

# A METHOD FOR SAR SPECKLE REDUCTION BASED ON PARTIAL DIFFERENTIAL EQUATION

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**ABSTRACT:**

Synthetic Aperture Radar (SAR) has been get broad recognition and application in many domains such as surveying and mapping. But the SAR speckles influence the interpretation of the image badly. In this paper, we discuss the second-order and the fourth-order partial differential equations(PDE), and we unite the two equations to reduce the speckles. Finally we use SAR image to do some experiment, the result shows that this equation combine the strongpoint of the second-order and the fourth-order partial differential equation, it can keep the edge of the SAR image and reduce the SAR speckle better. It avoids the block effect when we use second-order PDE to progress image and makes the image smoothing naturally.

## 1. INTRODUCTION

Synthetic Aperture Radar(SAR)is an active image system widely used in remote sensing applications and in course of SAR imaging the SAR images suffer from a special kind of noise, it called 'speckle'. The Speckle is common in the radar system, it influences the resolution of the image directly and degrade the ability of explore, analyse and interpretation for SAR image, hence increases great difficulty for the observer to extract fine detail and textural information from SAR images. If we process and analyse SAR image without speckle reduction, the influences of the speckle should come into the result and we will get wrong conclusion. Therefore how to reduce the speckle in the SAR images has become a necessary step in processing.

In recent years, Partial Differential Equation (PDE) has applied to the image progressing widely. Using PDE to reduce the speckle is the noise-smoothing methods which is getting attention widely, because PDE can keep the edge well when it reduces the noise.

## 2. PDE MODEL OF IMAGE DENOISING

### 2.1 second-order PDE model of image denoising

The image is a function which defines in a rectangle region and reflects the grey level changing in the realistic scene, that is the image  $f : x \in R^2 \rightarrow R$ , the gray level is  $f_0(x, y)$ .

If we import the time parameter 't' in the image equation, the changing of image can express with PDE as:

$$\frac{\partial f}{\partial t} = F[f(x, y, t)] \tag{1}$$

Where,  $F$  express some certain algorithm.

PDE can fall into linearity diffusion model, non-linearity diffusion model, and so on. And the solution  $f(x, y, t)$  of PDE is the image when the scale is  $t$ .

In 1990, Perona and Malik proposed a non-linearity anisotropic diffusion equation which is a better effect method of image smoothing. It is:

$$\frac{\partial f}{\partial t} = \text{div}(g(|\nabla f|)\nabla f) \tag{2}$$

Where,  $|\nabla f|$  is the norm of gradient,

$g(|\nabla f|)$  is named smoothing function for border stopping, it is use keeping edge.

The function  $g(x)$  is a decreasing function and  $g(0) = 1, \lim_{x \rightarrow \infty} g(x) = 0$ . In the edge of the image  $|\nabla f|$  is bigger commonly, thereby the edge region stop smoothing.

### 2.2 fourth-order PDE model of image denoising

But in using P-M equation to smooth the image, it appeared "blocky effects" sometimes, that is say, after image progress the grey level in some region is very closed or same.

So Yu-Li and M.Kaveh proposed a fourth-order partial differential equation. Using this equation, it can reduce the noise, at the same time it can keep the image edge better. The PDE is:

$$\frac{\partial f}{\partial t} = -\nabla^2 [g(|\nabla^2 f|)\nabla^2 f] = -\Delta [g(\Delta f)\Delta f] \tag{3}$$

Where,  $g(x) = \frac{1}{1 + \left(\frac{x}{k}\right)^2}$ ,  $k$  is a constant.

We can receive the better effect when we use fourth-order PDE in denoising image, the advantage of this equation is it removes the blocky effects that made by second-order nonlinearity diffusion equation and the disadvantage is it need more times iteration and this equation has no removed ability to pepper and salt noises.

### 3. MODEL OF COUPLED PDE

So, we study the coupled PDE based on second-order and fourth-order PDE. The equation is:

$$\frac{\partial f}{\partial t} = -\nabla[g_1(\Delta f)\nabla f + a \cdot g_2(\Delta f)\nabla\Delta f] \quad (4)$$

Where,  $a$  is the coupled coefficient

the function  $g_1$  and  $g_2$  is plus and non-increasing function.

Normally,

$$g_1(x) = \exp\left(-\left[\frac{x}{k_1}\right]^2\right)$$

$$g_2(x) = \frac{1}{1 + \left(\frac{x}{k_2}\right)^2}$$

Where,  $k_1$  and  $k_2$  are constant.

This equation unites the advantage of the second-order and fourth-order PDE, it can keep the image feature better. Using the anisotropic character of the coefficient in the Perona—Malik equation, we can easily remove the noise and keep the edge by a few iteration; Using the fourth-order PDE in image processing, we can avoid the blocky effects by second-order PDE and make the figure smoothing.

### 4. THE DIFFERENCE OF THE MODEL

The differential equation (4) may be solved numerically using an iterative approach. Assuming a space grid size of  $h$  and  $Ih \times Jh$  is the size of image support, we quantize the time and space coordinates as follows:

$$x = ih \quad (n = 0, 1, 2, 3\Lambda, I)$$

$$y = jh \quad (n = 0, 1, 2, 3\Lambda, J)$$

1) we calculate  $\Delta f$  of the image intensity function as

$$\Delta f = \nabla^2 f_{i,j}^n = \frac{f_{i+1,j}^n + f_{i-1,j}^n + f_{i,j+1}^n + f_{i,j-1}^n - 4f_{i,j}^n}{h^2}$$

with symmetric boundary conditions

$$f_{I+1,j}^n = f_{I,j}^n, \quad f_{-1,j}^n = f_{0,j}^n \quad (j = 0, 1, 2\Lambda J)$$

$$f_{i,-1}^n = f_{i,0}^n, \quad f_{i,J+1}^n = f_{i,J}^n \quad (i = 0, 1, 2\Lambda I)$$

2) we calculate the value of the following function:

$$c(\nabla^2 f) = g_2(\nabla^2 f)\nabla^2 f \quad c_{i,j}^n = c(\nabla^2 f_{i,j}^n)$$

3) we calculate the  $\Delta g(\Delta f)$  of as

$$\nabla^2 c_{i,j}^n = \frac{c_{i+1,j}^n + c_{i-1,j}^n + c_{i,j+1}^n + c_{i,j-1}^n - 4c_{i,j}^n}{h^2}$$

with symmetric boundary condition

$$c_{I+1,j}^n = c_{I,j}^n, \quad c_{-1,j}^n = c_{0,j}^n \quad (j = 0, 1, 2\Lambda J)$$

$$c_{i,-1}^n = c_{i,0}^n, \quad c_{i,J+1}^n = c_{i,J}^n \quad (i = 0, 1, 2\Lambda I)$$

Finally, the numerical approximation to the differential equation (4) is given as

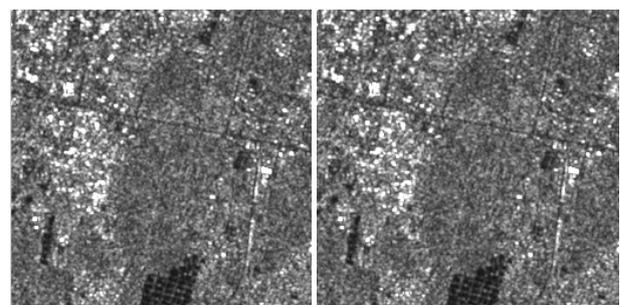
$$f_{i,j}^{n+1} = f_{i,j}^n + \frac{1}{4} \sum (g_1(\nabla f)\nabla f + a \cdot g_2(\Delta f)\nabla\Delta f)$$

### 5. EXAMPLES AND RESULTS

In this section, we evaluate the performance of our proposed denoising algorithm of couple PDE with two real SAR images and compare it with second-order and fourth-order PDE. We use the local Equivalent number of looks (ENL) to measure the noise reduction level. It is better at noise reduction if the ENL is higher. The equivalent number of looks can be defined in the way:

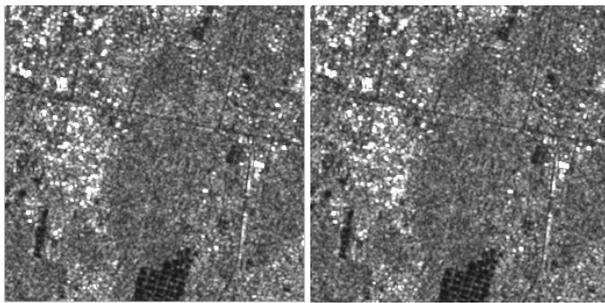
$$ENL = \frac{\mu^2}{\sigma^2}$$

where  $\mu$  is the mean of the points in the homogeneous areas of the recovered image and  $\sigma$  is the standard deviation.



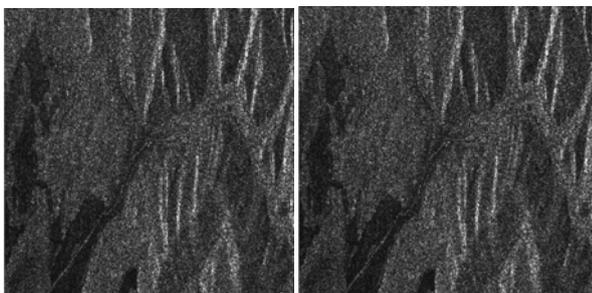
(a) Original SAR image  
ENL=7.5715

(b) Second-order PDE  
ENL=7.5814

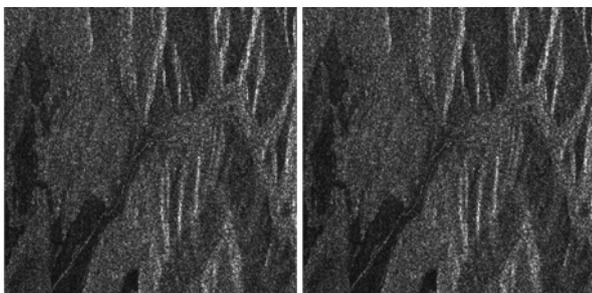


(c) fourth-order PDE ENL=7.5720 (d) couple PDE ENL=7.6205

Figure 1 SAR image and result image



(a) Original SAR image ENL=2.6804 (b) Second-order PDE ENL=2.6899



(c) fourth-order PDE ENL=2.6810 (d) couple PDE ENL=2.7017

Figure 2 another SAR image and result image

We compared the denoising performance on these image among (a) original image , (b) image based second-order PDE filter , (c) image based fourth-order PDE filter, and (d) image based couple PDE filter. And their ENL are listed under corresponding image. From the results, we can see that couple PDE has a superior performance than second-order and fourth-order PDE in SAR image denoising.

## 6. CONCLUSION

We have studied a new filtering algorithm to denoise SAR images based on the PDE. In order to keep the image feature better we apply the second-order and fourth-order PDE to SAR images. Experimental results indicate that this new algorithm has better satisfactory performance in terms of speckle reduction and detail preservation than other PDE. Furthermore,

it can be easily implemented and very robust. Therefore it has a wide applicability in practice.

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