The partition of  $U$  generated by  $IND(P)$  is denoted  $U / IND(P)$  (or  $U / P$ ) and can be calculated as follows:

$$U / IND(P) = \otimes \{U / IND(\{a\}) \mid a \in P\} \quad (2)$$

Where

$$A \otimes B = \{X \cap Y \mid \forall X \in A, \forall Y \in B, X \cap Y \neq \emptyset\} \quad (3)$$

If  $(x, y) \in IND(P)$ , then  $x$  and  $y$  are indiscernible by attributes from  $P$ . In words, for any selected subset of attributes  $P$ , there will be sets of objects that are indiscernible based on those attributes. These indistinguishable sets of objects therefore define an equivalence or indiscernibility relation, referred to as the  $P$ -indiscernibility relation.

### 2.2 Definition of rough set

Let  $X \subseteq U$ , be a target set that we wish to represent using attribute subset  $P$ . That is, we are told that an arbitrary set of objects  $X$  comprising a single class, and we wish to express this class (i.e., this subset) using the equivalence classes induced by attribute subset  $P$ . In general,  $X$  cannot be expressed exactly, because the set may include and exclude objects which are indistinguishable based on attributes  $P$ .

For example, consider the target set  $X = \{O_1, O_2, O_3, O_4\}$ , and let attribute subset  $P = \{P_1, P_2, P_3, P_4, P_5\}$ , the full available set of features. It will be noted that the set  $X$  cannot be expressed exactly because in  $[x]_P$ , objects  $\{O_3, O_7, O_{10}\}$  are indiscernible.

Thus, there is no way to represent any set  $X$  which includes  $O_3$  but excludes objects  $O_7$  and  $O_{10}$ .

However, the target set  $X$  can be approximated using only the information contained within  $P$  by constructing the  $P$ -lower and  $P$ -upper approximations of  $X$ :

$$\underline{P}X = \{x \mid [x]_P \subseteq X\} \quad (4)$$

$$\overline{P}X = \{x \mid [x]_P \cap X \neq \emptyset\} \quad (5)$$

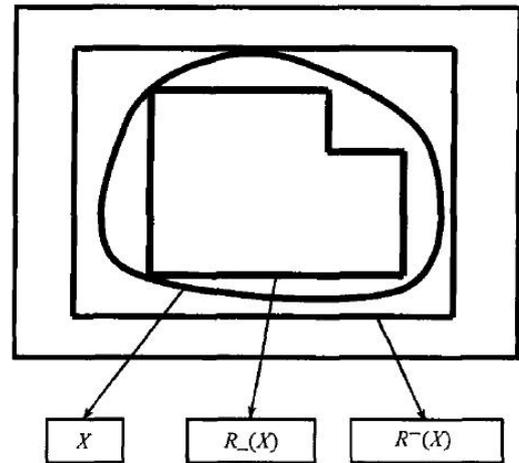


Figure 1. Diagram of Rough Set Concepts

### 2.3 Lower approximation and positive region

The  $P$ -lower approximation or positive region is the union of all equivalence classes in  $[x]_P$  which are contained by (i.e., are subsets of) the target set. In the example,  $\underline{P}X = \{O_1, O_2\} \cup \{O_4\}$ , the union of the two equivalence classes in  $[x]_P$  which are contained in the target set. The lower approximation is the complete set of objects in that can be positively (i.e., unambiguously) classified as belonging to target set  $X$ .

### 2.4 Upper approximation and negative region

The  $P$ -upper approximation is the union of all equivalence classes in  $[x]_P$  which have non-empty intersection with the target set. In the example,  $\overline{P}X = \{O_1, O_2\} \cup \{O_4\} \cup \{O_3, O_7, O_{10}\}$ , the union of the three equivalence classes in  $[x]_P$  that have non-empty intersection with the target set. The upper approximation is the complete set of objects that in  $U/P$  that cannot be positively (i.e., unambiguously) classified as belonging to the complement of the target set  $\overline{X}$ . In other words, the upper approximation is the complete set of objects that are possibly members of the target set  $X$ .

The set  $U - \overline{P}X$  therefore represents the negative region, containing the set of objects that can be definitely ruled out as members of the target set.

### 2.5 Boundary region

The boundary region, given by set difference  $\overline{PX} - PX$ , consists of those objects that can neither be ruled in nor ruled out as members of the target set X.

In summary, the lower approximation of a target set is a conservative approximation consisting of only those objects which can positively be identified as members of the set. (These objects have no indiscernible "clones" which are excluded by the target set.) The upper approximation is a liberal approximation which includes all objects that might be members of target set. (Some objects in the upper approximation may not be members of the target set.) From the perspective of U/P, the lower approximation contains objects that are members of the target set with certainty (probability = 1), while the upper approximation contains objects that are members of the target set with nonzero probability (probability > 0).

### 2.6 The rough set

The tuple  $\langle PX, \overline{PX} \rangle$  composed of the lower and upper approximation is called a rough set. Thus, a rough set is composed of two crisp sets, one representing a lower boundary of the target set X, and one representing an upper boundary of the target set X.

The accuracy of the rough set representation of the set X can be given (Pawlak 1991) by the following:

$$\alpha_p(X) = \frac{|PX|}{|\overline{PX}|} \quad (6)$$

That is, the accuracy of the rough set representation of X,  $\alpha_p(X)$ ,  $0 \leq \alpha_p(X) \leq 1$ , is the ratio of the number of objects which can be *positively* placed in X to the number of objects that can be *possibly* be placed in X. This provides a measure of how closely the rough set is approximating the target set. Clearly, when the upper and lower approximations are equal (i.e., boundary region empty), then  $\alpha_p(X) = 1$ , and the approximation is perfect. Whenever the lower approximation is empty, the accuracy is zero (regardless of the size of the upper approximation).

### 2.7 Definability

In general, the upper and lower approximations are not equal. In such cases we say that target set X is undefinable or roughly definable on attribute set P. When the upper and lower approximations are equal (i.e., the boundary is empty),  $\overline{PX} = PX$ , then the target set X is definable on attribute set P. We can distinguish the following special cases of undefinability (Pawlak, Wong, & Ziarko 1988):

Set X is internally definable if  $PX \neq \phi$  and  $\overline{PX} = U$ . This means that on attribute set P, there are objects which we can be certain belong to target set X, but there are no objects which we can definitively exclude from set X.

Set X is externally definable if  $\overline{PX} = \phi$  and  $PX \neq U$ . This means that on attribute set P, there are no objects which we can be certain belong to target set X, but there are objects which we can definitively exclude from set X.

Set X is totally non-definable if  $PX = \phi$  and  $\overline{PX} = U$ . This means that on attribute set P, there are no objects which we can be certain belong to target set X, and there are no objects which we can definitively exclude from set X. Thus, on attribute set P, we cannot decide whether any object is, or is not, a member of X.

## 3. REMOTE SENSING IMAGE CLASSIFICATION

### 3.1 Concept of remote sensing image classification

Remote sensing image classification is based on the value of data files. divide; The course of divide Sub-Pixel into many limited date type. Multispectral remote sensing image classification is based on Each pixel's multi-spectral vector data, As shown in Figure 2.1. Supposing multi-spectral images with n bands, while  $(i, j)$  location pixel in all bands's gray value can constitute a vector  $X = (x_1, x_2, \dots, x_n)^T$ , Which  $x_k$  states that the k bands on the image pixel gray value, X pixel called the eigenvalues, contain X dimensional space called the feature space,

So that n band multi-spectral images can be used dimensional feature space to a series of points to show. In the classification of remote sensing images, usually called target a certain type in images as mode, and Regarded belong to this type's Sub-Pixel as samples, called Multi-spectral vector  $X = (x_1, x_2, \dots, x_n)^T$  as Sample observations.

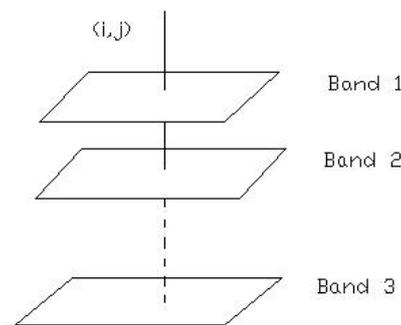


Figure 2. Examples of multi-spectral image

### 3.2 Remote sensing image supervised classification

Remote sensing image supervised classification idea is: First According to the category defined a priori knowledge discriminant function and the corresponding criterion, Which use a certain number of known types of samples (known as the training sample) of the observed values determined discriminant function parameters to be determined in the process as learning or training, Then take unknown types of sample observations into the category function, and then based on the sample of the types of criteria to judge each category.

### 3.3 Uncertainties in Remote Sensing Information Appraise Based on Theory of Rough Sets

One of the key points in research of uncertainty in remote sensing is measuring and visualizing the degree and spatial

distribute of uncertainty completely and accurately in the processing of remotely sensed image. In the classical fashions, e.g. error matrix and Kappa coefficient, the performance of the classification models is estimated directly on the training date. Whereas it is actually not appropriate. The error matrix based on the training date set can not be regarded as the measurement of overall accuracy of classification models, and these models's performance need to be evaluated on "out-of-sample-date" that have not been used in constructing the models. In this paper, we apply the rough sets theory as the application framework of measuring the attribute uncertainties in remote sensing information, and several measures are proposed for assessing the attribute uncertainties in sample area date and different spatial objects based on the scale of pixel, landcover class and the whole image in classified remotely sensed imagery. These measurements could measure effectively attribute uncertainties and facilitate to the propagation of error and uncertainties in classified remotely sensed date. Subsequently.

In remote sensing images, corresponding Rough set theory basic concepts is:  $S = (U, A)$  is constituted by the image information systems, among which on the domain of  $U$  is the image of all the pixel, a property set  $A$  can including different wavelengths gray value, texture, geometric characteristics and so on,  $U/A = \{R_1, R_2, \dots, R_m\}$  as one of category for remote sensing image, rough set  $X \subseteq U$  as the real world's features objectives or the feature category, belong to the target similar expression. Sub-Pixel set can not be completely accurate expression reality feature  $X$  in the world, so use the theory of the  $P$ -lower approximation and the  $P$ -upper approximation in the rough set approximate expression.

Use Rough Set to analysis remote sensing data's fuzzy uncertainty, The key is to find analysis object's  $P$ -lower approximation and  $P$ -upper approximation.

And then count up accuracy or roughness of rough set, and then conduct comparison between different objects, Thus use different target to control different approach's uncertainty, to meet the actual requirements.

#### 4. THE SAMPLE AREA DATA IN SUPERVISED CLASSIFICATION UNCERTAINTY MEASUREMENT

In the course of supervised classification First choice which can identify or other information can help determine the type of pixel building model, at the then on this basis classification. Therefore, in classification process choice sample data is vital importance.

District image samples and testing zone data in supervised classification can as decision tables concept in the theory of rough set.

Decision table is a kind of special and important knowledge system. Most decision-making can be express by the form of decision table, this tool plays an important role in the decision-making.

According to knowledge expression system decision table can defined as follows:

Definitions 1.1: In the theory of rough set decompose attribute information system into  $A = C \cup D$  ( $C \cup D = \phi$ ), therefore,

decision system can defined as  $T = (U, C \cup D, V, f)$ ,  $D$  is decision-making attribute set.

In the theory of rough set, decision table be designed to find dependence of properties between condition attributes and decision-making, and then contain a minimum description based on conditions and access attributes. It is the description framework of supervised learning. In decision table, Whether it is complete or not complete, decide attribute can always decide space division and a different concept. Decision-making attribute just have right region border region. The right region of the decision table is some basic knowledge of the sets, each of these elements can be exported a coordinated decision-making rules. On the contrary, Each border of the element can derived non-coordinated decision-making rules. Whether decision-making attributes of the border is empty, decide whether its Approximate rough. On the other word, Rough of the decision-making attributes reflect the decision-making rules non-coordination. So, the rough of decision attributes about approximation of the space can distinguish by distinguish to be measure.

In the supervised classification of the remote sensing images, because area of samples and testing zone data has already have designated by the user or by a higher resolution images available information, That area of samples and testing zone date each types of the Sub-Pixel as known, the category of This information is equivalent to decision-making attributes in information system, area of samples and testing zone data constitute the decision-making system of rough set theory. So can use decision-making attributes rough entropy to measurement uncertainties of the area samples and testing zone data

Definition 1.2:  $U$  as domain, the relationship  $P$  divide  $U$  into  $U/P = \{X_1, X_2, \dots, X_m\}$ , so the information Shannon of  $P$  can definition as:

$$E(P) = \sum_{i=1}^m \frac{|X_i|}{|U|} \left(1 - \frac{|X_i|}{|U|}\right) \quad (7)$$

Among which  $\frac{|X_i|}{|U|}$  means  $X_i$  the probability in the domine.

Information Shannon measure the information provided by the sources the average information size. Is one of uncertainty measurement of relationship (or knowledge).

From (1) we can kown:

$$E(P) = \sum_{i=1}^m \frac{|X_i|}{|U|} - \sum_{i=1}^m \left(\frac{|X_i|}{|U|}\right)^2 = 1 - \frac{1}{|U|^2} \sum_{i=1}^m |X_i|^2 \quad (8)$$

therefore can definable knowledge  $P$  Rough Shannon as:

$$E_R(P) = \frac{1}{|U|^2} \sum_{i=1}^m |X_i|^2 \quad (9)$$

Suppose a knowledge on the P rough set as U-domain, so roughness can be defined as  $\rho_p(X) = 1 - d_p(X)$ , among which

$$d_p(X) = \frac{|P_*(X)|}{|P^*(X)|}, \quad P_*(X) \text{ and } P^*(X) \text{ express set } X \text{ about}$$

the knowledge of P upper and lower approximate Set. Set roughness a certain extent reflected the uncertainty.

## 5. UNCERTAINTY MEASUREMENT OF THE CLASSIFICATION RESULTS

### 5.1 Error matrix and Kappa coefficient

Remote sensing data attribute uncertainty metric usually based the classification evaluation of the Sub-Pixel (Lunetta et al, 1991), Property the most frequent method evaluation attribute error is establish the error matrix analysis. Surface classification true value is often called as reference classification map, Through the classification algorithm of which through needs assessment contain the results be called the actual classification map.

Error matrix also known as Confusion matrix, It is one to express Sub-Pixel divided into which category and the ground test for this type 's comparison array (Congalton,1991). arrange of the matrix representative reference date, but trip representatives categories data classification by remote sensing data. The omission error and omission error of every category can directly contain from error matrix. In addition variety of precision measurement indicators can be calculated from error matrix, for example Overall accuracy, Producer's accuracy and User's accuracy (Story and Congalton, 1986).

Besides the description accuracy of measurement above, use all kinds of analysis technology on the basis of error matrix, can be used to compare different classifications the most commonly used analytical techniques is Kappa.

Kappa analysis technical is one of multivariate statistical analysis techniques, In statistical sense, it reflect how extent superior to the classification results, and can be used for comparison the error matrix between two classification rectangular array whether have distinct difference Kappa analyze result is KHAT statistics. From error matrix we can count KHTA overall statistical of value classification and Kappa coefficients and variance estimation method of different classification.

Although from error matrix we can get Overall accuracy, Producer's accuracy and User's accuracy and Kappa coefficient and so on, And has been become the core methods of remote sensing data classification accuracy evaluation. However, in practical application, there are still many problems.

### 5.2 The problem based on the method of Error matrix evaluation

Because the basis of error matrix is comparison between a certain types of realities ground samples and Categories, different sampling design and sample size direct relative with evaluation results. Secondly, use which sampling methods relative with further analysis based on error matrix, because different model require different sampling estimation method; Finally, sampling method determine samples spatial distribution, this directly affects the cost of accuracy evaluation. Remote sensing image

### 5.3 Classification evaluation based on the theory of rough set

Uncertainty indicator Based on the overall scale images, can use attribute shannon in rough set, approximate classification accuracy and approximate classification quality concepts.

The first measure is approximate classification accuracy under R,  $\xi$ :

$$\alpha_R(\xi) = \frac{\sum_{i=1}^n |RX_i|}{\sum_{i=1}^n |\overline{RX}_i|} \quad (10)$$

The second measure is approximate classification quality under R,  $\xi$ :

$$r_R(\xi) = \frac{\sum_{i=1}^n |RX_i|}{|U|} \quad (11)$$

Approximate classification accuracy description when using knowledge R classification objects, the percentage of right decision-making in decisions may be; Classification quality application use knowledge R the percentage can be accurately divide into  $\xi$ .

Approximate classification accuracy and approximate classification quality can be used as uncertainty measurements of remote sensing images, reflection overall uncertainty of image. So we can use mode(1) and mode (2) compare uncertainty in plenty of remote sensing classification appliances, Here between multiple classifiers still need to meet the partial sequence or Here between multiple classifiers still need to meet the sequence or partial sequence relations simulate sequence relations.

In the course of classification remote sensing images, Will produce various land cover types subordinate in image of each pixel. Use subordinate of each Sub-pixel. We can calculate the shannon of each sub-pixel, lower approximation and upper approximation in different land cover types, and then calculate uncertainty measure based on type and overall image.

## 6. CONCLUSION

To date, Remote Sensing technologies have been applied to environmental management, monitoring and control. Remote Sensing technologies are also being applied to monitor land desertification, land use cover, environment pollution, etc. Due to limitations in instrument and processing technology in RS, system errors and measurement errors may corrupt the data. Errors maybe associated with both attribute value and its location. After collecting spatial data, we then need to process, analyze and convert the data in order to make RS information understandable to users. When we produce the final RS products, newer-rors can be created as a result of spatial operations on maps or images that contain errors (spatial error propagation). If we ignore these errors in the RS products, it can result in economic loss. Though, the uncertainty of remote sensing information has been a hotspot in the spatial information process and many research have provided us lots of valuable results.

In this paper we deem uncertainty of remote sensing information attribute and the current classification accuracy this two remote sensing issues hot issues as study core, focusing on the topic on uncertainty expression and evaluation problem in remote sensing data classification, based on the theory of rough set measurement and evaluation data of the sample areas and different classification result of remote sensing image evaluation.

In the general, this paper discusses use theory of rough sets appraise uncertainties in remote sensing information .But there are still more problems that need to research deeply, such as the relativity of pixels, the vector of fuzzy weight, ect.

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