BUILDING ROOF SEGMENTATION AND RECONSTRUCTION FROM LIDAR POINT CLOUDS USING CLUSTERING TECHNIQUES

Aparajithan Sampath, Jie Shan

School of Civil Engineering, Purdue University, 1284 Civil Engineering Building, West Lafayette, IN 47906, USA - (asampath, jshan)@purdue.edu

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ABSTRACT:

This paper presents an approach to creating a polyhedral model of building roof from LiDAR point clouds using clustering techniques. A building point cloud is first separated into planar and breakline sections using the eigenvalues of the covariance matrix in a small neighbourhood. The planar components from the point cloud are then grouped into small patches containing 6-8 points and their normal vector parameters are determined. The normal vectors are then clustered together to determine the principal directions of the roof planes. Directly using a clustering algorithm on normal vectors presents difficulties due to a lack of a-priori information on approximate roof directions. Therefore, a potential based approach is used iteratively with the ***k***-means algorithm. This generates the necessary planar parameters, and segments the LiDAR roof points. For reconstruction, a plane adjacency matrix is created for the roof using the segmented roof points. Planes that intersect each other are identified and breaklines and roof vertices are generated by solving the intersecting planar equations. A vector polyhedral model of the roof is created.

1. INTRODUCTION

Extracting interest features from an airborne Laser (or LiDAR) point cloud is not a trivial task. This paper adapts techniques from data mining and models building roof under the assumption that they are polyhedral planes. Regions on the roof such as breaklines, small chimneys etc are considered its “non-planar” parts. LiDAR returns from roof tops are segmented such that each return is mapped to a single roof plane. This is accomplished by first detecting points that lie close to breaklines in the roof structure and removing them from initial analysis (Section 2) using the properties of the principal components within a kernel (Kernel-PCA). The remaining planar points are used to determine the equations of the roof planes, by using clustering techniques (Section 3). It is shown that LiDAR points on the building roof can be successfully segmented into different planes (Section 4). Finally, the segmented roof points are used to determine the roof’s plane-segment equations and a vector building model is generated (Section 5).

There are several ways to approach the problem of building roof segmentation, modelling, and reconstruction. Some methods assume a basic building model, or a combination of several basic models to fit the data. These are called model driven approach which is based on specific assumptions made on the building models. Mass and Vosselman (1999) utilized the invariant moment technique to determine the roof parameters of regular building types using the raw laser scanning data.

Data-driven methods do not assume any underlying building model, but make use of the redundancy created from a dense point cloud to determine the extant model. Finding the component planes is the key issue in reconstructing the buildings. Rottensteiner and Briese (2003), Rottensteiner et al (2005) generate a digital surface model and determine a few “homogenous pixels”, i.e., pixels which are most likely to be planar. Connected homogenous pixels were used as initial seeds to generate planar regions. Statistical tests were used to determine thresholds at various stages. Peternell and Steiner (2004), Forlani et al.(2005) also used a similar approach. Al-Harthy and Bethel (2004) used moving windows, instead of an explicit grid to determine the slopes in x, y, and the z intercept for points. Then a region growing approach was used to extract planar segments. Further methods to extract surfaces are reviewed in Vosselman et al. (2004).

Data mining approaches mainly use classification or clustering techniques to seek patterns such as planar segments in the data set for building extraction. Sampath and Shan (2006), Nizar et al. (2006) etc have demonstrated such an approach to building reconstruction. Nizar et al. defined feature vectors for each point based on a tangential plane and a height difference measure. They used these four quantities to cluster points into surface classes and separate similar surface classes based on their spatial proximity. However, LiDAR returns from non-planar parts, even in a predominantly planar building, (from trees, chimneys, small dormers) etc may affect the clustering process. Fitting a plane at roof edges, roof ridges or trees would lead to errors in clustering. Interested readers can also refer to Tarsha-Kurdi et al. (2007), where Hough transforms were combined with RANSAC for building reconstruction. Brenner (2005) summarized various building reconstruction techniques using image, LiDAR and map data that have been suggested by different authors. Interested readers may also refer Brenner and Haala (1999), Vosselman and Dijkman (2001), Overby et al.(2004), Hoffman et al.(2002), Schwalbe et al.(2005). Schenk and Csathó (2002, 2007), Vögtle and Steinle (2000), Rottensteiner et al.(2005), Sohn and Dowaman (2007).

2. BREAKLINE DETECTION

The motivation to detect breaklines is, in part to exclude LiDAR returns from them from taking part in the clustering process that is described in detail in the next section. Normal vectors to non-breakline points on the roof are determined and clustered. The presence of breaklines, and equally importantly, returns from trees, vertical portions of walls etc causes the normal vectors to be noisy. The process of removal of breaklines also removes returns from trees etc. Breakline detection is an important element in building extraction, because it implicitly follows the process of human perception, as Schenk and Csathó (2002, 2007) pointed out. A kernel (neighbourhood) based PCA method is adopted to determine the location of these breaklines, in an interesting manner. Breakline detection has been studied by Briese (2006), Yokohama (2006). Fransens (2003) discuss a methodology based on principal components for determining flat regions in a point cloud.

In principle, any point in an n-dimensional linear space can be defined as a linear combination of mutually orthogonal basis vectors. A principal component analysis of a given set of (3D) points would reveal to us the direction of these mutually orthogonal vectors (eigenvectors), and their relative importance or strength (the eigenvalues). If we can show that a group of points can be represented by only two rather than three mutually orthogonal basis vectors (i.e., the third coefficient in the linear combination is zero), then this group of points lies on a plane. By the same logic, for breakline points they need all three basis vectors. Let a group of points **P** be represented by $\mathbf{X}_i \ (i = 1,..., p)$ in $\Re^3$ and let $\overline{\mathbf{X}}$ be their mean vector, then the covariance matrix $\mathbf{\Sigma_{XX}}$ is calculated as

$$\mathbf{\Sigma_{XX}} = \sum_{i=1}^{p} \left(\mathbf{X}_i - \overline{\mathbf{X}}\right)\left(\mathbf{X}_i - \overline{\mathbf{X}}\right)^T \qquad (1)$$

We can then determine the eigenvalues $\lambda_1, \lambda_2, \lambda_3$ where $\lambda_1 \le \lambda_2 \le \lambda_3$ and their corresponding eigenvectors $\Lambda_1, \Lambda_2, \Lambda_3$ for the matrix $\mathbf{\Sigma_{XX}}$. A small valu`e for λ_1 indicates that the dimension of the basis space for the points in **P** is 2, instead of 3. That is, it is possible to represent the points in **P** with just two basis vectors. This indicates that the region represented by the points in **P** is planar. On the other hand, if all three eigenvalues are significant, then the points in **P** require all three coordinates to define, and hence do not lie on a plane. The conclusion would be that these points lie near a breakline. The above principle is summarized as: if $[\lambda_1 /(\lambda_1 + \lambda_2 + \lambda_3)] < \varepsilon$, (where ε is a small quantity) then the point set **P** is planar; otherwise it is on the breaklines. An example of the separation of breaklines and planar regions is shown in Figure 1, where the dark points are LiDAR returns from planar regions, and bright points are returns from locations near breaklines.

3. ROOF SEGMENTATION - CLUSTERING

Having detected and (temporarily) excluded the non-planar points from consideration, we are left to deal only with points that are planar. To segment these planar points into their individual roof segments, their normal vectors are collected and clustered. Consider a group of neighbourhood points **P** in the planar point dataset. If the covariance matrix of these points is calculated, and the eigenvalues and eigenvectors generated for this matrix, the eigenvector corresponding to the smallest eigenvalue represents the normal $\mathbf{N} = (N_X, N_Y, N_Z)^T$ to the least squares plane that can be fitted to the points in **P**. Therefore, **N** represents the normal vector of the whole roof segment to which this small neighbourhood of points belongs to. Such a normal vector $(\mathbf{N})$ uniquely determines the direction of a roof plane and thus is selected as the feature vector for clustering.

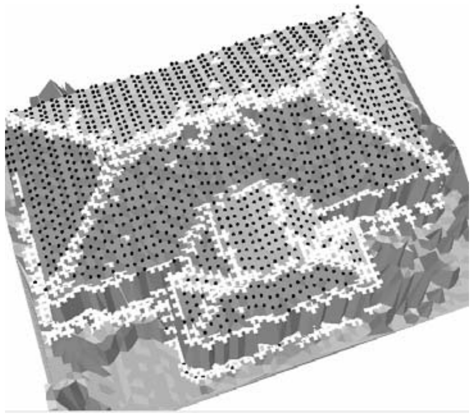

Figure 1. Detected breakline points (light) and planar points (dark)

3.1 K-means clustering

K-means clustering algorithm is a simple non-parametric unsupervised technique to determine clusters in the input dataset. Mathematically, the k-means algorithm partitions the data points in feature space into k parts by minimizing the following objective function

$$\Phi = \sum_{j=1}^{k} \sum_{i=1}^{m} (\mathbf{X}_i - \overline{\mathbf{X}}_j)^2 \qquad (2)$$

where $\mathbf{X}_i$ refers to the $i-th$ feature vector assigned to $j-th$ cluster, $\overline{\mathbf{X}}_j$ refers to the mean feature vector of cluster j , k is the number of clusters, m is the total number of data points assigned in cluster j . For a detailed discussion on the k -means clustering algorithm, please refer to Jain et al. (1998) and Tibushirani at al (2001). K-means algorithm, however, requires us to know the number of clusters, and their approximate locations a-priori. In our case, knowing the number of clusters would mean knowing the number of different directions to which the planes in the building point to, something that is not known. This also means that the approximate cluster locations are also unknown. The next section deals with this problem.

3.2 Cluster centres and the number of clusters

To determine the cluster centres and the number of clusters, an approach first introduced by Yager and Filev (1994) and later developed by Chiu (1994) is used. Chiu assigns each data point a potential value, based on its location with respect to its neighbouring points. The point with the highest potential is considered as a cluster centre. Chiu assigns density potential to each point $\mathbf{X}_i$ as

$$p_i = \sum_{j=1}^{n} \exp\{-\frac{4}{r_a^2}\left\|\mathbf{X}_i - \mathbf{X}_j\right\|^2\} \quad (3)$$

Here n is the number of points within the neighbourhood defined by the radius r_a centred at $\mathbf{X}_i$. Points outside this neighbourhood are not considered for potential calculation. The first cluster centre is thus chosen as the point that has the greatest potential. Once a point is chosen, it is undesirable to choose the next cluster centre very close to the first one. Therefore, the potential for each point in the neighbourhood is reduced as a function of its distance to the first cluster centre

$$p_{m\_new,j} = p_{m\_old,j} - p_i \exp\{-\frac{4}{r_b^2}(\mathbf{X}_i - \mathbf{X}_j)^2\} \quad (4)$$

Move the texts below out of math.

$\mathbf{X}_{\mathrm{i}}$: *The* current potential center

$\mathbf{X}_{\mathrm{j}}$: *Location* of any point

within a radius $\mathrm{r_b}$ of point $\mathbf{X}_{\mathrm{i}}$ ('m')

$p_{m\_new,j}$: *New* Potential of any point 'm'

$p_{m\_old,j}$: The old Potential of the point 'm'

p_i : The potential of the cluster center.

Notice that in equation 4, the neighbourhood for reducing the potential is r_b. Chiu recommends a ratio of 1.5 between r_b and r_a, which is found to be satisfactory in this study. The process of acquiring new clusters stops when the cumulative potential becomes too small.

It is clear that in this method the value of the radius r_a is crucial for the clustering results. A smaller r_a will yield a higher number of clusters and vice versa. Since we are operating in the feature vector domain, it is difficult to design a reasonable threshold for r_a. To overcome the problem, the method described by Chiu is iteratively implemented. Starting from a smaller radius and increasing it gradually, less and less number of cluster centres will be obtained. The cluster centres generated for each r_a are used as the input to the k-means clustering algorithm. As the result of clustering, a likelihood estimate for each cluster will be produced. It measures the compactness of the clustering and can thus be used to determine the optimal value of r_a, and hence the number of clusters. Figure 2 plots the likelihood estimates with respect to the number of clusters.

The error of likelihood estimates, alternatively termed as dissimilarity measure, has been defined as the average cluster-centre to data point distance. As is shown figure 2, the likelihood errors fall sharply onto a point (# Cluster = 4), after which the decrease becomes stable. This effect is likened to an elbow, and the "elbow joint" is considered to be a good estimate of the number of clusters in the data set. The reader is referred to Tibshirani et al (2001) for a more detailed explanation. The cluster centres, corresponding to the number of clusters defined by the "elbow joint" are also noted. Since each cluster represents a group of planar patches that are facing the same direction, we have the normal directions to each plane of the roof.

At the end of this process, we know the number of clusters, the cluster centres, and the constituent planar patches for each cluster. These correspond, in lidar data space, respectively to the number of planar faces with the same normal vector, the normal vector for each distinct roof planes, and the point patches that form a roof plane.

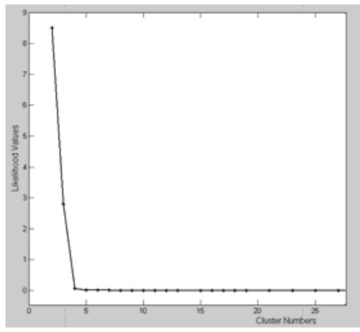

Figure 2. Error (likelihood) estimate vs. the number of clusters

3.3 Separation of parallel and coplanar planes

The clustering process returns the planes with the same normal vectors $\mathbf{N} = (N_X, N_Y, N_Z)^T$. Within each of these clusters, parallel planes have to be segmented. Also to be segmented are plane segments that might mathematically possess the same equation, but are spatially separated. If we write the equation of a plane as $N_X X_C + N_Y Y_C + N_Z Z_C + D_n = 0$, then separating parallel planes means that we determine different values for the parameter *Dn*, which would indicate different parallel planes within a single cluster represented by $\mathbf{N} = (N_X, N_Y, N_Z)^T$. We take each patch that went into the clustering algorithm, and determine its centroid (X_C, Y_C, Z_C). The distance from the coordinate origin to this patch is computed as

$$\rho = (N_X X_C + N_Y Y_C + N_Z Z_C) / \sqrt{N_X^2 + N_Y^2 + N_Z^2} \quad (5)$$

If a given cluster has parallel planes, the values of ρ will vary significantly among the patches. Parallel planes can thus be separated based on these different ρ values. If we use unit eigenvectors, then the quantity $\sqrt{N_X^2 + N_Y^2 + N_Z^2}$ equals unity, and the quantity ρ is the same as *–Dn*. For this research, a value of ρ =1 meter is used to separate parallel planes. At this stage, all the planes in the roof have been determined, i.e. the set of parameters $\mathbf{N}_k = (N_X, N_Y, N_Z)_k^T$ and its corresponding D_k for each plane is known. The non-planar (breakline) points that have been discarded in section 2 are assigned to the planes by back substituting their coordinates to these parameters, and assigned to the plane with minimum offset. Non-planar points that might actually be reflected returns from trees can be eliminated at this stage if the offset value is over a threshold. A large minimum offset value indicates that the point is a return from a tree or a pipe etc.

Building roof can have two or more planar segments that are mathematically the same, but spatially separated. Such coplanar segments can be separated in the conventional data space based on the concept of density clustering and connectivity analysis. In figure 3, points A and B are directly density connected, and point C is density connected to points A and B. However, point D is not density connected or density reachable to any of the points A, B or C. Therefore, point D lies in a separate cluster from A, B and C. This concept depends on the neighbourhood defining radius R. For a further reading in this topic, readers are referred to Ankerst et al. (1999). In this case, clusters of points that lie on the same mathematical plane, but are spatially separated. Using an appropriate radius (usually slightly less than twice the point spacing), these clusters can be easily separated.

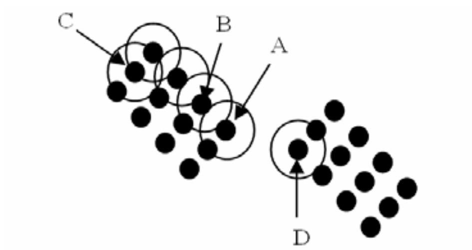

Figure 3. Direct density connection (A and B), density reachable (A and C), and not density reachable (A and D)

4. CLUSTERING RESULTS AND DISCUSSIONS

For this research, the ground point spacing is around 1.0 m. Figure 4 shows some results of the process on point clouds over a few buildings. Color coded roof points of several buildings are shown in this figure. The results indicate that almost of all points are classified correctly to their respective planes. The segmentation of the building in figure 4b shall be used as example for discussion.

4.1 Discussion of the process

Beginning with $\bar{\lambda}$, the relative eigenvalue, it should be noted that for a perfect planar region $\bar{\lambda}$ should be zero. LiDAR datasets contain some noise and the errors associated with each measurement preclude this possibility of a zero value for $\bar{\lambda}$. It can be proved that this value is the ratio of the sum of the distance of the points to the least squares plane, to the sum of the distance of the points from their centroid. The quantity $\bar{\lambda}$ is compared with a small threshold $\bar{\lambda}_T$, below which the point patch shall be considered planar. A value that is very close to zero shall result in a planar regions being classified as breaklines, and a very large threshold shall result in the opposite. A value of $\bar{\lambda}_T = 0.005$ is found giving satisfactory results.

Figure 5a (left) shows a plot of normal vector values for all of the planar patches in clustering. Each dot represents a planar patch and its location stands for the direction of the patch normal vector $\mathbf{N} = (N_X, N_Y, N_Z)^T$.

The dots seem to form distinct clusters. Some outliers, probably caused by patches formed by LiDAR points near the breaklines, are also seen. The k -means and Chiu's density based clustering can successfully determine the number and location of these clusters. In the elbow-joint graph (Figure 5-b) that plots the likelihood values for different number of clusters, the correct number of clusters (5) is chosen at the place where the distinct change occurs.

The final results (figure 4b as the example) are obtained by separating parallel planes based on their distances to the coordinate origin, and separating coplanar segments using density connectivity analysis (section 3.3).

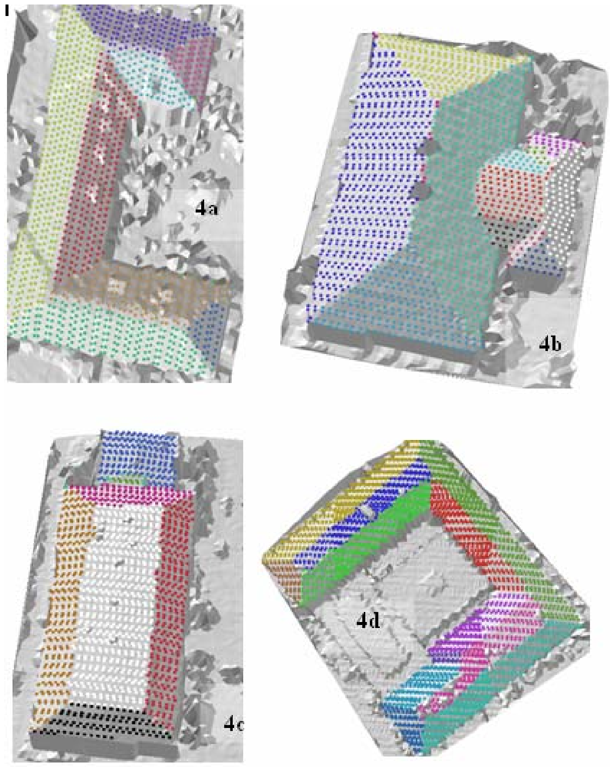

Figure 4. Segmented planar roof elements

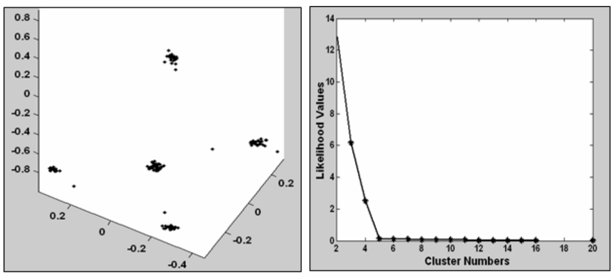

Figure 5. Plot of the direction cosines for each planar patch (a, left) and the elbow-joint graph (b, right).

Mis-clustering can occur for planar patches that are near non planar regions of the roof, such as those near chimneys, breaklines, pipes, trees etc. These can cause many outlier points that interfere with the clustering algorithm.

5. BUILDING MODEL RECONSTRUCTION

Once the roof has been segmented, we can create an adjacency matrix. The adjacency matrix is a neighbourhood map for the plane segments on a roof. Equations of adjacent roof planes are solved to obtain breaklines and roof vertices. The figure below (figure 6) shows a vectorised building (same as the one in figure 4b) and shall be used for illustrative purposes.

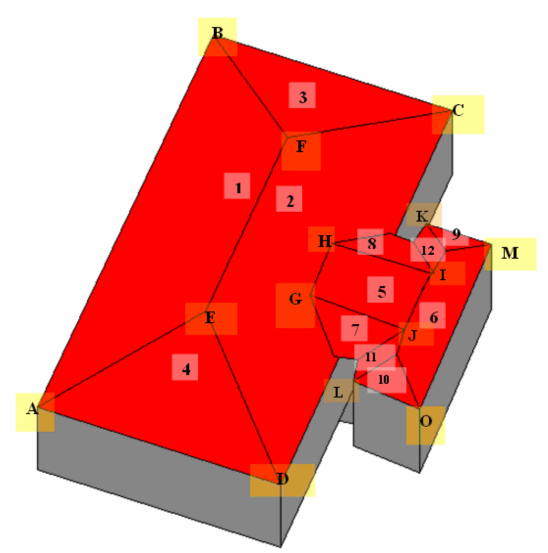

Figure 6. A vectorised building labelled with vertices and planes

5.1 Plane adjacency matrix

To determine the adjacency of any pair of planes pA and pB, the distance between them is first determined. The distance is defined as

$$D_{pA-pB} = \min_{\bar{a} \in pA} \left\{ \min_{\bar{b} \in pB} \{ d(a, \bar{b}) \} \right\} \quad where$$

$$d(\bar{a}, \bar{b}) = \sqrt{(X_{\bar{a}} - X_{\bar{b}})^2 + (Y_{\bar{a}} - Y_{\bar{b}})^2 + (Z_{\bar{a}} - Z_{\bar{b}})^2}$$

$$where\ \bar{a}(X_{\bar{a}}, Y_{\bar{a}}, Z_{\bar{a}}) \text{ and } \bar{b}(X_{\bar{b}}, Y_{\bar{b}}, Z_{\bar{b}})$$

Here, $\bar{a}$ is any point in pA and $\bar{b}$ is any point in pB**.** If the quantity D_{pA-pB} is less than twice the point spacing of the LiDAR point cloud, we deem the planes pA and pB to be neighbors. For the building in figure 6, the plane adjacency matrix is shown in table 3. Consider Plane 1 in the adjacency matrix. Looking at the row (for plane 1), it becomes clear that it is adjacent to planes 2, 3 and 4. The vertices are named E and F and represent the breakline E-F in figure 6. E-F is the intersection of planes 1 and 2. To determine these vertices, we need the planes that are common to both planes 1 and 2. We determine this from the adjacency matrix by looking at the rows of planes 1 and 2, and determining that planes 3 and 4 are common to both planes 1 and 2. Using the equations of the planes {1, 2 and 3} vertex F is obtained and E is obtained using equations of planes {1, 2 and 4}. To determine the coordinates of vertices A, B, C, D etc, each of which has only two intersecting planes, we enforce a boundary constraint on the breakline. For example, for breakline A-E, A is obtained by determining the breakline A-E from equations of planes 1 and 4, and constrained such that it lies on the boundary of both planes 1 and 4. In this manner, each vertex is defined by at least three planes, and each breakline is defined by its two intersecting planes.

Sgmt_No	1	2	3	4	5	6	7	8	9	10	11	12
1		*	*	*								
2	*		*	*	*		*	*				
3	*	*										
4	*	*										
5		*				*	*	*			*	*
6					*		*	*	*	*	*	*
7		*			*	*					*	
8		*			*	*						*
9						*						*
10						*					*	
11					*	*	*			*		
12					*	*		*	*			

Table 1. Plane adjacency matrix

6. CONCLUSIONS

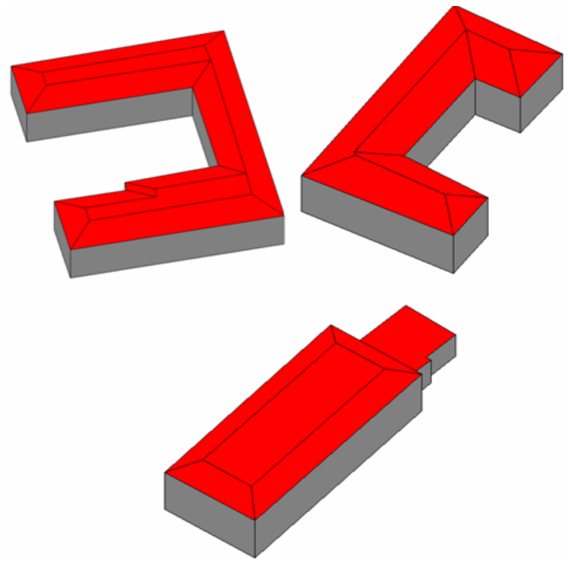

Figure 7. Reconstructed building models

We have presented a method to segment LiDAR point cloud from roof-tops of buildings into different roof planes. We demonstrate a method of determining points that lie along the breaklines, i.e. along intersection of two (or more) roof planes by using eigenvalue and eigenvector analysis. As a consequence, we show that LiDAR returns from trees, vertical portions of walls etc are also categorized under breaklines, so we term them together as non-planar regions. The planar sections of the LiDAR returns are divided to planar patches characterized by its normal vector and distance to the coordinate origin. To group the patches into planar surfaces, an iterative combination of density potential based clustering with the k-means clustering are investigated, which yields promising results of moderately complex buildings. 3D vector models of the buildings are reconstructed by generating a plane adjacency matrix and determining the breaklines and vertices of the roof by using the planar parameters derived previously. A few more results of reconstructed buildings are shown in figure

7 These results correspond to the remaining three buildings shown in figure 4.

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