A QUATERNARY PROTOTYPE FOR SPATIOTEMPORAL ANALYSIS OF PERMANENT SCATTER INTERFEROMETRY

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ABSTRACT:

In this paper, a set of quaternary arc time series of the double phase differences formed by a PS (Permanent Scatter) with its surrounding four PSs in each quadrant are processed together, where the spatial constrains on the parameters are included directly in the adjustment model. Equations of this spatiotemporal analysis model are formulated. A simulation example using this new method is presented. It shows that a priori information of the crustal deformation can be integrated into the integer least squares adjustment model to improve the accuracy of parameters estimated.

1. INTRUDUCTION

Repeat-pass satellites SAR interferometry (InSAR) technology has been used for providing EDMs with meter accuracy and terrain deformations with millimetric accuracy (Hanssen, 2001). It has significant advantages over traditional geodetic methods for its larger spatial coverage with high spatial resolutions and all weather running. InSAR technology has been used for crustal deformation monitoring, such as ground subsidence, slope slides, volcanoes and so on. However, the essential limitations of InSAR are due to temporal and geometrical decorrelation and atmospheric inhomogeneities effects on interferometric phases. In 1999, a new interferometric method based on permanent scatters, named PS-InSAR is proposed (Ferretti, et al., 1999). This method uses the long time reliable coherence properties of PSs to overcome the temporal and geometrical decorrelation and also uses the time series of interferometric phase differences of adjacent PSs to eliminate the effects of the atmospheric inhomogeneities. Actually, in Permanent Scatter Interferometry (PSI), a stack of N differential interferograms of PSs are analyzed for phase unwrapping and deformation parameters estimation. The conventional method processes the time series of phase differences of the adjacent PSs (usually called as double difference of arcs) using the Integer Least Squares (ILS) method, such as the LAMBDA (Least squares AMBiguity Decorrelation Adjustment method) (Teunissen, 1995). Then the spatial closure conditions among arcs are applied for validations and corrections of phase ambiguities and parameters of models estimated (Kampes, 2006). And the temporal and spatial information in the interferograms are used separately. This PSI method sometimes fails to give correct estimations, so an integrated spatiotemporal analysis method is expected to be able to solve this kind of problem more efficiently.

Considering a set of quaternary arcs of time series radiated from one chosen PS bearing both spatial and temporal information of model parameters, we use these quaternary arcs of time series as an elementary adjustment cell for double difference phase ambiguity estimation. At first a prototype of the quaternary spatiotemporal adjustment model is given. Then a simulated example is demonstrated and the results are obtained. At last a conclusion of this research is given.

2. QUATERNARY ADJUSTMENT MODEL

Supposing we have N+1 SLC SAR images, based on the optimal baseline (spatial and temporal) distribution (Adam, et al., 2004), one image is chosen as master and the others as slaves. Each slave image has been coregistered with the master and N interferograms are obtained. With methods based on temporal stability of amplitudes or phases of a pixel, the PS candidates can be obtained (Kampes, 2006; Hooper, 2006). On each PSs, the wrapped phase ϕ_x^k in differential interferogram k can be decomposed to

$$\phi_x^k = W\{\phi_{x,t}^k + \phi_{x,d}^k + \phi_{x,a}^k + \phi_{x,n}^k\} \quad (1)$$

where $W\{\cdot\}$ is the wrapping operator, $\phi_{x,t}^k$ is the phase caused by uncompensated topography, $\phi_{x,d}^k$ is the phase caused by displacement of the target in the time between master and corresponding slave image acquisitions, $\phi_{x,a}^k$ is the phase caused by atmospheric delays, and $\phi_{x,n}^k$ is the additive noise term, the subscript x presents the position of the PS.

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Normally pairs of nearby PSs are chosen to form a number of arcs, the phase differences of these arcs are thought to be atmospheric delay free, i.e.

$$\phi_{x,y}^{k} = W\{\phi_{y,t}^{k} - \phi_{x,t}^{k} + \phi_{y,d}^{k} - \phi_{x,d}^{k} + \phi_{xy,n}^{k}\} \qquad (2)$$

where x , y are nearby points. According to Kampes (2006), the phase differences of arcs caused by topographic error and deformation can be written as the following:

$$\phi_{y,t}^{k} - \phi_{x,t}^{k} = \beta_{x}^{k} \cdot \Delta h_{x,y} \qquad (3)$$

$$\phi_{y,d}^{k} - \phi_{x,d}^{k} = -\frac{4\pi}{\lambda} T^{k} \Delta v_{x,y} \qquad (4)$$

where $\Delta h_{x,y}$ is the topographic error difference between x and y and $\Delta v_{x,y}$ is the SOL(Sight of Looking) rate difference between x and y , and a linear SOL rate is assumed for the deformation; T^{k} is the time baseline, and equals acquisition time difference between slave image k and the master image; β_{x}^{k} is an attitude to phase changing factor which is a function of interferometric baseline and radar wave length. So the observation equation of the wrapped double phase difference for the arc time series is

$$\phi_{x,y}^{k} = -2\pi a^{k} + \beta_{x}^{k} \cdot \Delta h_{x,y} - \frac{4\pi}{\lambda} T^{k} \Delta v_{x,y} + e^{k} \qquad k = 1,...N \qquad (5)$$

where a^{k} is the integer ambiguity and e^{k} is white noise. This is an underdetermined problem with N equations for N+2 unknown parameters. Pseudo observations which composed of assumed, or statistical variances of topographic errors and deformation rates have been added for solving this problem in Kampes (2006) with a LAMDA method. There is no spatial information has been applied in these pseudo observations. In this paper we expanded the equation (5) to a set of quaternary arc series as in Fig.1. Then we have 4N equations for 4N+8 unknown parameters.

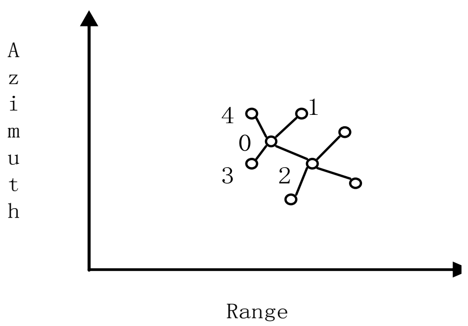

Figure.1 Quaternary arcs by PSs

For solving this quaternary arc underdetermined equations, similar pseudo observations with value of zeros and a priori variances for topographic errors and deformation rates are added, however the spatial correlations of deformation rates are considered and relevant variance and covariances for deformation rate differences are formulated under assumption of constant strain rate tensor. The equation (5) can be written in a matrix form as:

$$y_1 = A_1 a + B_1 b + e \qquad (6)$$

where,

$$y_1 = [\phi_{0,1}^{1}, \phi_{0,1}^{2}, ...\phi_{0,1}^{N}, ...\phi_{0,4}^{1}, \phi_{0,4}^{2}, ...\phi_{0,4}^{N}]^{T}{}_{4N\times 1},$$
$$a = [a_{0,1}^{1}, a_{0,1}^{2}, ...a_{0,1}^{N}, ...a_{0,4}^{1}, a_{0,4}^{2}, ...a_{0,4}^{N}]^{T}{}_{4N\times 1},$$
$$b = [\Delta h_{0,1}, \Delta h_{0,2}, \Delta h_{0,3}, \Delta h_{0,4}, \Delta v_{0,1}, \Delta v_{0,2}, \Delta v_{0,3}, \Delta v_{0,4}]^{T}{}_{8\times 1},$$
$$e = [e_{0,1}^{1}, e_{0,1}^{2}, ...e_{0,1}^{N}, ...e_{0,4}^{1}, e_{0,4}^{2}, ...e_{0,4}^{N}]^{T}{}_{4N\times 1},$$

they are wrapped phase difference vector, unknown ambiguity vector, unknown topographic error and deformation rate vector, and white noise error vector respectively; A_1 , B_1 are respectively $4N \times 4N$ and $4N \times 8$ coefficient matrix.
The pseudo observation equations are added as:

$$y_2 = A_2 a + B_2 b + e_2 \qquad (7)$$

where A_2 is an $8 \times 4N$ matrix with all elements zero, B_2 is an 8×8 identity matrix, and $y_2 = 0$. The covariance matrix of y_1 and y_2 are assumed as following:

$$Q_{y_1} = \begin{bmatrix} Q_{ifg}^{arc1} & 0 & 0 & 0 \\ 0 & Q_{ifg}^{arc2} & 0 & 0 \\ 0 & 0 & Q_{ifg}^{arc3} & 0 \\ 0 & 0 & 0 & Q_{ifg}^{arc4} \end{bmatrix} \qquad (8)$$

$$Q_{y_2} = \begin{bmatrix} \sigma_h^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \sigma_h^2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma_h^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma_h^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sigma_d^2 & \sigma_{12} & \sigma_{13} & \sigma_{14} \\ 0 & 0 & 0 & 0 & \sigma_{21} & \sigma_d^2 & \sigma_{23} & \sigma_{24} \\ 0 & 0 & 0 & 0 & \sigma_{31} & \sigma_{32} & \sigma_d^2 & \sigma_{34} \\ 0 & 0 & 0 & 0 & \sigma_{41} & \sigma_{42} & \sigma_{43} & \sigma_d^2 \end{bmatrix} \qquad (9)$$

where Q_{ifg}^{arci} is variance matrix of the i^{th} arc phase difference observations which can be estimated from error propagation law (Kampes, 2006); σ_h is the standard deviation of topographic error differences, σ_d^2 is the variance of deformation rate difference and σ_{ij} is the covariance of deformation rate difference between the i^{th} and the j^{th} arc. The SOL deformation rate difference of each arc caused by a constant surface deformation rate can be deduced (Malvern, 1969)

$$\Delta v_{xy} = (\dot{\varepsilon} L_{xy}) \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix} \sin\theta \qquad (10)$$

where $\dot{\varepsilon}$ is strain rate tensor, L_{xy} is the arc geometric vector, θ is radar side looking angle. Then the variance matrix for the quaternary arc phase differences in SOL can be deduced based on error propagation law using equation (10), it can be written approximately as

$$D = (\sin\theta\sigma_{\dot{\varepsilon}})^2 \begin{bmatrix} d_{11} & d_{12} & d_{13} & d_{14} \\ d_{12} & d_{22} & d_{23} & d_{24} \\ d_{13} & d_{23} & d_{33} & d_{34} \\ d_{14} & d_{24} & d_{34} & d_{44} \end{bmatrix} \qquad (11)$$

where $\sigma^2_{\dot{\varepsilon}}$ is variance of components of strain rate which are assumed to be equal accuracy and independent, and

$$d_{ij} = L_i L_j \cos(\alpha_i - \alpha_j) \qquad (12)$$

where L_i, L_j are length of the i^{th} and the j^{th} arc respectively; α_i is azimuth angle of the i^{th} arc vector with respect to the range axis positive direction. This variance matrix D integrated a priori information of constant strain rate and will take place the sub matrix [σ_{ij}] in equation (9) during the ambiguity determination in the following simulation example.

3. SIMULATION EXAMPLE

The simulation scenario is same as that in Kampes (2006), except that the LOS deformation rate is simulated by a constant strain rate model, see equation (10). ERS satellite parameters are used in the simulation. Input data is simulated at 1000 points, for an area of approximately $10 \times 10 km^2$, of 31 SAR images. The 31 SLC SAR images are ordered in their acquisition times and the middle acquisition time image are used as the master image of interferometry. Totally 30 interferograms are obtained. We randomly choose a set of quaternary arcs, see Fig.2, to get the four arc time series. Table 1 lists the relevant values in simulation data set. The ILS method are used to solve for the ambiguity and model parameters. The covariance matrix of pseudo observations are formulated by (9) and (11). The true and estimated parameters are listed in Table 2, and the histogram of data residuals are drawn in Fig.3.

Parameter	Value
Span of perpendicular baseline	1636.2m
Span of temporal baseline	7.96year
Maximal DEM error	24.9685m
Minimal DEM error	-24.9752m
Uniform dilation strain rate	5.0E-5/year
Phase noise level	20 degrees

Table 1 The relevant values in simulation interferometric data

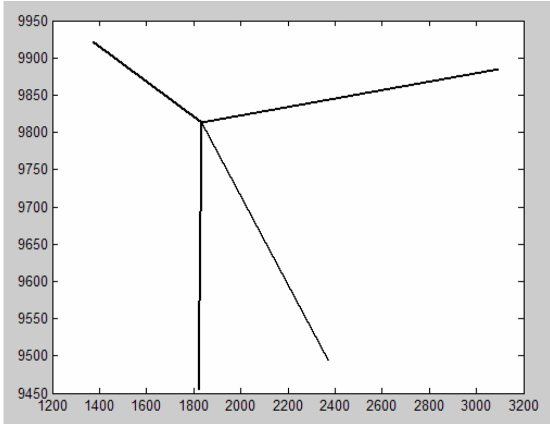

Figure.2 Quaternary arcs chosen

	Arc one		Arc two		Arc three		Arc four	
	$\Delta h_{x,y}$ (m)	$\Delta v_{x,y}$ (mm/y)	$\Delta h_{x,y}$ (m)	$\Delta v_{x,y}$ (mm/y)	$\Delta h_{x,y}$ (m)	$\Delta v_{x,y}$ (mm/y)	$\Delta h_{x,y}$ (m)	$\Delta v_{x,y}$ (mm/y)
True	25.989	-24.5	-13.647	9.1	-5.990	-0.3	-4.718	-10.4
Est.	25.860	-24.6	-13.353	8.9	-5.984	-0.3	-5.019	-10.5

Table 2 True and estimated parameters

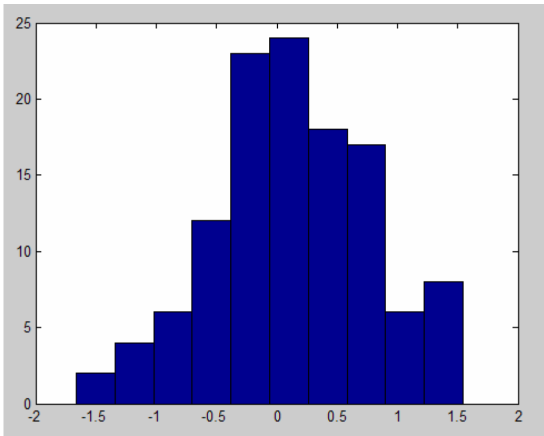

Figure. 3 Histogram of data residuals

From Table 2, we can see that the estimated SOL rate differences are almost same as the true values. However, the estimated DEM error differences are large in somewhat, the maximum estimated DEM error is about 0.3m, which is about 5 cycles of phase ambiguity. Fig.3 shows the histogram of the 120 data residuals. Most of them are located between -0.5rad and 0.5rad, this is comparable with phase noise level added (noise standard deviation is set to be 20 degrees) in the simulation data.

4. CONCLUSION

We have developed an adjustment model of quaternary arcs for realizing an integrated spatiotemporal analysis for double difference phase unwrapping. Because the double difference phase unwrapping is generally an underdetermined ILS problem, a priori information of unknown parameters is necessary for getting solution. Under the assumption of a constant strain rate of deformation, the variances of SOL rate differences are deduced as in equation (9) to (12), and are incooperated into the phase unwrapping solving using the LAMBDA method. Further research may include more radiation arcs at a PS so as to include more bearing spatial information in double difference phase unwrapping.

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REFERENCES

Adam, N., Kampes, B. M. and Eineder, M., 2004. The development of a scientific persistent scatterer system: Modifications for mixed ERS/ENVISAT time series. *ENVISAT & ERS Symposium, Salzburg, Austria, 6-10 September, 2004.* pp. 1-9.

Ferretti, A., C. Prati and F. Rocca. 1999. Permanent Scatterers in SAR Interferometry. *In: Proc. IGARSS 1999, Hamburg, Germany, 28 June-2 July 1999*, pp.1528-1530.

Ferretti, A., Prati, C. and Rocca, F. 2001. Permanent scatterers in SAR interferometry. *IEEE Transactions on Geoscience and Remote Sensing*, 39(1), pp.8-20.

Kampes, B. M., 2006. *Radar Iterferometry: Persistent Scatterer Technique*. Springer.

Hanssen, R. F., 2001. *Radar Interferometry:Data Interpretation and Error Analysis*. Kluwer Academic Publishers. Dordrecht.

Malvern, L. E., 1969. *Introduction to the mechanics of a continuous medium*. Prentice-Hall, Inc., Englewood Cliffs, New Jersey.

Teunissen, P. J. G. 1995. The least-squares ambiguity decorellation adjustment: a method for fast GPS integer ambiguity estimation. *Journal of Geodesy*, 70(1－2), pp.65－82.