

# GENERAL ARCHITECTURE DESIGN OF LUNAR PROJECTION SYSTEM

Lü xiaohua<sup>a</sup>, Deng Shujun<sup>a,b</sup>, Ma Jun<sup>a</sup>

<sup>a</sup>Institute of Surveying and Mapping, Information Engineering University, 66 Longhai Road, Zhengzhou, 450052, China – chxydsj@163.com

<sup>b</sup>RS and GIS Institute of Peking University, Beijing, 100871, China - chxydsj@163.com

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### ABSTRACT:

With constant development of the lunar exploration and deep space exploration, the research scope of traditional earth mapping has been promoted to stretch from earth surface and terrestrial space to the moon even other major planets and interstellar space. In the process of lunar mapping, the most fundamental and forward-looking tasks are to design lunar map projection and to construct mathematical foundation for the lunar surface. In this thesis, the ways to acquire, handle and express multi-source information of lunar surface have been put forward, the lunar map projection, which is multi-layer, multi-scale, multi-style, and adaptable to entire lunar space, regional lunar space and large-and-medium scale, has been designed, and the basic framework has been established preliminarily for lunar space information projecting system.

## 1. INTRODUCTION

Lunar exploration is the first step for human to step towards immense cosmos out of the earth cradle, as well as a historical beginning for deep space exploration. In the 21st century, as the only means to protect earth, exploit outer-space resources, and realize sustainable development, lunar exploration draws attention from countries all over the world once again, and has become a stage for them to exhibit comprehensive strengths. With constant development of lunar exploration technique, the lunar mapping work, which provides technical support, shoulders a momentous and intricate task. The science and technology for earth mapping will stretch to lunar surface and deep space.

In the process of lunar mapping, it is of important theoretical significance and practical application value to establish a lunar centre geodetic coordinate system, research a plane coordinate system which is adaptable to lunar mapping, design a lunar projection system, provide mathematical foundation for lunar mapping, and offer plane coordinate standard for handling, expression and application of lunar space information and thematic information. Besides, it is also of practical urgency significance by providing a prominent supporting technique base for tri-dimensional lunar surface mapping, lunar resources research, soft-landing of lunar probe, evolution and formation of lunar structure and development of deep space exploration<sup>[4]</sup>. This thesis proposes the basic principles for lunar map projection design. On the basis of analyzing and comparing the shapes, features of deformation and application situation of the quadrillage of the existing map projection, it selects the ways to acquire, handle and express multi-source information of lunar surface and designs the lunar map projection, which is multi-layer, multi-scale, multi-style and adaptable to entire lunar space, regional lunar space and large-and-medium scale, and preliminarily establishes the basic framework for lunar space information projecting system.

## 2. BASIC PRINCIPLES FOR LUNAR PROJECTION SYSTEM DESIGN

For solving the contradiction of inextensible projection original surface and map plane, map projection provides important theories and methods. It is a main content of map mathematical foundation and important norm of geographical space information.

After several hundred years' observation and study, man has accumulated some basic understanding about the moon. The lunar shape is close to sphere, the difference of three semi-axis is only about 1km, comparing with the earth, the lunar equatorial radius is 1737km, about 1/4 of that of the earth, and the flattening is about 1/10 of that of the earth, which can also be regarded as ellipsoid of rotation<sup>[2][3][8]</sup>.

Similar to three-dimensional geocentric geodetic coordinate system on the earth, it builds three-dimensional lunar centre geodetic coordinate system. The grid origin is located at the lunar mass center; Z-axis and lunar rotation axis conform and directly cross at the lunar equatorial plane. Define the lunar geographic coordinate system: equator northward, the latitude is positive value, southward is negative value; on the cross section formed by X-Z-axis on the moon, eastward of the longitude is positive and westward is negative<sup>[2]</sup>.

The design of lunar map projection and the building of mathematical foundation can follow the pattern of the building of the map projection system on the earth, and comply with the following principles.

(1) Fully consider the basic factors, such as the usage, using features and scale of the lunar map, the shape, size, and space character of geographic position of drawing area, compromising map arrangement and special requirements for the shape of quadrillage.

(2) The type of projection is not suitable to be excessive, each projection needs to complement for each other, with integrity

on the structure and comparability and harmony on the expression.

(3) The selection and application of lunar projection shall strive for basic conformation with other developed countries in the world and the versatility in the world; meanwhile, it shall sufficiently embody our characteristics, adapt to the present geographical spatial information mathematical foundation, and innovate in the process of inheritance.

(4) The building of lunar projection system and lunar mathematical foundation shall make for the treatment and expression of multi-source lunar spatial information (including vector data, grid data, GPS data and so on) to adapt to the ceaseless progress of lunar exploration technique and continuous development of subject specialty.

Therefore, the lunar map projection should include at least integrative description of lunar global projection on the surface of the moon, which can adopt normal cylindrical projection, pseudo-cylindrical projection, and poly-conic projection and so on; of lunar regional projection applied in the region on the lunar surface whose area is not very large, which can adopt azimuthal projection and conic projection and so on; equiangular crosscut cylindroid projection and universal polar stereographic projection of the two poles applied in the lunar map with large and medium scale. The three series of projection form the complete lunar map projection system.

### 3. SELECTION AND DESIGN OF LUNAR PROJECTION SYSTEM

#### 3.1 Lunar Global Projection

There exists several ways to describe the lunar global projection, however, considering the requirements of integrative display and expression of the moon, the bipolar treatment shall be appropriate, various deformation shall be moderate, with simple shape of quadrillage and good integral visual effect, therefore, it is suitable to choose normal cylindrical projection, pseudo-cylindrical projection or poly-conic projection.

##### 3.1.1 Normal Cylindrical Projection

In the normal cylindrical projection, the parallel projection is a parallel straight line, and meridional projection is another group of parallel straight lines orthogonal with the parallel of latitude. Set the central meridional projection as X-axis and equatorial projection as Y-axis, the general formula of coordinate normal cylindrical projection is as follows:

$$\left. \begin{aligned} x &= f(B) \\ y &= c \cdot (L - L_0) \end{aligned} \right\} \quad (1)$$

In this formula,  $c$  is projective constant, referring to the radius formed by the cylinder tangent or secant to the latitude circle of the moon, i.e.  $c = a$  or  $c = N_0 \cos B_0$ .

When  $x = c \cdot \ln U$ ,  $U = \operatorname{tg}\left(\frac{\pi}{4} + \frac{B}{2}\right) \left(\frac{1 - e \sin B}{1 + e \sin B}\right)^{\frac{e}{2}}$ , we get normal equiangular cylindrical projection, i.e. Mercator's

projection; When  $x = S_m$  and  $x = \frac{l}{c} F$ , we get normal equalarea and equidistant cylindrical projection respectively, of which,  $S_m$ ,  $F$  respectively represent longitude arc length of the lunar surface from equator to latitude  $B$  and the trapezoidal area of lunar surface from equator to latitude  $B$  with longitude difference of 1 radian.

The deformation of normal cylindrical projection is only relevant to latitude. Deformations in south and north symmetrical to equator are identical. There is no deformation on standard parallel, the further from it, the more obvious the deformation is. The normal equiangular cylindrical projection ensures that the position on the picture is in line with the actual position. The projection of loxodrome is a straight line, but the deformation increases rapidly as latitude grows high. If standard parallel is chosen at  $\pm 30^\circ$ , then deformation extent between  $\pm 40^\circ$  will be within  $\pm 13\%$ , between  $\pm 60^\circ$ ,  $73\%$ . We can see that deformation of area is more than twice of that of length. The normal equalarea cylindrical projection keeps the area undeformed, but angles deformation increases. The normal equidistant cylindrical projection has various but relatively slight deformations, and the shape of quadrillage is graticule. If normal cylindrical projection is chosen as entire lunar projection, standard parallel  $\pm 30^\circ$  or  $\pm 35^\circ$  with simple quadrillage and fine universality, and range in latitude direction can be larger than  $360^\circ$  with a fair holistic effect, but deformation in high latitudinal area becomes bigger with both poles emanative and polar regions unmeasured.

##### 3.1.2 Pseudo-cylindrical Projection

It defines parallel projection as a set of parallel straight line, of which interval changes with the latitude; after the central meridional projection, it is the straight line vertical to latitude line, other meridional projections are curves symmetric to central meridian. It takes the central meridian as X-axis, equator as Y-axis, and under the condition of undeformed area and parallel length, then, the coordinate formula of equalarea pseudo-cylindrical projection is:

$$\left. \begin{aligned} x &= \int_0^B \frac{a \cdot (1 - e^2)}{(1 - e^2 \sin^2 B)^{3/2}} dB \\ y &= \frac{a \cdot \cos B}{(1 - e^2 \sin^2 B)^{1/2}} \cdot l \end{aligned} \right\} \quad (2)$$

When taking the moon as a sphere with radius as  $R$ , from formula (2), we can see

$$\left. \begin{aligned} x &= R \cdot \varphi \\ y &= R \cdot \lambda \cdot \cos \varphi \end{aligned} \right\} \quad (3)$$

The projection determined by formula (3) is the famous Sanson projection, which has no distortion on the central meridian and equator, if far away from the central meridian and equator, the distortion will increase. The distortion of meridional length is less than 1.0711 in most of areas in the global range, the maximum reaches to 2.2969; the angular distortion is less than

76°, the maximum can reach to 115°. The meridional shape is sine curve, the pole projection is a point, the integral globular and cubic effect of the quadrillage is strong, and the integrality is good. Such as figure 1 (a), it fits mathematical foundation of the whole moon used for small scale. In a similar way, it can get the equalarea pseudo-cylindrical projection with pole projection as a line and that with longitude shape as elliptic curve, such as figure 1 (b) and (c), it can design and apply the poly-conic projection with random features.

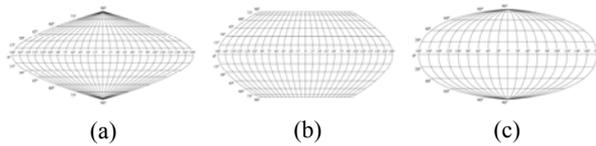


Figure 1 Equalarea pseudo-cylindrical projection

### 3.2 Regional Lunar Map Projection

The regional projection is mainly applied in the regional map on the lunar surface area with airstop and space station on the lunar surface as the centre and with moderate area, the scope of latitude and longitude differences is from several degrees to over ten degrees, the scale generally is from one in hundreds of thousands to one in several millionth. The regional lunar map projection chooses azimuthal projection, conic projection and so on.

#### 3.2.1 Lunar Azimuthal Projection

The lunar azimuthal projection is to assume that there is a plane tangent (secant) to a certain point (a small circle) on the moon, and formed by the projection from lunar surface to plane with geometry perspective theory or mathematical method. Taking the moon as a sphere with radius as  $R$ , its projective almucantar is concentric circle, and vertical circle is the radius of concentric circle. In the coordinate system with central meridional projection as  $X$ -axis and projection center point as origin, the general formula is:

$$\left. \begin{aligned} \rho &= \frac{R \sin Z (R \cos Z_0 + D)}{R \cos Z + D} \\ \delta &= \alpha \\ x &= \rho \cdot \cos \delta \\ y &= \rho \cdot \sin \delta \end{aligned} \right\} \quad (4)$$

In the formula:  $D$  is the distance between the viewpoint vertical to perspective axis on projection surface and lunar centre;  $Z_0$  is a constant determined by tangent or secant position;  $R$  is the radius of the moon.  $(\alpha, Z)$  is a spherical coordinate with the tangent point of plane and lunar surface (small entelechy) as new pole, which is given in the following formula:

$$\left. \begin{aligned} \cos Z &= \sin \varphi \sin \varphi_0 + \cos \varphi \cos \varphi_0 \cos(\lambda - \lambda_0) \\ \operatorname{tga} &= \frac{\cos \varphi \sin(\lambda - \lambda_0)}{\sin \varphi \cos \varphi_0 - \cos \varphi \sin \varphi_0 \cos(\lambda - \lambda_0)} \end{aligned} \right\} \quad (5)$$

When  $D = 0$ ,  $D = R$ ,  $D > R$ , lunar gnomonic projection, lunar stereographic projection, and lunar external projection with diverse properties and purposes can be gotten respectively. Besides, equalarea and equidistant lunar azimuthal projection can also be gotten through projection terms.

In the lunar azimuthal projection, the deformation near the centre point is relatively slight, the azimuth keeps correct from centre point to any point in the region. While in the stereographic projection, the length deformation about 30° away from the centre point is controlled within 7.2%, the area deformation within 15%, no angular deformation and the position on the map is in conformity with actual position. The orthodrome projection of lunar gnomonic projection is a straight line. There is no deformation of distance and direction from the centre point to any point within the equidistant lunar azimuthal projection. These features are extremely favorable for astronauts to determine the orientation and positioning on the lunar surface.

When azimuthal projection is applied to lunar mapping, the maximum extent generally represents the disc-shaped area visible on the lunar surface. Suppose the projection centre point is placed on the equator, we can get a normal side-faced hemispheric projection; if on the two poles, polar normal azimuthal projection. The American army once adopted the perspective external projection with  $D = 1.53748 R$  to draw lunar map, which was later called AMS projection.

#### 3.2.2 Lunar Equiangular Poly-conic Projection

The poly-conic projection is a projection with conical surface as projection plane, projecting the quadrillage of lunar surface onto conical surface according to equiangular condition, then unfolding along certain generatrix into a plane, of which, the parallel projection is a concentric circular arc and longitude projection is the radius of the concentric circular arc. In the coordinate system with central meridional projection as  $X$ -axis and intersection point of lowest parallel in the projection area and central meridian as origin, the calculating formula is:

$$\left. \begin{aligned} \rho &= \frac{C}{U^\alpha}, \quad \delta = \alpha \cdot l \\ x &= \rho_s - \rho \cdot \cos \delta \\ y &= \rho \cdot \sin \delta \end{aligned} \right\} \quad (6)$$

In this formula,  $\alpha$ ,  $C$  are both projective constants, and are determined by the method of identical absolute value of the length deformation of bounding parallel and equator, i.e

$$\left. \begin{aligned} \alpha &= \frac{\ln r_s - \ln r_N}{\ln U_N - \ln U_s} \\ C &= \frac{1}{\alpha} \sqrt{r_N r_m U_N^\alpha U_m^\alpha} = \frac{1}{\alpha} \sqrt{r_s r_m U_s^\alpha U_m^\alpha} \end{aligned} \right\}$$

In this formula,  $B_m = \frac{1}{2}(B_s + B_N)$ ,  $B_s$ ,  $B_N$  refer to northern and southern latitude degree in this projection area respectively.

Normal conic projection is adaptable to draw maps of the mid-latitude region stretching along the direction of parallel, compared with the tangent conical projection, secant conic projection can reduce the length deformation in mapping region and effectively improve the deformation distribution. In regional lunar mapping, if choose equiangular secant conic

projection, do zoning as per latitude difference of 4°, the projection deformation is very slight. In each projection zone, the standard parallel is at about 35' inside the northern and southern bounding parallel, the maximum length deformation is ±0.3%, maximum area deformation ±0.6% with no angular deformation. The quadrillage is of simple shape and easy to calculation, and it is in light of topographic map projection with scale 1/1,000,000 of our country, so it can be extensively applied to mathematical foundation for lunar maria map and lunar terrain map in the mid-latitude region of the moon.

### 3.3 Lunar Map Projection of Large-and-medium Scale

In order to get a comprehensive, accurate and multi-scaled impression of geographic features of lunar surface and spatial relationship among them, usually the series maps need to be made with scale over 1:1,000,000. The maps with different scales meet different purposes, and they are comparable. Thus, projections of this kind demand slight deformation and high accuracy, the ability to cover main regions of the entire moon and meet the need of multi-scale mapping. After a comprehensive consideration about the application situation of lunar projection of this kind in foreign countries and the present status of our country's topographic map projection with series scales, it is suggested that UTM should be adopted between latitude ±80°, and universal polar lunar surface projection in the polar zones above 80°, so that the projection of entire lunar map is successive and universal

#### 3.3.1 Universal transverse Mercator Projection (UTM)

UTM projection is apprehended as equiangular crosscut cylindroid projection in the respect of geometry. The cylinder crosscuts two almucantars of the moon, with two secants undeformed after projection, and the length ratio on the central meridian is reduced to 0.9996. In the coordinate system with central meridional projection as X-axis and equatorial projection as Y-axis, the calculating formula is:

$$\left. \begin{aligned} x &= 0.9996 \cdot \left[ S_m + \frac{1}{2} N t \cos^2 B l^2 + \frac{1}{24} N t (5 - t^2 + 9 \eta^2 + 4 \eta^4) \cos^4 B l^4 \right. \\ &\quad \left. + \frac{1}{720} N t (61 - 58 t^2 + t^4 + 270 \eta^2 - 330 t^2 \eta^2) \cos^6 B l^6 + \dots \right] \\ y &= 0.9996 \cdot \left[ N \cos B l + \frac{1}{6} N (1 - t^2 + \eta^2) \cos^3 B l^3 + \frac{1}{120} N (5 - 18 t^2 + t^4 \right. \\ &\quad \left. + 14 \eta^2 - 58 t^2 \eta^2) \cos^5 B l^5 + \dots \right] \end{aligned} \right\} \quad (7)$$

In this formula,  $\eta = e' \cos B$ ,  $t = \tan B$ ,  $e'$  is the second eccentricity of the ellipsoid.

The deformation of UTM projection is mainly determined by longitude difference. In order to ensure the map accuracy, UTM projection is suggested to adopt the method of zoning along the parallel 6° or 3° in the topographic map with basic scale, which is applied on earth in our country. UTM is better than Gauss-Kruger projection, and makes the whole deformation in the projection zone less than 1/1000, no deformation of two secants on the equator about ±180km (longitude difference  $\approx \pm 1^\circ 40'$ ) away from the central meridian, and makes the deformation distribution near latitude 40° more reasonable and preferably improves the deformation in low latitude region, but the two poles are beyond expression.

#### 3.3.2 Universal Polar Lunar Surface Projection

Universal polar lunar surface projection refers to the normal equiangular azimuthal projection applied to North and South poles with lunar surface as the ellipsoidal surface, the plane is secant with North and South Pole of the moon at certain latitude.

When the length ratio at the pole is less than 1, and the calculating formula is:

$$\left. \begin{aligned} \rho &= \frac{C}{U} \\ \delta &= l \\ x &= x_0 + \rho \cdot \cos \delta \\ y &= \rho \cdot \sin \delta \end{aligned} \right\} \quad (8)$$

In this formula,  $C = N_0 \cdot \cos B_0 \cdot U_0$

Universal polar lunar surface projection has no angular deformation, and the area deformation is twice of that of length. When the latitude of secant parallel is chosen at 70°, pole length deformation is about -3%; at latitude 55°, +6.6%. The USA applies this projection as the mathematical foundation for polar topographic map with pole length as 0.994 and the latitude of secant parallel as 81°07'. Universal polar lunar surface projection is complementary to UTM projection in polar region.

## 4. CONCLUSION

Lunar projection is the main content to build mathematical foundation for lunar surface map, and its selection and design is a systematic engineering which is quite complicated as well as the fundamental part of lunar mapping technique. The basic difference between lunar mapping and earth mapping is that lunar surface cannot be directly used by man, and the data observation is acquired mainly through the space probe. Therefore, the multi-layer lunar projection system that adapts to lunar regional space features is just a preliminary framework, which basically follows the thought of earth map projection. With continuous improvement of lunar exploration technique, man's understanding of the moon is continuously deepening and the research scope has stretched from geometrical space to physical space, from lunar surface to the inner side<sup>[1]</sup>. The lunar space information projection system is to be further improved in the application.

The mapping science and technology penetrate through the entire process of lunar exploration. As the research scope is stretching from earth surface and terrestrial space to the moon even other major planets and interstellar space, the lunar mapping, based on earth mapping, will inevitably break through several leading techniques, intersect and integrate with such subjects as earth informatics and space physics, and the theory and technique of deep space mapping, which mainly researches on the moon and planets, is to be formed and established gradually.

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