A ROBUST ESTIMATION ALGORITHM OF EPIPOLAR GEOMETRY THROUGH THE HOUGH TRANSFORM

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ABSTRACT:

Fundamental matrix is an algebraic expression of the epipolar geometry, and plays a very important role in the computer vision. It is all along an important research topic that how to raise the precision and robustness of the fundamental matrix estimation in the field of computer vision. On the basis of this point, this paper firstly analyzes the character and function of the fundamental matrix, and proves that if there is more matching points than three in a single two-dimension line, the extra points will be regarded as redundant points in theory. Those points will not be used as matching points. Then in order to eliminate those collinear redundant points at the same time of establishing the corresponding relationship among those matching points, especially on images which there are many line features, the author introduces the Hough transform algorithm. Finally, the experiments show the average epipolar distance and residual errors of the fundamental matrix which is estimated with the improved algorithm in which the redundant points are eliminated is less than that of using the previous algorithm .The accuracy and robustness of estimation are improved evidently.

1. INTRODUCTION

Epipolar geometry relationship is the only information of two uncalibrated images obtained from the same scene but different views. It can be expressed by a 3×3 matrix whose rank is 2, which is fundamental matrix F . The fundamental matrix plays an important role when to solution the matching relationship, camera inner parameter, motion parameter and stereo reconstruction from uncalibrated images. For this reason, the fundamental matrix estimation has become an important direction in computer vision field for many workers(Faugeras O,1993; Deriche R,1994; Faugeras O,1992a; Quan Long ,Kanade Takeo,1997; Hartley R, Zisserman,2000). Longuet-Higgins brought forward 8-point algorithm in 1981[6], which is a linear algorithm and easily realized but difficultly to be used in practice for its exceptional sensitivity to noise. In 1992, Faugeras proposed a nonlinear algorithm(Faugeras O, 1992b),which not only equalized the instability of the 8-point algorithm, but also improved the accuracy of the fundamental matrix evidently. Soon, Hartley found an improved 8-point algorithm(Hartley R,1995). The new algorithm can reduce the influence of noise through the translation to two-dimension data and scale transform. Its effect is equal to the iteration method which is used as the best algorithm at that time. In spite of that, it should be studied further on the stability of epipoles and the accuracy of fundamental matrix. Now, there are some nonlinear algorithms in common use ,such as M-estimate method, LMeds method and so on .These algorithms all transform the problem into the solution of the optimization problems. But if there are lots of matching point, the nonlinear algorithm has a complicated calculation and is difficult to get a best global solution.

Usually if we have got 8 matching points or more, the available algorithm is 8-point algorithm or the improved 8-point algorithm. Their advantage is that they are all linear algorithm and easy to be realized, but their disadvantage is any two-dimension data is seen to be equal, and the result will be more accurate with matching points adding, taking little about the collinear redundant points. The experiments prove that the collinear redundant points shouldn't be ignored in time of estimating the fundamental matrix, especially of images which there are many line features. If eliminating all collinear redundant points from matching points one time, the accuracy and robustness of the fundamental matrix estimation are improved evidently.

2. SEVERAL BASIC CONCEPTS

2.1 Epipolar geometry and fundamental matrix

As seen in Fig.1, the projection points of the 3D spacial point X is x on image I and x' on image I' . x and x' are corresponding matching points. C and C' are optical centers of two cameras respectively. The points e and e' which are the intersected points of the line CC' with image I and I', are the epipoles of two images. The radial line $< e', x' >$ is called epipolar line of the point x on image I . $< e, x >$ has the same definition. The function of the fundamental matrix is to map the points on image I to be epipolar line on image I'. The definition equations are:

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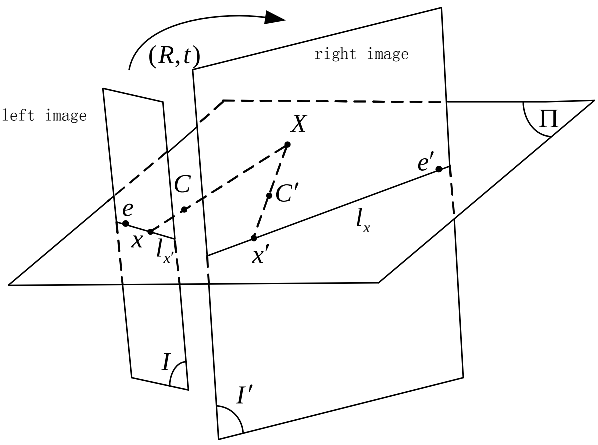


Figure.1 Epipolar geometry

$$Fx = < e', m' > = [e'] \times m' \quad (1)$$

$$F^T x' = < e, m > = [e] \times m \quad (2)$$

Where $[e]_\times$ is dissymmetry matrix, $[e]_\times = \begin{bmatrix} 0 & -e_3 & e_2 \\ e_3 & 0 & -e_1 \\ -e_2 & e_1 & 0 \end{bmatrix}$. That is to say, the corresponding matching point x' of point x must be on epipolar line Fx ; contrarily, x must be on epipolar line $F^T x'$. So we get the equations as follows:

$$x'^T Fx = x^T F^T x' = 0 \quad (3)$$

2.2 ISPRS Affiliation (optional)

From equations (3) we know, if x_i and x_i' are corresponding matching points, then $x_i'^T Fx_i = 0$. Because of the noise and bad data, it is usually $x_i'^T Fx_i \neq 0$. So, we define the residual errors:

$$r_i = \left| x_i'^T Fx_i \right| \quad (4)$$

and epipolar distance:

$$d_i = \left(\frac{1}{(Fx_i)_1^2 + (Fx_i)_2^2} + \frac{1}{(F^T x_i')_1^2 + (F^T x_i')_2^2} \right) (x_i'^T Fx_i)^2 \quad (5)$$

Where $(Fx_i)_j \ (j = 1,2)$ is the j[th] component of the vector Fx_i.

2.3 Key Words

That if there is more matching points than 3 in a single two-dimension line , the extra points will not be used as matching points ,is so-called collinear point redundancy. To prove this, we introduce a concept – line parameter λ. Suppose that l_{ab} represents the line determined by points a and b , and then any point m in line l_{ab} excluding a and b can be expressed as

$$m = a + \lambda(b - a), \text{ and } \lambda \neq 1 , \quad (6)$$

Obviously, m and λ are corresponding one by one. If m_1 , m_1' , m_2 and m_2' are four different matching points in image I_1 and I_2 , and m_1 , m_2 is in line l_{ab} and m_1' , m_2' in line $l_{a'b'}$. Their line parameter is λ_1 , λ_2 , λ_1' and λ_2' respectively, then we can get equations as follows according to equation (6) :

$$m_1 = a + \lambda_1(b - a) , \quad m_2 = a + \lambda_2(b - a)$$

$$m_1' = a' + \lambda_1'(b' - a') , \quad m_2' = a' + \lambda_2'(b' - a')$$

and we can also get according to equation (3) that

$$a'^T Fa = 0 , \quad b'^T Fb = 0$$

$$(a' + \lambda_1'(b' - a'))^T F(a + \lambda_1(b - a)) = 0 \quad (7)$$

$$(a' + \lambda_2'(b' - a'))^T F(a + \lambda_2(b - a)) = 0 \quad (8)$$

then through settling equations (7) and (8),we can get bellow equations:

$$\lambda_1(1 - \lambda_1')a'^T Fb + \lambda_1'(1 - \lambda_1)b'^T Fa = 0 \quad (9)$$

$$\lambda_2(1 - \lambda_2')a'^T Fb + \lambda_2'(1 - \lambda_2)b'^T Fa = 0 \quad (10)$$

In equations (9) and (10),if $b'^T Fa = 0$ or $a'^T Fb = 0$, they are all equal to zero, namely that equations (9) and (10) are equivalence, then we can conclude that (7) and (8) are equivalence, and this implies that there is one point among the four points is redundant. If $b'^T Fa \neq 0$ and $a'^T Fb \neq 0$, there will be

$$\frac{\lambda_1(1 - \lambda_1')}{\lambda_1'(1 - \lambda_1)} = \frac{\lambda_2(1 - \lambda_2')}{\lambda_2'(1 - \lambda_2)} = -\frac{b'^T Fa}{a'^T Fb} \quad (11)$$

Obviously this equation expresses that (7) and (8) are still equivalence. Therefore, if we choose more points than 3, we will meet the redundancy problem of matching points(Wang Wei,Shen Peiyi,Wu Chengke,1998). The matching point redundancy not only raises the calculation largely, but also brings the instability of the fundamental matrix. Consequently, this paper introduces a constraint which eliminates the collinear redundant points on the base of some classical matching algorithm, which not only saves the operation time, but also improves the robustness of the fundamental matrix estimation.

2.4 Abstract

The Hough transform is proposed by Hough in 1962, and it is used to detect curves on images such as line, circle, parabola and ellipse whose shape can be expressed by some function. Its use is successful in many fields such as image analysis, pattern recognition and so on. Its fundamental theory is that transforms curves (include straight line) in image space to that in parameter space, and then through detecting the maximum point in parameter space, qualifying the description parameter of the curves, sequentially, extracting the regular line on image(Zhang Zuxun, Zhang Jianqing, 1996).

The line mode adopted in line Hough Transform is $\rho = x\cos\theta + y\sin\theta$, where the perpendicular distance from the origin to the line is ρ, and θ is the angle from the perpendicular to positive direction of x-axes. As seen in Fig.2:

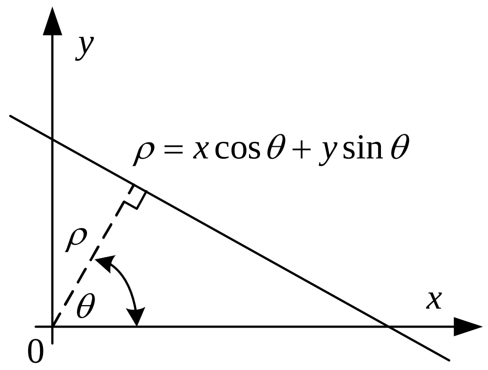


Figure.2 Line mode in Hough transform

To any point (x, y), the Hough Transform maps it to a point in a sinusoidal line in parameter space (θ, ρ). As a line in image space can be made curtain uniquely by a couple of parameters (θ_0, ρ_0), after transforming each point in the line to point in parameter space , each sinusoidal line that the point is in passes the point (θ_0, ρ_0) certainly, and the coordinate of the point in the parameter plane (or space) is the line's parameter in the image space. Through this, the problem of detecting lines in image space is converted to be that of detecting collinear points in parameter space. But because of the noise and the position error of feature point, the mapping curves don't all pass the point rigorously but form a peak, and of course, as long as we get the point of the peak, we can certain the line's parameter.

3. THE ELIMINATION OF THE COLLINEAR MATCHING POINT ON BASE OF THE HOUGH TRANSFORM

According to section 2.3, the collinear redundant matching points not only raise computation largely, but also cause the instability of the fundamental matrix estimation. So, this paper, before estimating the fundamental matrix F, eliminates all the redundant points in matching points one time. This course is as follows:

(1) Firstly, detect the maximum of x-axes and y- axes among all the matching points on left image, namely $\min_x$、$\max_x$、$\min_y$ and $\max_y$, and then with the new coordinates origin which is the top-left corner $(\min_x, \min_y)$ of the rectangle R made up of the above four maximums, transform the coordinates of the matching points according to the two equations $x_i^n = x_i^o - \min_x$, $y_i^n = y_i^o - \min_y$, and in sequence, store the new coordinates (x_i^n, y_i^n) into an array P, where i is the number of matching points and (x_i^o, y_i^o) the former coordinates.

(2) With the Hough transform described at section 2.4, initialize a two-dimension accumulative matrix $H(\theta_i, \rho_j)$ in transform space ρ, θ, where the quantization number in direction ρ is the pixel in diagonal direction of rectangle R, and that in direction θ is l_θ ($l_\theta = 180/\Delta$, Δ is the interval and it is one degree in this paper)

(3) Search every element in array P in sequence. To each element, add one to the corresponding point in accumulative matrix H and record the index of each point whose corresponding point is this at the same time, in order to be used for eliminating the redundant points next step.

(4) Do threshold detection to the accumulative matrix H and pick up the indexes at each point with record more than 3, and then order the matching points which have those indexes according to matching intensity. Remain the first three points with the maximal intensity and other points are eliminated as redundant points;

(5) Do above steps with the right image.

4. THE STEPS OF THE ALGORITHM AFTER IMPROVEMENT

The main steps of the improved algorithm are as follows:

(1) Detect the interesting points with Harris operator;

(2) Pick-up initial matching points with correlative coefficient matching based on gray degree;

(3) Eliminate the ambiguous matching points with relaxation algorithm(Zhengyou Zhang, Rachid Deriche et al, 1995);

(4) Eliminate the collinear redundant points with the improved method in this paper;

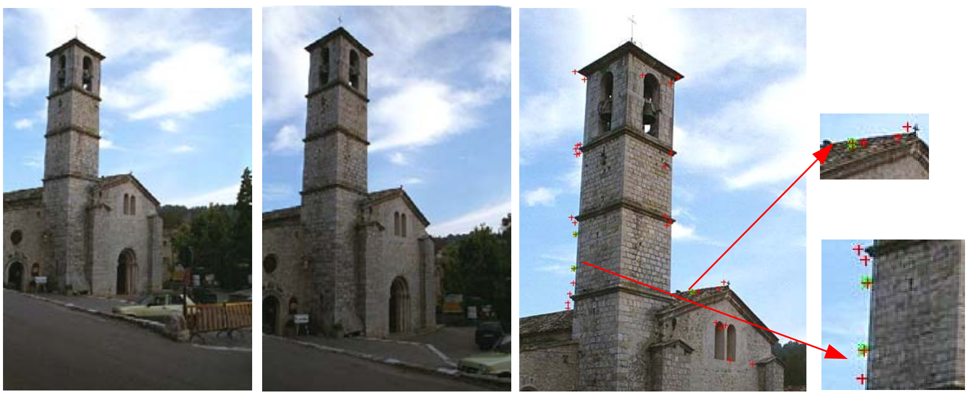
(5) Calculate the fundamental matrix F .

5. TTHE EXPERIMENT AND CONCLUSION

In order to prove the existence of collinear redundant points and its effect on the estimation precision and the robustness of the fundamental matrix estimation, this paper chooses two images shown in Fig.3 (a) and (b) to do the experiments. The experiments adopt 8-point algorithm and improved 8-point algorithm which are easy to realize but sensitive to noise respectively. And just for this, the two methods are easier to reflect the effect of the existent collinear redundant points to the accuracy of the fundamental matrix estimation.

(d) and (e) in Fig.3 show the detected collinear redundant points after local magnification. And Fig.4 is the residual errors variation gram of the two algorithms before and after eliminating the collinear redundant points. Table 1 and table 2 show partial experiment data before and after eliminating the collinear redundant points. We know, from section 2.2, the average epipolar distance reflects the mean value of the distances from the epipolar lines calculated from the current fundamental matrix to the matching points, while the average residual error reflects the mean of the absolute values of $x'^T Fx$ which is also calculated from the current fundamental matrix. The smaller the two values are, the better the estimation accuracy of the fundamental matrix is. Data in table 1 and table 2 and statistical data in Fig.4 all show that, if there are no redundant points among given matching points, the average epipolar distance and the average residual error change little before and after eliminating the redundant points, while if there are the redundant points, the accuracy of the fundamental matrix estimation before eliminating the redundant points is evidently not as good as that after eliminating them.

The theory and the results of experiments all prove that the redundant collinear points certainly exist among the matching points which are used to estimate the fundamental matrix, and decrease the accuracy and robustness of the fundamental matrix estimation. But eliminating the collinear redundant points can improve the accuracy and robustness of the fundamental matrix estimation evidently.



(a) and (b) are the original images; (c) is image with matching points(the red "+") and redundant collinear points(the green circle); (d) and (e) are local enlarged images in order to discern the collinear redundant points.

Figure.3 Experiment Images

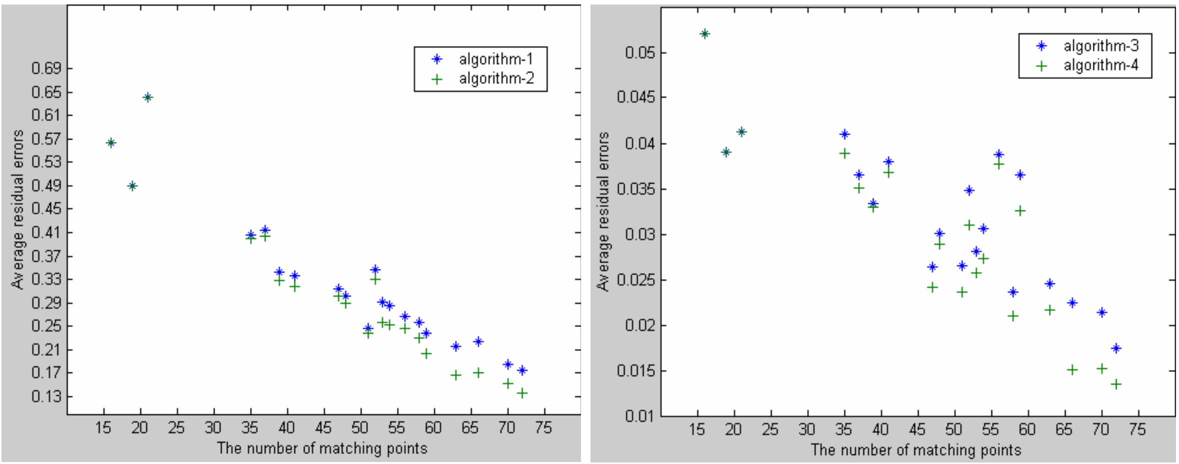
Matching points	The 8-point algorithm		The new 8-point algorithm	
	Errors	Distance	Errors	Distance
21	0.6312	0.5986	0. 6312	0.5986
35	0.4053	0.6162	0.3994	0.5625
48	0.2987	0.5868	0.2891	0.5569

Table.1 The residual errors comparison of the 8-point algorithm and the new 8-point algorithm

Matching points	The improved 8-point algorithm		The new improved 8-point algorithm	
	Errors	Distance	Errors	Distance
21	0.0413	0.3672	0.0413	0.3672
35	0.04093	0.2307	0.03894	0.2136
48	0.0312	0.1253	0.028907	0. 1018

Table.2 The residual errors comparison of the improved 8-point algorithm and the new improved 8-point algorithm

Note: Matching points denotes the pair of matching points; The new 8-point algorithm and the new improved 8-point algorithm denote the 8-point algorithm and the improved 8-point algorithm after eliminating the collinear redundant points through the Hough transform respectively.



(a)The 8-point algorithm (b) The improved 8-point algorithm

Algorithm-1 denotes the 8-point algorithm; Algorithm-2 denotes the 8-point algorithm after eliminating the collinear redundant points through the Hough transform; Algorithm-3 denotes the improved 8-point algorithm; Algorithm-4 denotes the improved 8-point algorithm after eliminating the collinear redundant points through the Hough transform.

Figure 4 The diagram of the residual errors variation

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