

# TOPOLOGY-PRESERVING NETWORK SNAKES

M. Butenuth

Technische Universität München, Remote Sensing Technology  
Arcisstr. 21, D-80333 München, Germany-  
matthias.butenuth@bv.tum.de

**KEY WORDS:** Computer Vision, Active Contour Models, Optimization, Segmentation, Networks, Topology, Roads

## ABSTRACT:

Active contour models have been part of extensive research since their introduction in the late eighties, because their concept of coupling the image data with shape control is a powerful method to delineate non-rigid curves. Various applications and enhancements have been examined using this technique, for example the development of multiple and coupled active contour models. Latest research deals with the introduction of topology into the model of active contours: network snakes exploit the given topology during the energy minimization process leading to improved results concerning the delineation of arbitrary networks and object borders of adjacent objects. However, the utilization of topology requires obviously a given correct topology and, moreover, presumes the preservation of this initial topology over the period of optimizing the energy functional. This fact can not be guaranteed in general, because close contour parts can merge or nodes with higher degrees can move around each other. Thus, a new topology-preserving energy is introduced in this paper to enhance the procedure of network snakes. The preservation of the topology is demonstrated with a synthetic example and with real application scenarios, in particular, the refinement of road networks in aerial images. The results demonstrate the general functionality and transferability of the proposed new topology-preserving energy. Concluding remarks are given at the end to point out further investigations.

## 1. INTRODUCTION

Active contour models have received a large popularity during the last two decades as powerful, physics-based technique for representing, reconstructing, recognizing and manipulating non-rigid curves or surfaces from images and image sequences. A large variety of applications utilize the method of active contours, which is reflected in numerous contributions comprising for example the detection of man-made objects such as roads (Laptev et al., 2000; Peteri et al., 2003) or buildings (Fua, 1995), the tracking of objects in image sequences (Delagney et al., 1995; Paragios and Deriche, 2000) or medical image applications (McInerney and Terzopoulos, 1996; Singh et al., 1998).

Two directions of active contour models have been developed as complement to each other: the firstly proposed model of *parametric active contours* (Kass et al., 1988; Blake and Isard, 1998) and the later introduced model of *geometric active contours* (Caselles et al., 1993; Malladi et al., 1995). The idea behind both concepts is quite similar: the coupling of the image data with an internal energy in an energy minimization framework regarding parametric active contours or the combination of the level set method with the curve evolution theory concerning geometric active contours. The conform goal of both models described with the explicit or implicit representation is to enable and control a smooth curve delineating non-rigid object contours. The elementary difference of both models is the *explicit representation* of parametric active contours compared to the *implicit representation* of geometric active contours, which involves several properties of the respective model. Parametric active contours allow for a direct interaction, e.g. by a user, while geometric active contours are parameterized after the evolution of the contour and a direct interaction is more difficult. On the other hand, the implicit representation enables topological changes during the evolution naturally, whereas the topology of

parametric active contours is rigid and a splitting or merging is complicated. When the object topology is known or the topology should be exploited during the optimization, the explicit concept is better suited. Otherwise, the delineation of objects with an unknown topology can be performed better with the implicit concept. Parametric active contours are sensitive to the initialization and require a starting point near to the object of interest, because the contour is locally optimized. In contrary, geometric active contours are relatively independent to the initialization, e.g. a circle within the object or the image border can be chosen. The global deformation causes less efficiency compared to the parametric model, even fast marching methods increase the computational speed.

In general, both models are only defined for *single closed* object boundaries, while newer research deals with the development of *multiple active contours* to facilitate solutions dealing with more than one object. An exemplary approach regarding the detection of intersections to control connected components automatically for eliminating overlaps of object contours is addressed in (Delingette and Montagnat, 2001). Another approach regarding multiple 3D deformable models is addressed in (Lachaud and Montanvert, 1999). Further problems occur, when several objects of interest are near to each other and partially touch each other: latest research of *coupled active contour models* incorporates topology in form of a penalty force during the optimization of all contours taking into account that objects cannot merge (Zimmer and Olivo-Marin, 2005). An adaptive adjacency graph is introduced in (Jasiobedzki, 1993) to extract *networks of active contours*. This work connects active contours at nodes during the deformation. The connectivity of the graph is achieved by imposing external energies in the form of constraints or springs to keep the adjacent contours together. However, a clear mathematical basis of the nodes with a degree unequal two is not given and, thus, vacuums can appear leading to an incorrect topology.

A new method of *network snakes* incorporating a complete topological and shape control during the optimization has been introduced in (Butenuth, 2007). Based on the concept of parametric active contours a new mathematical definition is proposed, which enables the optimization of arbitrary networks and the delineation of adjacent objects with only one boundary in between. Possible applications are the extraction of field boundaries from high resolution satellite imagery (Butenuth et al., 2007) or the delineation of adjacent cells in microscopic cell imagery (Butenuth and Jetzek, 2007). The exploitation of the topology during the energy minimization turns out to be a powerful method to deal with noise or disturbances in the imagery.

However, the utilization of the *topology* requires obviously a given network which is assumed to be topologically correct and, moreover, presumes the *preservation* of this initial topology over the period of optimizing the energy functional. This fact can not be guaranteed in general, because close contour parts can merge or nodes with higher degrees can move around each other. These undesired effects involve, that the criteria of a planar graph can no longer be satisfied. Thus, in this paper an enhanced *topology-preserving energy* is introduced to preserve the topology during the complete processing to avoid touching or overlapping contour parts loosing the initial correct topology.

In the next section the method of network snakes is briefly outlined, while Section 3 focuses on the new topology-preserving energy. The general functionality is demonstrated using a synthetic example and, in Section 4, it is exemplarily applied to the refinement of two road networks in a suburban and an open landscape environment to demonstrate the usability of the proposed new method. The refinement of roads with the use of road hypotheses generated from a GIS database is accomplished within the work of (Bordes et al., 1997). The delineation of road networks in urban or suburban areas is the content of current research due to the complexity of the scenario caused by disturbing local context objects such as buildings, trees, cars and their shadows, for example addressed in (Hinz and Baumgartner, 2003; Gautama et al., 2006). Finally, concluding remarks are given and further investigations are discussed in Section 5.

## 2. NETWORK SNAKES

A traditional parametric active contour, often called snake, is defined as a parametric curve  $C$

$$C(s, t) = (x(s, t), y(s, t)) , \quad (1)$$

where  $s \in [0,1]$  is the arc length,  $t$  is the time or iteration number and  $x$  and  $y$  are the image coordinates of a closed 2D-curve (Kass et al., 1988). The total energy functional  $E(C(s))$ , to be minimized, is defined as

$$E(C(s)) = \int_0^1 [E_{img}(C(s)) + E_{int}(C(s)) + E_{con}(C(s))] ds . \quad (2)$$

The energy functional consists of the *image energy*  $E_{img}(C(s))$  given an optimal description of the object of interest in the image, the *internal energy*  $E_{int}(C(s))$  introducing modeled object knowledge concerning the shape behavior or movement of the

object and, finally, the *constraint energy*  $E_{con}(C(s))$  having the possibility to insert any external constraints to the energy functional. A solution of the energy functional can be derived by solving the corresponding Euler equations (Kass et al., 1988) yielding in

$$\frac{\partial E_{img}}{\partial C} - \alpha \frac{\partial^2 E}{\partial C_{ss}} + \beta \frac{\partial^4 E}{\partial C_{ssss}} = 0 . \quad (3)$$

The derivatives are approximated with finite differences since they cannot be computed analytically:

$$\begin{aligned} & \frac{\partial E_{img}}{\partial C} \\ & + \alpha((C_i - C_{i-1}) - (C_{i+1} - C_i)) \\ & + \beta(C_{i-2} - 2C_{i-1} + C_i) - 2\beta(C_{i-1} - 2C_i + C_{i+1}) \\ & + \beta(C_i - 2C_{i+1} + C_{i+2}) = 0 . \end{aligned} \quad (4)$$

With  $\frac{\partial E_{img}}{\partial C} = f_C(C)$  Equation 4 can be rewritten in matrix form as

$$AC + f_C(C) = 0 . \quad (5)$$

$A$  is a pentadiagonal band matrix, which depends only on the parameters  $\alpha$  and  $\beta$ . A final solution can be derived by the introduction of a step size  $\gamma$  and the time derivatives, followed by a subsequent matrix inversion getting

$$C_t = (A + \gamma I)^{-1}(\gamma C_{t-1} - \kappa f_C(C_{t-1})) , \quad (6)$$

where  $I$  is the identity matrix and  $\kappa$  is an additional parameter in order to control the weight between internal and image energy. The introduction of the *topology* to the model of parametric active contours regarding the *internal energy*  $E_{int}(C(s))$  is identified as the crucial point in (Butenuth, 2007). The approximated derivatives with finite differences are required to control the shape of the contour (cf. Equation 4), but they are not usable in the common way at nodes with a degree of  $\rho(C) \neq 2$ . The utilized derivatives are not defined, because the

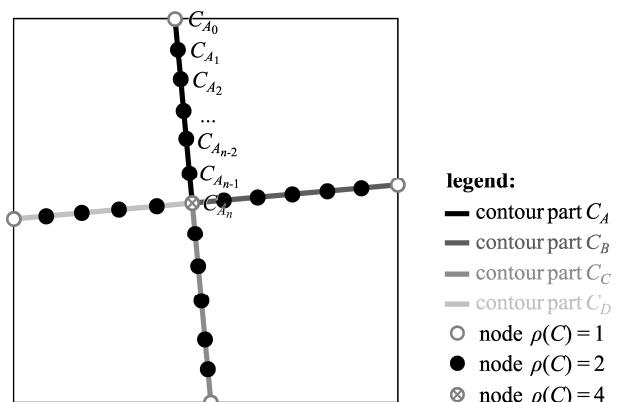


Figure 1. Topology of network snakes

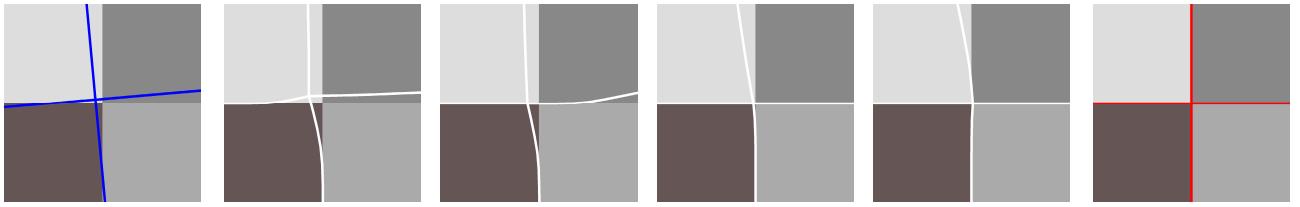


Figure 2. Synthetic example of a network snake: initialization (blue), optimization steps (white) and result (red)

required neighboring nodes are either not available at open contour ends (nodes with degree  $\rho(C) = 1$ ) or exist multiple times (nodes with degree  $\rho(C) > 2$ ), cf. the center of Figure 1 for a node with a degree of  $\rho(C) = 4$ . The solution proposed in (Butenuth, 2007) divides the given initial graph into separate contour parts  $C_A, \dots, C_Z$  connected at nodes  $C_n$ . In Figure 1 a synthetic example is taken to exemplify the contour parts of the required network: the four contour parts  $C_A, C_B, C_C$  and  $C_D$  are obtained describing a part of a contour network representing for example the boundaries between adjacent objects, each of them is depicted with a separate gray value. In general, the contour parts meet with their respective end points  $C_{A_n}, \dots, C_{Z_n}$  in the common node  $C_n$  in such a way, that the end points of the contour parts define an identical point, i.e.  $C_n = C_{A_n} = C_{B_n} = \dots = C_{Z_n}$ . Thus, the node  $C_n$  is contained in each connected contour part representing the topology and, simultaneously, depends on the specific shape model of each contour part.

The *first term* of the internal energy, weighted by the parameter  $\alpha$ , cannot support the control of the internal energy in the vicinity of  $C_n$  during the energy minimization when developing network snakes (cf. Equation 4). The finite differences of the first term approximating the required derivatives are only available for the two nodes  $C_{n-1}$  and  $C_n$  but not for  $C_{n+1}$ , because this node exists multiple times for the respective connected other contour parts ( $\rho(C) > 2$ ) or is just not available at open contour ends ( $\rho(C) = 1$ ). Thus, shape control is not possible here and the first term is not considered during the energy minimization in the vicinity of nodes with a degree of  $\rho(C) \neq 2$ . The *second term* of the internal energy, weighted by the parameter  $\beta$ , can aid the control of the shape behavior in the vicinity of  $C_n$  partly. This second term is rewritten using the available finite differences for the nodes  $C_{n-2}, C_{n-1}$  and  $C_n$  of each contour part separately to control the curvature of the contour network at nodes with a degree of  $\rho(C) \neq 2$ . The nodes  $C_{n+1}$  and  $C_{n+2}$  can not support the internal energy in the vicinity of  $C_n$ , because these nodes belong to the multiple available connected further contour parts or just not exists. Consequently, the new energy definition for *network snakes* valid for every nodes  $C_n$  with a degree  $\rho(C) \neq 2$  and the direct vicinity is defined at the common nodes with  $C_n = C_{A_n} = C_{B_n} = \dots = C_{Z_n}$  as

$$\begin{aligned} \beta(C_{A_n} - C_{A_{n-1}}) - \beta(C_{A_{n-1}} - C_{A_{n-2}}) + f_{C_A}(C_A) &= 0 \\ \beta(C_{B_n} - C_{B_{n-1}}) - \beta(C_{B_{n-1}} - C_{B_{n-2}}) + f_{C_B}(C_B) &= 0 \\ \beta(C_{C_n} - C_{C_{n-1}}) - \beta(C_{C_{n-1}} - C_{C_{n-2}}) + f_{C_C}(C_C) &= 0 \quad (7) \\ \dots \\ \beta(C_{Z_n} - C_{Z_{n-1}}) - \beta(C_{Z_{n-1}} - C_{Z_{n-2}}) + f_{C_Z}(C_Z) &= 0 , \end{aligned}$$

where the terms  $f_{C_A}(C_A), \dots, f_{C_Z}(C_Z)$  represent the image energy at the respective contour parts (Butenuth, 2007). Now, all contour parts  $C_A, \dots, C_Z$  intersect in the joint and sole node  $C_n$  and can be optimized *simultaneously* when minimizing the energy functional of network snakes. The energy definition of Equation 7 allows for an energy minimization controlling the shape of each contour part *separately* up to the common node  $C_n$  without interacting concerning their particular shape. At the same time, the exploitation of the topology is ensured during the energy minimization process.

In Figure 2 an exemplary synthetic example is given to demonstrate the general functionality of network snakes. Starting from an initial contour network (blue), the contour optimizes step by step (white) to the desired result (red). The given initial topology is maintained during the energy minimization process and, moreover, is exploited. For example, the contour part  $C_A$  (top) is not close to the desired object boundary at the beginning, but the exploitation of the topology as a result of the connection of the contour part to the network pulls the contour part  $C_A$  to the true object boundary (cf. Figure 2).

### 3. TOPOLOGY-PRESERVING NETWORK SNAKES

The exploitation of the topology within the framework of network snakes emphasizes as powerful enhancement compared to traditional parametric active contours to extract arbitrary networks and the borders of adjacent objects. In particular, contour parts not initialized close to the desired true object boundary or underlying blurry image data can only result in good object delineations utilizing the given topology. Thus, the requirements concerning the topology are high during the energy minimization process, i.e. *conserving* the given topology, which is assumed to be correct. This fact can not be guaranteed in general, because close contour parts can merge or nodes with higher degrees can move around each other leading to wrong results. These undesired effects involve, that the criteria of a planar graph can no longer be satisfied. For solving this problem, an enhanced *topology-preserving energy* is introduced in this paper to preserve the topology during the complete processing. It avoids touching or overlapping contour parts, i.e. loosing the given topology. The general goal is to satisfy the criteria of a planar graph during the energy minimization to guarantee an optimal exploitation of the topology.

The crucial point regarding the definition of a topology-preserving energy introduced to the whole energy functional is to avoid merging or overlapping contour parts. Thus, the positioning of neighboring contour parts within the network is monitored and controlled during the processing by an additional *topology energy*  $E_{topo}(C(s))$ , which is defined as

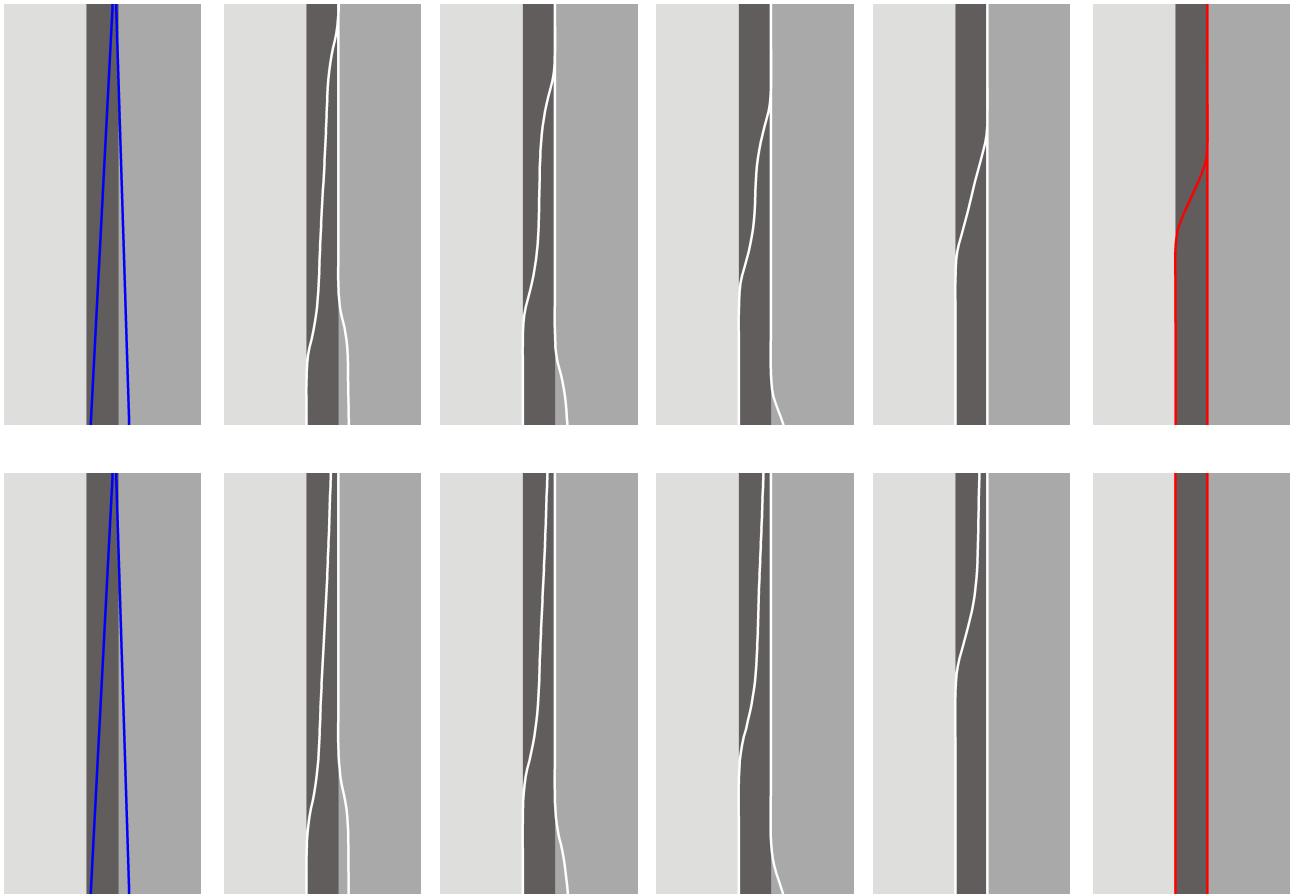


Figure 3. Synthetic example with parametric active contours without the topology-preserving energy (top) and including the topology-preserving energy (bottom): initialization (blue), optimization steps (white) and result (red)

$$E_{topo}(C(s)) = \frac{1}{d(C(s))^2} . \quad (8)$$

The parameter  $d(C(s))$  with  $0 < d < d_{max}$  describes the Euclidean distance between two neighboring contour parts. Consequently, a convergence of two contours becomes more and more expensive within the energy minimization process and, thus, inhibits the merging. The distance is introduced as a non-linear force, because the interesting point of control arises only when object contours are close to each other. The weighting of the topology-preserving energy  $E_{topo}(C(s))$  compared to the other energy terms of the total functional has to be done in a manner, that it is activated only when two contours converge. The behavior of two neighboring contour parts far away from each other is not influenced, because the image and internal energy superpose the proposed topology-preserving energy at those parts. In addition, the distance is restricted by an upper limit  $d_{max}$ , because neighboring contours will only influence each other within a specific spacing. In the vicinity of nodes with a degree of  $\rho(C) \neq 2$  representing the connectivity of a contour network the topology-preserving energy is not considered due to the intended convergence. The distance of neighboring contour parts must be calculated at each iteration step, but depending on the application a speed up with less frequent calculations is possible.

In Figure 3 a synthetic example of a traditional parametric active contour as part of a contour network is depicted (top).

Starting point are two separate contours (blue), which merge very fast during the optimization (white) leading to an obviously wrong result (red). The reason is very plausible, because the left contour moves to the closest object border, which is at the top the right one. The existence of a second contour is without any consequence here, because this fact is not considered in the traditional framework of network snakes.

The introduction of the new topology-preserving energy to the minimization process yields in a correct result as shown in Figure 3 using the same synthetic example (bottom). The starting point are again the two separate contours (blue), but now the topology-preserving-energy avoids the merging of the contour parts (white). Instead, each contour moves step by step to the desired true object boundary deriving the assumed correct result (red). Thus, the new energy definition enables the exploitation of the topology during the energy minimization in arbitrary kinds of networks preserving the initial topology over the complete period of optimization.

#### 4. EXEMPLARY RESULTS

The proposed new topology-preserving energy introduced to the framework of network snakes is examined in this section with two real application scenarios. The goal is to demonstrate the capability and transferability of the new approach in complex environments in terms of the refinement of road networks. The first example deals with the optimization of a road network in a



Figure 4. Refinement of a road network in a suburban area with network snakes including the topology-preserving energy: initialization (blue), optimization steps (white) and result (red)

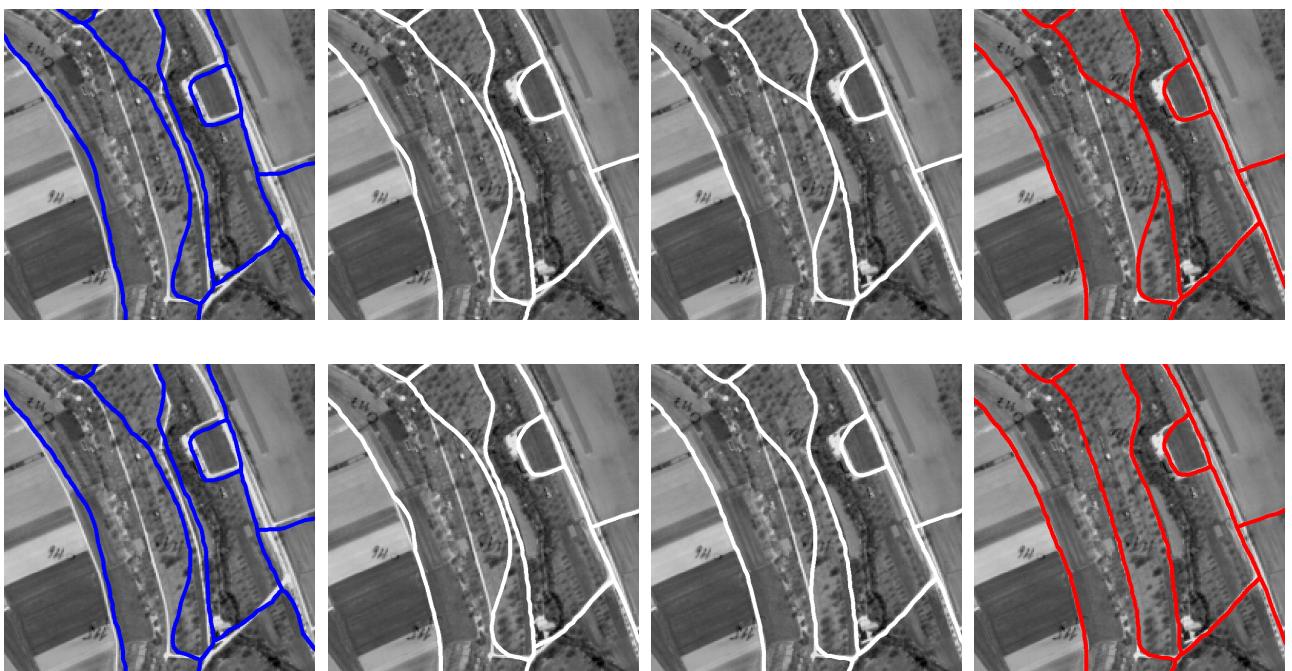


Figure 5. Refinement of a road network in open landscape with network snakes without the topology-preserving energy (top) and including the topology-preserving energy (bottom): initialization (blue), optimization steps (white) and result (red)

suburban area. The reason for the complexity are the local context objects such as buildings, trees, cars and their shadows, which hamper the detection of roads due to caused occlusions of the road or similar object representations compared to the road model. In Figure 4 an exemplary part of a suburban scene of a CIR-aerial image with a ground resolution of 1.0 m is depicted, superimposed is a road network (blue) taken from a GIS or a alternative manually defined initial contour. The given topology is assumed to be correct, but the geometric correctness is unsatisfying. Starting from the initialization the contour network moves step by step (white) to the expected final result (red). The junction in the top left part of the example given in Figure 4 moves to the true position only caused by the exploitation of topology during energy minimization. The road is in most parts occluded by trees or their shadows, but the neighboring road parts support the road network refinement. The two junctions in the lower right part are very close to each other defined in the initialization. This fact can cause a problem, because a local minimum in the image energy could pull both junctions to one point. The topology-preserving energy prevents such an undesired effect and enables the later influence of the neighboring road parts leading to the correct result.

A second example delineating a road network in an open landscape environment using a panchromatic aerial image with a ground resolution of 2.0 m is shown in Figure 5. First, traditional network snakes are applied to this example (top): starting point is a given contour network (blue), where two contour parts in the center merge during the optimization (white) leading to an obviously wrong result (red). The reason is the fact that one initial contour part is closer to a neighboring road part presented in the image leading to a movement to the wrong result. Second, the proposed new topology-preserving energy is considered during the energy minimization (Figure 5, bottom): again, starting point is the initial road network (blue), but now the new energy term preserves the given topology in terms of a planar graph preventing a merging of both contour parts (white). Step by step, the adjacent and already correctly optimized parts of the contour network pull the whole contour part to the correct result (red). Thus, the proposed new topology-preserving energy can support the method of network snakes to enable the exploitation of the topology during the energy minimization preserving the initial topology.

## 5. CONCLUSIONS

A new topology-preserving energy is introduced in the framework of network snakes. The benefit of network snakes is the exploitation of the topology during the energy minimization to delineate arbitrary networks and borders of adjacent objects. The important point is to overcome poor or fragmented object representations within the imagery utilizing the topology of the objects of interest and to enable initializations far away, because adjacent edges in the graph could help to push every contour part to the respective object boundary due to their connectivity. This fact requires a given correct initial topology and, in addition, the preservation during the optimization process. This fact can not be guaranteed in general, why a new topology-preserving energy is proposed in this paper. The shown synthetic example and the real application scenarios regarding the refinement of road networks in suburban and open landscape environments demonstrate the general functionality and transferability of the enhanced method leading to reasonable results in complex areas.

The question to which extent the use of the topology can allow for coarser initializations of the contour or less concise object characteristics within the image will be part of further research. In addition, the examination of the topology-preserving energy within this framework has to be regarded concerning different application scenarios. Furthermore, strategies to speed up the required calculation of the distances between two neighboring contours have to be investigated.

An unsolved problem up to now is the influence of a wrong topology, because the initial given topology is assumed to be correct. The question, how a wrong topology affects the final result of the correct parts of the graph will be part of further analysis. Moreover, the exploitation of the topology combined with the detection of topology errors and their elimination is a challenging research task being currently unsolved.

## REFERENCES

- Blake, A., and Isard, M. 1998. Active Contours. Springer, 352 p.
- Bordes, G., Giraudon, G., and Jamet, O., 1997. Automatic Road Extraction from Grey-Level Images Based on Object Database. In: D. M. McKeown, J. C. McGlone, O. Jamet (eds), Integrating Photogrammetric Techniques with Scene Analysis and Machine Vision III. SPIE 3072, pp. 110-118,
- Butenuth, M., 2007. Segmentation of Imagery Using Network Snakes. Photogrammetrie Fernerkundung Geoinformation 1/2007, pp. 7-16.
- Butenuth, M., Gösseln, G. v., Tiedge, M., Heipke, C., Lipeck, U., and Sester, M., 2007. Integration of Heterogeneous Geospatial Data in a Federated Database. ISPRS Journal of Photogrammetry & Remote Sensing 62(5), pp. 328-346.
- Butenuth, M., and Jetzek, F. 2007. Network Snakes for the Segmentation of Adjacent Cells in Confocal Images. In: A. Horsch, T. M. Deserno, H. Handels, H. Meinzer, and T. Tolxdorff (eds.), Bildverarbeitung für die Medizin 2007. Springer, pp. 247-251.
- Caselles, V., Catté, F., Coll, T., and Dibos, F., 1993. A Geometric Model for Active Contours in Image Processing. Numerische Mathematik 66(1), pp. 1-31.
- Delagnes, P., Benois, J., and Barba, D., 1995. Active Contours Approach to Object Tracking in Image sequences with Complex Background. Pattern Recognition Letters 16(2), pp. 171-178.
- Delingette, H., and Montagnat, J., 2001. Shape and Topology Constraints on Parametric Active Contours. Computer Vision and Image Understanding 83(2), pp. 140-171.
- Fua, P., 1995. Parametric Models are Versatile: The Case of Model Based Optimization. In: International Archives of Photogrammetry and Remote Sensing III/2, pp. 828 - 833.
- Gautama, S., D'Haeyer, J., and Philips, W., 2006. Graph-Based Change Detection in Geographic Information Using VHR Satelite Images. International Journal of Remote Sensing 27(9), pp. 1809-1824.
- Hinz, S., and Baumgartner, A., 2003. Automatic Extraction of Urban Road Networks from Multi-View Aerial Imagery. ISPRS Journal of Photogrammetry & Remote Sensing 58(1-2), pp. 83-98.
- Jasiobedzki, P. 1993. Adaptive Adjacency Graphs. In: B. C. Vemuri (eds.), Geometric Methods in Computer Vision II. pp. 294-303.
- Kass, M., Witkin, A., and Terzopoulos, D., 1988. Snakes: Active Contour Models. International Journal of Computer Vision 1(4), pp. 321-331.
- Lachaud, J., and Montanvert, A., 1999. Deformable Meshes with Automated Topology Changes for Coarse-to-fine Three-Dimensional Surface Extraction. Medical Image Analysis 3(2), pp. 187-207.
- Laptev, I., Mayer, H., Lindeberg, T., Steger, C., and Baumgartner, A., 2000. Automatic Extraction of Roads from Aerial Images Based on Scale Space and Snakes. Machine Vision and Applications 12(1), pp. 23-31.
- Malladi, R., Sethian, J. A., and Vemuri, B. C., 1995. Shape Modeling with Front Propagation: A Level Set Approach. IEEE Transactions on Pattern Analysis and Machine Intelligence 17(2), pp. 158-175.
- McInerney, T., and Terzopoulos, D., 1996. Deformable Models in Medical Image Analysis: A Survey. Medical Image Analysis 1(2), pp. 91-108.
- Paragios, N., and Deriche, R., 2000. Geodesic Active Contours and Level Sets for the Detection and Tracking of Moving Objects. IEEE Transactions on Pattern Analysis and Machine Intelligence 22(3), pp. 266-280.
- Peteri, R., Celle, J., and Ranchin, T., 2003. Detection and Extraction of Road Networks from High Resolution Satellite Images. In: Proceedings International Conference on Image Processing, pp. 301-304.
- Singh, A., Goldgof, D. B., and Terzopoulos, D. 1998. Deformable Models in Medical Image Analysis. IEEE Computer Society Press, 388 p.
- Zimmer, C., and Olivo-Marin, J., 2005. Coupled Parametric Active Contours. IEEE Transactions on Pattern Analysis and Machine Intelligence 27(11), pp. 1838-1842.