K-ORDER-BASED TOPOLOGICAL RELATIONS OF GEO-ENTITIES IN 3D RASTER SPACE

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KEY WORDS: GIS, Model, Spatial, Raster, Three-dimensional, Algorithms

ABSTRACT:

The theoretical foundations and application availability of topological spatial relationship research for three dimensional Geo-entities in raster space are discussed. The digital topology theory is applied to define the interior, boundary and *k*-order six neighborhood of a 3D geo-entity which is represented with regular hexahedron tessellation. A new 9-intersection model, which replaces the original interior I, the boundary B and the exterior E of an entity in 9-I model with I_6, B_6 and E_6^k respectively, is presented. A computable method based on relation database query language SQL (Structured Query Language) is adopted for the model computation, topological query and analysis. The experimental analytic results show that the topology analysis model can be properly used for topological relations query and analysis of 3D Geo-entities represented in raster space, and it can also be applied for the reasoning of metric relation, direction relation and other spatial relationships.

1. INTRODUCTION

Topological relations denote the unchanged characteristics under the topological transform, such as translation, rotation and scale (Egenhofer 1989). Early in 1988, the American National Science Foundation (NSF) began to sponsor the American National Center for Geographic Information & Analysis (NCGIA) on researches on several theoretical issues of spatial relationship (NCGIA, 1989). Topological relations are one of the most foundational and most important aspects in spatial relationships, which play the key roles in many fields such as spatial data modeling, spatial query, spatial analysis, spatial reasoning and cartographic generalization. In the latter twenty years, researches on topological relations have attracted many specialists and a large number of papers have been published, which mainly concentrate on topological data model (Wu Lixin, 2004) and formal representation framework (Egenhofer, 1991; Li Chenming, Chen Jun, 1997) of topological relations. The topological data model usually embodies local topological relations between inner parts of an entity, while the formal representation framework of topological relations, which is named as Entity Topology, emphasizes on global topological relations between geological entities. Currently, the 9-intersection model presented by Egenhofer is widely used in two dimensional space. However, there are some imperfections in theory and application in this model, such as linear dependency between three sets of geometric elements of an entity, difficult definition for the exterior of an entity and worse computability. Hence, the 9-intersection model can not be directly used for Geo-entity topology research unless some improvements are made (Zhao Renliang, 2002; Chen Jun, Guo Wei, 1998).

With continuous development of human beings cognition and behaviors, the human beings has gradually changed its attention on geo-objects from two dimensional plane to three dimensional space, and from Earth surface to underground space. As a result of human beings activities, the entities on Earth surface are usually artificial entities with regular structure, accurate position and unambiguous boundary, which can be properly represented with vector data structure. While the underground space is usually invisible and continuous; many kinds of entities that embodied in underground space are usually with fuzzy boundary and may invade into each other. In this situation, the boundaries of underground Geo-entities can not be properly represented with geometric elements; especially the trend relation between inner parts of an entity and the relation between adjacent Geo-entities can not be indicated. However, the digital space based on raster tessellation is much more suitable for the representation of the continuity and structured variety of underground space.

2. 3D RASTER TOPOLOGY

2.1 Raster Data Model

There are several kinds of raster data model for Geo-entity tessellation such as tetrahedron, regular hexahedron, three Dimensional Voronoi etc. Compared to other data models, regular hexahedron has many good characteristics such as easy tessellation algorithm, simple data structure, smaller data storage and strict neighborhood definition, especially has good performance in indicating the spatial structured variety of Geo-entities. Besides, it has already been widely used as a three dimensional block model and has been proved with good applicability in attribute interpolation and resources estimation. A three dimensional spatial interpolation method such as inverse distance weighting method or Kriging method can be applied to combine the attribute variety of Geo-entities intensively with the spatial position of each hexahedron to

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indicate the spatial position relevancy and attribute variety trend between different Geo-entities.

2.2 Neighborhood Model of Regular Hexahedron

In order to study the topological relationships of entities, which are represented by vector data structure, a famous model (9-Intersection Model, 9-I Model) was presented, in which the topological space of an entity is partitioned to three parts: Interior, Boundary and Exterior based on point-set topology theory and the nine intersections with each part of two entities, are computed. The content (whether null or not) and the dimension (1D, 2D or 3D) of the nine intersections are studied to analyze the topological relation between two entities. Anyway, the 9-I Model can not be directly applied for querying and analyzing the topological relations between entities which are represented by raster data structure (Egenhofer, 1990, 1991, 1993) in that the raster space is an open-set, so the boundary of an entity is not a closed Jordan curve but numbers of inter-neighbored raster cells, which can not partition the interior of an entity from its exterior space unless the neighborhood connectivity is defined.

In two dimensional space Z^2, supposing that X is a sub-set of Z^2, then for arbitrary x $(i, j) \in Z^2$, the 4 neighborhood and 6 neighborhood (Hou Miaole, 2005; P. K. SAHA, 1996; ROSENFELD, 1974) are respectively defined as follow (Fig. 1):

N_4 (x) = {N, S, W, E};
N_8 (x) = {N, NW, NE, W, E, S, SW, SE}.

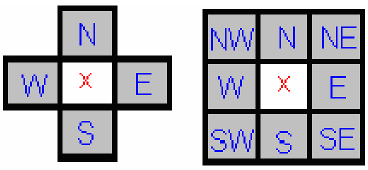

Figure 1 The N_4 and N_8 in 2D raster space

In three dimensional space Z^3, supposing that X is a sub-set of Z^3, then for arbitrary x $(i, j) \in Z^3$, the 6 neighborhoods (face-connected hexahedrons), 18 neighborhoods (edge-connected hexahedrons) and 26 neighborhoods (vertex-connected hexahedrons) are respectively defined as follow (Fig. 2 and 3):

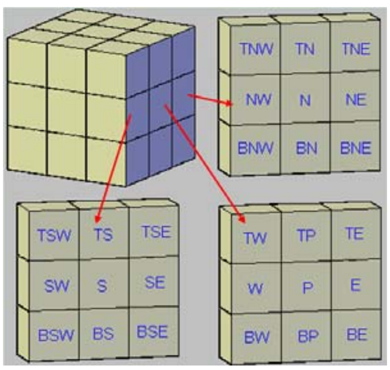

Figure 2 The 3*3*3 neighborhood of a raster cell (P. K. SAHA, 1996)

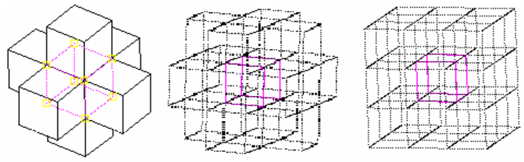

Figure 3 N_6, N_{18} and N_{26} model in 3D raster space

N_6 (x) = {TP, BP, N, S, W, E};
N_{18} (x) = {TN, N, BN, NW, NE, TW, TP, TE, W, E, BW, BP, BE, TS, SW, S, SE, BS};
N_{26} (x) = {TNW, TN, TNE, NW, N, NE, BNW, BN, BNE, TW, TP, TE, W, E, BW, BP, BE,TSW, TS, TSE, SW, S, SE, BSW, BS, BSE}.

Compared to N_{18} and N_{26} neighborhood model, N_6 is more suitable for representing the attribute relations between 3D raster cells. Hence, N_6 is selected as the connectivity definition model to define the boundary of a 3D entity.

3. *K*-ORDER NEIGHBORHOOD OF 3D RASTER ENTITY

3.1 Definition of Interior and Boundary

According to N_6 neighborhood model defined above, cells of a 3D raster entity can be partitioned to two parts: the interior and the boundary. With N_6 model, for each cell (named as e) of a 3D raster entity (named as E), if any cell of its N_6 is not a member cell of E, then this raster cell e is defined as a boundary cell of E. The collection of these boundary cells composes the boundary of the entity, which is named as six neighborhood boundary (B_6):

$$B_6 = \{e \mid e \in E,\ \exists\, x \in N_6(e),\ x \notin E\}. \qquad (1)$$

Contrarily, for each cell (named as e) of a 3D raster entity (named as E), if all the cells of its N_6 are member cells of E, then this raster cell e is defined as an interior cell of E. The collection of these interior cells composes the interior of the entity, which is named as six neighborhood interior (I_6):

$$I_6 = \{e \mid e \in E,\ \forall\, x \in N_6(e),\ x \in E\}. \qquad (2)$$

3.2 Definition of *K*-Order Neighborhood

In order to indicate the attribute relevancy between a Geo-entity and its neighborhood Geo-entities, the spatial relationship between them must be analyzed firstly. In this paper, the k-order buffers of a Geo-entity which are generated with six-connectivity is named as k-order six-neighborhood (E_6^k) and they are defined as follow:

1.E_6^1 can be defined based on I_6 and B_6, which is the collection of the N_6 cells of B_6 that are not member cells of I_6 and B_6;

2. E_6^2 can be defined based on E_6^1, which is the collection of the N_6 cells of E_6^1 that are not member cells of 1-order N_6 inclusion (the Geo-entity itself and its E_6^1);

3. E can be defined based on E_6^{k-1}, which is the collection of the N_6 cells of E_6^{k-1} that are not member cells of (*k*-1)-order N_6 inclusion (the Geo-entity itself and its E_6^1, E_6^2 … E_6^{k-1}).

As shown in Fig.4, a vector Geo-entity which is surrounded by 114 triangles was transformed to a 3D raster Geo-entity which is consists of 1652 regular hexahedrons. According to the definition of E_6^k, this Geo-entity is defined as in the followings: I_6 (992 cells), B_6 (660 cells), E_6^1 (792 cells) and 1-order inclusion (2444 cells).

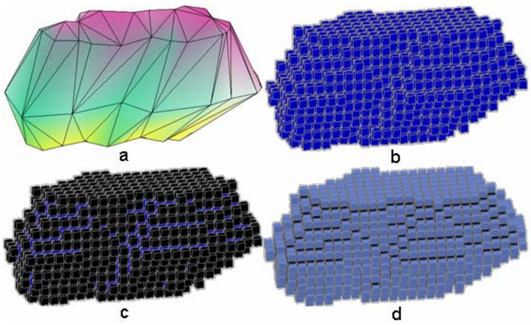

Figure.4 The 1-order N_6 inclusion of a 3D raster entity (a. vector entity; b. raster entity; c. the I_6 and B_6 of raster entity; d. 1-order N_6 inclusion)

4. *K*-ORDER 6 NEIGHBORHOODS-BASED 9-INTERSECTION MODEL

A new topological relationship analyzing model improved from 9-intersection model is presented and named as *k*-order six neighborhoods-based 9-intersection model (*K*6N9-I model). In this model, the original interior, boundary and exterior of 9-I model are replaced with I_6, B_6 and E_6^k respectively. The new model is shown as:

$$\begin{bmatrix} \partial A_6 \cap \partial B_6 & \partial A_6 \cap B_6^{\circ} & \partial A_6 \cap B_6^k \\ A_6^{\circ} \cap \partial B_6 & A_6^{\circ} \cap B_6^{\circ} & A_6^{\circ} \cap B_6^k \\ A_6^k \cap \partial B_6 & A_6^k \cap B_6^{\circ} & A_6^k \cap B_6^k \end{bmatrix}$$

Where

∂A_6 = the 6-connectivity boundary of Geo-entity A

A_6° = the 6-connectivity interior of Geo-entity A

A_6^k = the *k*-order N_6 of Geo-entity A

∂B_6 = the 6-connectivity boundary of Geo-entity B

B_6° = the 6-connectivity interior of Geo-entity B

B_6^k = the *k*-order N_6 of Geo-entity B

5. TOPOLOGICAL RELATIONS COMPUTATION AND ANALYSIS EXPERIMENT BASED ON K6N9-I MODEL

Seven Geo-entities with different shapes and different sizes were represented in raster space with regular hexahedrons, and its I_6, B_6 and E_6^1 were computed and defined (Fig. 5).

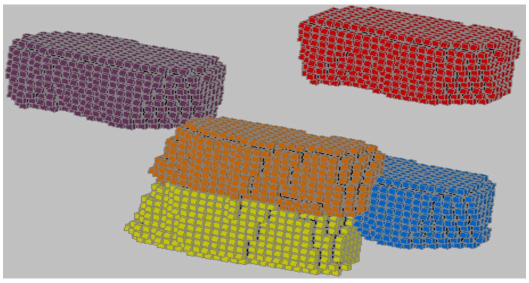

Figure 5 The raster representation and its 1-order N_6 inclusions of 3D entities

An experimental system for *K*6N9-I topological relations computation and for the analysis of 3D raster Geo-entities was developed based on Visual Basic program language and relation database query language: SQL (Structured Query Language). The content and count of each model element was computed, and the count was shown in the dialogue textbox (Fig. 6).

The counts of model elements were analyzed to build up the mapping regulation between quantitative results and qualitative topological relationships. Several basic topological relationships such as disjoint, meet, intersect, cover, contain and equal were queried after computation and analysis (Fig. 7).

6. CONCLUSIONS AND DISCUSSIONS

In this paper, the digital topology theory is applied to deal with Geo-entities in underground space which are represented in raster space with regular hexahedron tessellation. A new 9-intersection model, named as *K*6N9-I model, is presented to replace the original interior, boundary and exterior of 9-I model with I_6, B_6 and E_6^k respectively. The *K*6N9-I model is proved to be suitable for topological relations query and analysis of Geo-entities in raster space. A computable method for topological query and analysis based on relation database query language SQL (Structure Query Language) is presented. The experimental analytic results show that the topology analysis model can be properly used for topological relations query and for the analysis of 3D raster Geo-entities, and it can also be applied for the reasoning of metric relation, direction relation and other relations.

The *K* value and hexahedron scale are two import roles of *K*6N9-I model. However, only 1-order N_6 (E_6^1) of a Geo-entity was generated in this paper. In the next research stage, *K* will be set with different values to study refined topological relations and *K*-order-based neighborhood relations between Geo-entities. On the other hand, the method for the interior and the boundary definition of a Geo-entity under different hexahedron scales will also be studied.

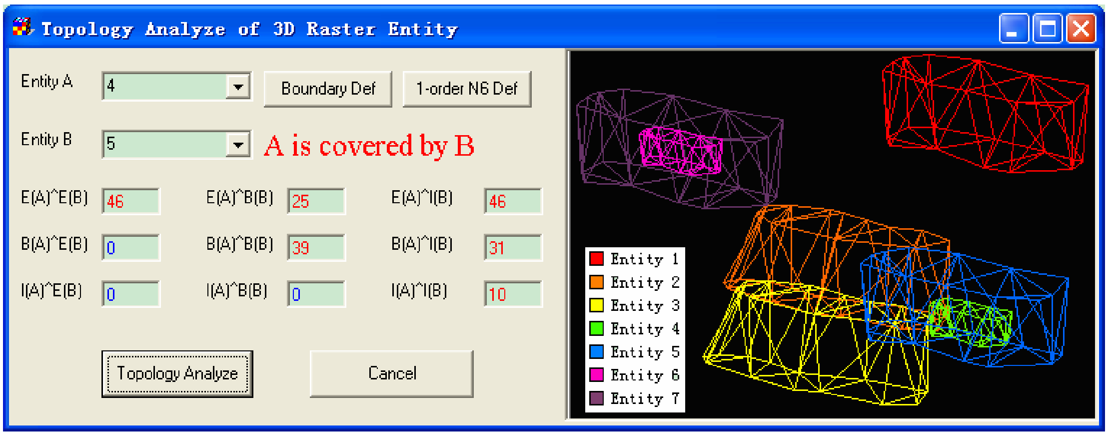

Figure 6 A topological analysis system based on K6N9-I model

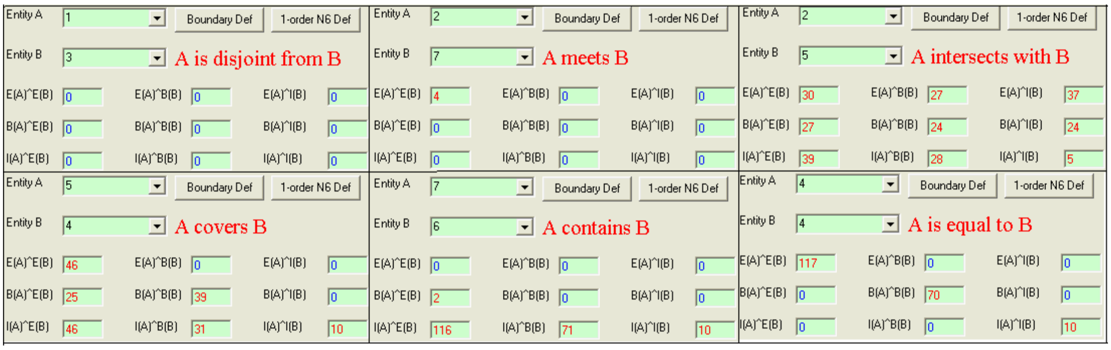

Figure.7 The computed results of basic topological relations based on K6N9-I model

ACKNOWLEDGEMENT

This research is jointly supported by the National 863 High-Tech Program of China (No. 2007AA06Z108, 2006AA12Z216), the Natural Science Fund of China for

Distinguished Young Scholars (No. 50525414), the National Natural Science Foundation (No. 40571137).

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