

ACCURACY EVALUATION OF RATIONAL POLYNOMIAL COEFFICIENTS SOLUTION FOR QUICKBIRD IMAGERY BASED ON AUXILIARY GROUND CONTROL POINTS

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ABSTRACT:

Relative to the rigorous physical model, rational polynomial coefficient (RPC) has been adopted as an alternative common sensor model data for image Geometric correction exploitation. In this paper, based on collected QuickBird imagery in Shanghai region, the iterative least-squares solution with regularization(ILSR) is derived to determine the RPCs by using 50 fair distributed ground control points (GCPs) firstly. Two methods are then used to refine determined RPCs under different circumstance as: 1) when both the original and the additional GCPs are available, the RPCs will be recomputed using the batch iterative least-squares solution with regularization (BILSR) method; and 2) when only the new GCPs are available, incremental discrete Kalman filtering (IDKF) method has been described. Meanwhile, check points are used to evaluate their geo-positioning accuracy, and their comparison is conducted. Finally, some conclusion is then achieved when handing the high resolution imagery in metropolitan area.

1. INTRODUCTION

Satellite Imagery such as QuickBird, IKONOS has been widely used with the development of high resolution satellite technology. Collinearity based rigorous sensor model is the basis of geometric positioning for high resolution satellite imagery (HRSI). Dependence on physical parameters and satellite orbit parameters makes the rigorous sensor model much more complicated, thus hard to be applied with. The RPC, a mathematical model which is sensor independent and not rigorous, has been used widely by satellite companies for the survey process of HRSI and as the alternative of the rigorous sensor model. The image coordinates are denoted as the third polynomial expression in RPC. RPC, by providing a simple and exact relation for vendors and customers to describe the relationship of object and image, has been successfully employed in the terrain modeling, orthographic projection and feature extraction. A lot of research work has been done about the geometric correction and 3D reconstruction of IKONOS imagery using RPC (Tao and Hu, 2001,

2002 (1), 2002 (2);

Dowman, 2000; Fraser, 2002; Clive, 2002).

Two methods are used for the calculation of RPC, terrain dependant approach and terrain independent approach (Yong Hu et al., 2004). The terrain dependent approach, without setting up grids, is to obtain GCP through topographical measurement or field survey to fit the imagery geometry using sufficient parameters. Its accuracy is determined by hypsography and the GCP number and distribution (Fraser, 2006; Liu, 2006). The relativity between the RPC parameters may

result in the singularity of design matrix for normal equation. The regularization method can improve the condition number of the design matrix, thus avoiding the numerical instability of least square solution (Tao and Hu, 2001).

The RPC direct correction method was put forward to improve the positioning accuracy and meet the demand of high accuracy users. Different mathematical methods were applied for the RPC accuracy improvement when the physical sensor model was unknown. When the original and auxiliary GCP were both available, BILSR was used to recalculate the RPC (Hu and Tao, 2002; Di et al., 2003). Here the original GCP denotes the GCP used for calculating the original RPC, while the auxiliary GCP means the auxiliary collected GCP that never used for RPC calculation. The correction process is to include all the GCP into the RPC solution with different power to the new and original GCP. When there are only auxiliary GCP, the IDKF can be employed to improve the RPC accuracy (Hu and Tao, 2002; Bang et al., 2003), which means the accuracy of RPC is improved through the inclusion of new GCP with proper power.

Based on the QuickBird imagery in Shanghai, China, this paper mainly discusses the solution of RPC and accuracy after correction applied in metropolitan area without obvious hypsography. Experiences of application for similar data could be learned from the models and data this paper employed.

2. RPC MATHEMATICAL MODEL

The RPC of QuickBird imagery denotes the image coordinates as the ratio of polynomials based on the variable of longitude, latitude and height, which is as equation (1):

$$r_n = \frac{p_1(P_n, L_n, H_n)}{p_2(P_n, L_n, H_n)} = \frac{\sum_{i=0}^3 \sum_{j=0}^3 \sum_{k=0}^3 a_i P_n^i L_n^j H_n^k}{\sum_{i=0}^3 \sum_{j=0}^3 \sum_{k=0}^3 b_i P_n^i L_n^j H_n^k}$$

$$c_n = \frac{p_3(P_n, L_n, H_n)}{p_4(P_n, L_n, H_n)} = \frac{\sum_{i=0}^3 \sum_{j=0}^3 \sum_{k=0}^3 c_i P_n^i L_n^j H_n^k}{\sum_{i=0}^3 \sum_{j=0}^3 \sum_{k=0}^3 d_i P_n^i L_n^j H_n^k}$$

(1)

The image coordinate (r_n, c_n) and ground coordinate (P_n, L_n, H_n) , whose value are within $[-1, +1]$, are both the standardization coordinates through translation and scale, for the purpose of reduce the rounding errors in calculation because of quantitative difference. The unit of (r_n, c_n) is pixel; P_n, L_n are the coordinates of WGS84 with unit degree; H is the geodetic height with unit meter. (P_n, L_n, H_n) can be expressed as equation (2):

$$p_1 = a_1 + a_2 L_n + a_3 P_n + a_4 H_n + a_5 L_n P_n + a_6 L_n H_n + a_7 P_n H_n + a_8 L_n^2 + a_9 P_n^2 + a_{10} H_n^2 + a_{11} P_n L_n H_n + a_{12} L_n^3 + a_{13} L_n P_n^2 + a_{14} L_n H_n^2 + a_{15} L_n^2 P_n + a_{16} P_n^3 + a_{17} P_n H_n^2 + a_{18} L_n^2 H_n + a_{19} P_n^2 H_n + a_{20} H_n^3$$

(2)

Where: a_t is the polynomial coefficients ($t = 1, 2, \dots, 20; i, j, k = 0, 1, 2, 3$), other polynomials have the similar expression.

The terrain dependent approach is to calculate the 80 parameters of RPC using GCP from field survey. For connivance in the following expression, the (P_n, L_n, H_n) are shorted for (P, L, H) , and (r_n, c_n) for (r, c) , so equation (1) and (2) can be expressed as (3):

$$r = \frac{(1 \ L \ P \ H \ \dots \ P^2 H \ H^3)(a_1 \ a_2 \ \dots \ a_{20})^T}{(1 \ L \ P \ H \ \dots \ P^2 H \ H^3)(1 \ b_1 \ \dots \ b_{19})^T}$$

$$c = \frac{(1 \ L \ P \ H \ \dots \ P^2 H \ H^3)(c_1 \ c_2 \ \dots \ c_{20})^T}{(1 \ L \ P \ H \ \dots \ P^2 H \ H^3)(1 \ d_1 \ \dots \ d_{19})^T}$$

(3)

Linearization of equation (3) (Tao and Hu, 2001) we could get equation (4):

$$v_r = \left[\frac{1}{B} \ \frac{L}{B} \ \frac{P}{B} \ \frac{H}{B} \ \dots \ \frac{P^2 H}{B} \ \frac{H^3}{B} \ -\frac{rL}{B} \ -\frac{rP}{B} \ \dots \ -\frac{rP^2 H}{B} \ -\frac{rH^3}{B} \right] \square J - \frac{r}{B}$$

$$v_c = \left[\frac{1}{D} \ \frac{L}{D} \ \frac{P}{D} \ \frac{H}{D} \ \dots \ \frac{P^2 H}{D} \ \frac{H^3}{D} \ -\frac{cL}{D} \ -\frac{cP}{D} \ \dots \ -\frac{cP^2 H}{D} \ -\frac{cH^3}{D} \right] \square K - \frac{c}{D}$$

(4)

Where:

$$B = (1 \ Z \ Y \ X \ \dots \ Y^3 \ X^3) \square (1 \ b_1 \ \dots \ b_{19})^T$$

$$J = (a_0 \ a_1 \ \dots \ a_{19} \ b_1 \ b_2 \ \dots \ b_{19})^T$$

$$D = (1 \ Z \ Y \ X \ \dots \ Y^3 \ X^3) \square (1 \ d_1 \ \dots \ d_{19})^T$$

$$K = (c_0 \ c_1 \ \dots \ c_{19} \ d_1 \ d_2 \ \dots \ d_{19})^T$$

(5)

Suppose there are n GCP, the error equation is:
 $V = WTI - WG$

Where:

$$W = \begin{bmatrix} \frac{1}{B_1} & \dots & 0 & 0 & \dots & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & \dots & \frac{1}{B_n} & 0 & \dots & 0 \\ 0 & \dots & 0 & \frac{1}{D_1} & \dots & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & \dots & 0 & 0 & \dots & \frac{1}{D_n} \end{bmatrix},$$

$$T = \begin{bmatrix} 1 & \dots & -r_1 H_1^3 & 0 & \dots & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 1 & \dots & -r_n H_n^3 & 0 & \dots & 0 \\ 0 & \dots & 0 & 1 & \dots & -c_1 H_1^3 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & \dots & 0 & 1 & \dots & -c_n H_n^3 \end{bmatrix}$$

$$I = [a_1 \ \dots \ a_{20} \ b_1 \ \dots \ b_{20} \ c_1 \ \dots \ c_{20} \ d_1 \ \dots \ d_{20}]^T$$

$$G = [r_1 \ \dots \ r_n \ c_1 \ \dots \ c_n]^T$$

(6)

If the n observations are unit weight observations, then the normal equation is:

$$T^T W^2 T I - T^T W^2 G = 0$$

(7)

Then we get the coefficient matrix I:

$$I = (T^T W^2 T)^{-1} T^T W^2 G \quad (8)$$

3. REGULARIZATION METHOD

3.1 Regularization Model

The denominators B_i and D_i ($i = 1, \dots, n$) change quickly in quantity when the GPS input distributed unevenly in calculation, which makes matrix T ill-condition in the equation, and thus the matrix $T^T W^2 T$ singularity. This case often happens when the rank of RPC polynomial is higher (eg, more than 2), which may result in not convergent during iteration.

A unit matrix E could be added by the regularization method to improve the condition number of matrix $T^T W^2 T$. $T^T W^2 T$ is a symmetric nonnegative definite matrix, and the eigenvalue of matrix $T^T W^2 T + h^2 E$ is in the range of $[h^2, h^2 + \|T^T W^2 T\|]$, so the condition number of which will no bigger than $(h^2 + \|T^T W^2 T\|) / h^2$, on the contrary, it will reduce with the increase of h^2 . A regularization rule method is employed in this paper to improve the condition number of matrix to obtain stable numerical solution. The normal equation is calculated through iteration and can be ended when the condition be met (Neumaier, 1998).

$$(T^T W^2 T + h^2 E)I - T^T W^2 G = 0 \quad (9)$$

$$I_{(0)} = 0, W_{(0)} = W(I_{(0)}) = E$$

$$I_{(s)} = I_{(s-1)} + (T^T W_{(s-1)}^2 T + h^2 E)^{-1} T^T W_{(s-1)}^2 v_{(s-1)}$$

$$W_{(s)} = W(I_{(s)}), v_{(s)} = G - T I_{(s)} \quad (10)$$

Where:

E is the unit matrix, h is the regularization parameter, s is the number of iteration.

When the pixel measurement error is known, the covariance matrix P of parameter I could be calculated from the following equation:

$$P = (T^T W^2 T + h^2 E)^{-1} T^T W^2 R_G W^2 T (T^T W^2 T + h^2 E)^{-1} \quad (11)$$

There are many ways to obtain the regularization parameter h. Different h will get different result, of which the optimal value is attained by trial method, here L curve method (Neumaier, 1998) is employed. To get the optimal value of h, different value was tested in formula (9). The third power RPC was employed in the test, 50 GCP and their corresponding image points were selected for calculation of RPC, while 29 CkP were used for accuracy assessment. The result was shown as figure 1. Based on the test, less than 10 iteration times may get good convergence under most conditions when h was in the range of 0.009 and 0.1. We also found that as long as h was in the range, the accuracy was not sensitive to specific h, that is to say the results were within 0.01 pixel (see table 1). So h=0.05 was used in the following text.

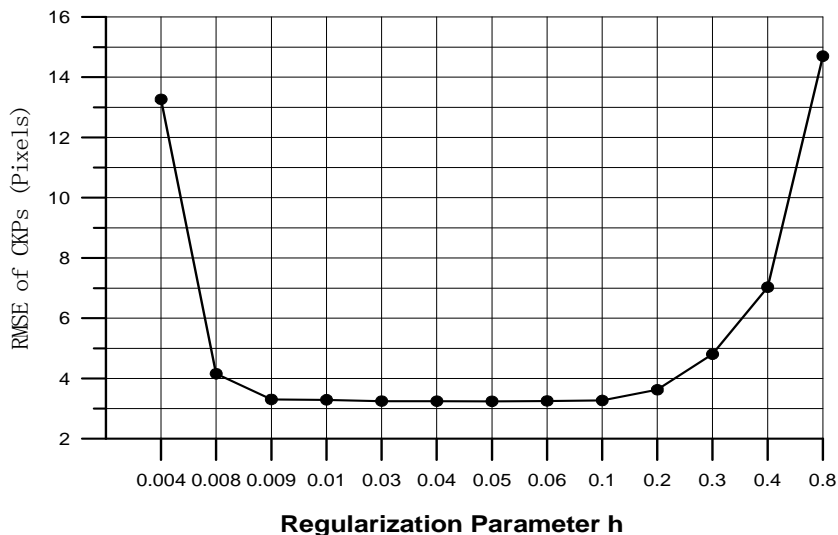


Figure 1 Calculation for h----L Curve Figure

h Value	Matrix Singularity (Y/N)	Iteration Number	CkP Point Position Error (pixel)	h Value	Matrix Singularity (Y/N)	Iteration Number	CkP Point Position Error (pixel)
<0.004	Y	Not Convergence	N	0.05	N	5	3.2425
0.004	N	25	13.266	0.06	N	6	3.256
0.008	N	14	4.159	0.1	N	10	3.272
0.009	N	3	3.305	0.2	N	20	3.626
0.01	N	3	3.293	0.3	N	20	4.806
0.03	N	4	3.2498	0.4	N	27	7.030
0.04	N	4	3.2476	0.8	N	30	14.699

Table 1 Positioning Accuracy with different h values

3.2 Calculation of RPC

The test imagery is QuickBird imagery of Shanghai area, of which the collection time is February 15, 2004, the coordinates of lower left corner (31.14796° , 121.424732°) ,the coordinate of upper right corer (31.299428° , 121.61359°) . The image collection azimuth is 355.3 degree; the elevation angle is 68.2 degree. 50 GCP distributed evenly were

selected from the overall 139 surveyed points to calculate the initial RPC, the distribution of which was shown in figure 2. 26 points were selected randomly from the rest surveyed points as CkP for accuracy analysis, of which the distribution was as figure 3.

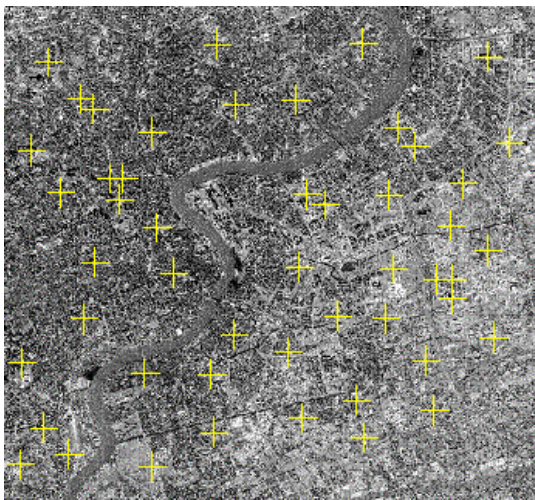


Figure 2 GCP Distribution

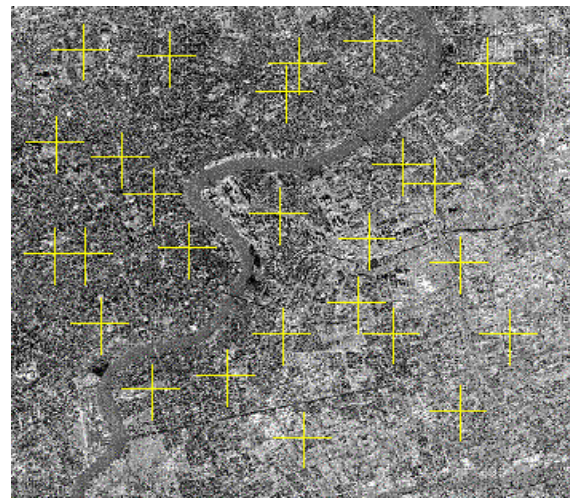


Figure 3 CKP Distribution

Taking $h = 0.05$ into formula (10) to get 80 RPC parameters, we can get the image coordinated by adding the parameters and ground points (P, L, H) of the 26 CkP into formula (1). The differences between calculated image coordinates and corresponding image points of CkP were shown in table 2:

Results and Conclusion of tests:

Error Type		Accuracy (pixel)	Error Type		Accuracy (pixel)
Absolute Error	Max Row Error	4.092	Relative Error	Row Error	1.059
	Row Mean Square Deviation	1.864		Column Error	2.055
	Max Column Error	7.060		Point Position Error	2.312
	Column Mean Square Deviation	2.653			
	Point Position Error	3.243			

Table 2 The positioning accuracy of TD method

3D grid points cannot be established without physical model, so traditional methods (eg, field survey, map measurement or DEM) should be employed for the obtaining of GCP and CkP.

Under this case, the result depends on the hypsography, number and distribution of GCP. It is a popular positioning method when the rigorous sensor model is unavailable or the accuracy is not demanding.

4. RPC CORRECTION METHOD

Here the coorection method means to apply different mathematical models without changing the RPC model to the 80 parameters to get the updated RPC parameters. If both the GCP calculating the RPC and the auxiliary GCP are available, BILSR is applied for a group of new RPC, otherwise if only the auxiliary GCP available, IDKF is employed.

4.1 BILSR Method

Both the original and new GCP are used in this method in batch process for the updated RPC. All the GCP are used in the equation 10 with different power for each point.

time, there will be flexibility for the calculation if a non-zero Q_k is provided (Hu and Tao,2002).

The process (equation 13) and linearization (equation 14) are realized by adding new GCP using increment based on Kalman Filter to improved the initial RPC accuracy. Traditionally, Kalman Filter is used for the complicated time problems. Kalman Filter is used to space domain is based on its recursion character for the new GCP are obtained in sequence.

- 1) Calculation of initial value and covariance matrix

$$\begin{aligned} I_k^- &= I_{k-1} \\ P_k^- &= P_{K-1} + Q_{k-1} \end{aligned} \quad (14)$$

Where '-' means the value is the previous result of the new one.

- 2) Calculation of increment of Kalman Filter

$$K_k = P_k^- T_k^T (T_k P_k^- T_k^T + R_k)^{-1} \quad (15)$$

- 3) Updating I_k^- by adding new GCP

$$I_k = I_k^- + K_k v_k, v_k = G_k - T_k I_k^- \quad (16)$$

- 4) Calculation of updated I_k covariance

$$P_k = (E - K_k T_k) P_k^- \quad (17)$$

There will be only one process from step (2) to (4) for the RPC updating if all new GCP are considered to a whole group. If the

4.2 IDKF Method

Increment is used in this method for the accuracy improvement when the 80 parameters and the matrix P (in equation 11) are both available. With the new GCP, the RPC accuracy can be updated using this method.

The value of RPC iteration is stable, of which the process and expression are as following (Hu and Tao, 2002)

$$I_{k+1} = I_k + w_k \quad (12)$$

$$G_k = T_k I_k - v_k = T_k I_k + v_k \quad (k=1,2,\dots) \quad (13)$$

Where:

Equation 13 is transformed from equation 7, which denotes the linearity relation between the observations and parameters.

w_k is the noise vector, or white noise with known covariance matrix Q_k ; v_k is the measure error of image points, it is considered to be white noise with the known covariance matrix R_k of new GCP. Vector w_k and v_k are independent. R_k is usually based on experiences and tests in calculation. Test results show that even though RPC changes very little every GCP are divided into several groups, then repeated processes are needed. From the above we can see that the initial value of RPC covariance is very important for it decides the sensibility of new GCP and its covariance.

5. RPC CORRECTION AND ACCURACY ANALYSIS

The image is the same as mentioned above. 50 GCP distributed evenly were selected from the overall 139 surveyed points to calculate the initial RPC, the distribution of which was shown in figure 2. 26 points were selected randomly from the rest surveyed points as GCP and CkP for accuracy analysis, of which the distribution was as figure 4.

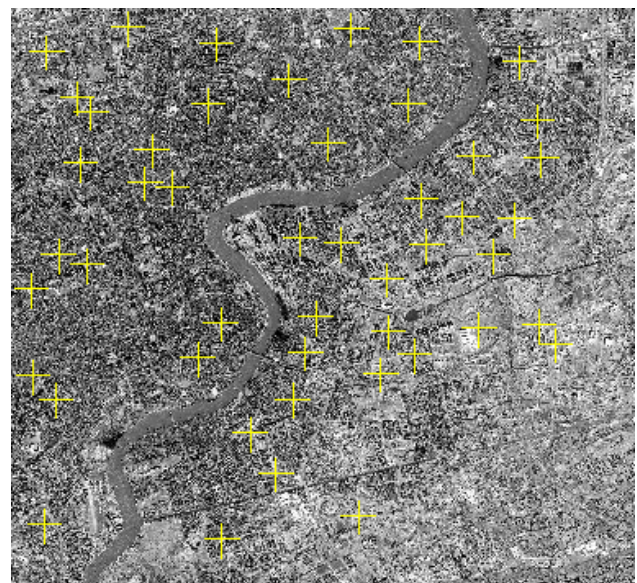


Figure 4 Distribution of 49 Auxiliary GCP

9 of the 49 points are selected auxiliary GCP, the remaining 40 as CKPs. Equation 10 was used for calculation of RPC. 50 GCP were used to get the initial value of RPC, and then BILSR and IDKF were both used to improve the RPC accuracy. The points were added into the equation one by one in IDKF for the use of 9 GCP (Because the auxiliary GCP usually have higher accuracy, they have higher power in calculation). In this test, all the GCP were collected at the same time with the same way, so they all had the same power. So the test here is to determine improvement efficiency of BILSR and IDKF for RPC accuracy after increase of GCP. The covariance matrix is set to $Q = 10^{-6}$ to test the accuracy of updated RPC. Image points of 40 CkP were obtained using updated RPC according to equation (1). The errors of calculated image points and measured ones were used for analysis.

The original row mean square deviation, column mean square deviation and point position error were 1.536, 3.431, 3.760 pixels, respectively. The updated row mean square deviation, column mean square deviation and point position error with BILSR using 9 GCP were 1.467, 3.315, 3.625 pixels, respectively. The updated errors with IDKF using 9 GCP were 1.494, 3.320, 3.641 pixels, respectively. The RPC accuracy was improved within 0.2 pixels after the addition of 9 GCP, for 9 GCP was not notable compared with 50 GCP when it comes to adjustment (figure 5).

The results of BILSR and IDKF are as table 3, where the mean square deviation, max absolute error and point position error of CkP in both row and column directions are given.

Number of New GCP	BILSR: pixel					IDKF: pixel				
	Row		Column		Point Position Error	Row		Column		Point Position Error
	RMS	MAX	RMS	MAX		RMS	MAX	RMS	MAX	
0	1.536	2.743	3.431	11.492	3.760	1.536	2.743	3.431	11.492	3.760
1	1.536	2.748	3.426	11.484	3.754	1.531	2.743	3.426	11.484	3.752
2	1.541	2.759	3.398	11.355	3.731	1.549	2.756	3.396	11.352	3.732
3	1.534	2.756	3.420	11.430	3.749	1.531	2.751	3.422	11.426	3.749
4	1.549	2.830	3.448	11.498	3.780	1.552	2.768	3.437	11.479	3.771
5	1.523	2.816	3.449	11.514	3.770	1.544	2.762	3.457	11.523	3.786
6	1.536	2.832	3.445	11.517	3.781	1.553	2.761	3.447	11.456	3.781
7	1.497	2.708	3.373	11.335	3.691	1.515	2.743	3.375	11.328	3.699
8	1.483	2.691	3.342	11.174	3.657	1.497	2.721	3.345	11.185	3.665
9	1.467	2.674	3.315	11.120	3.625	1.494	2.733	3.320	11.125	3.641

Table 3 Results of BILSR and IDKF by adding GCP by sequence

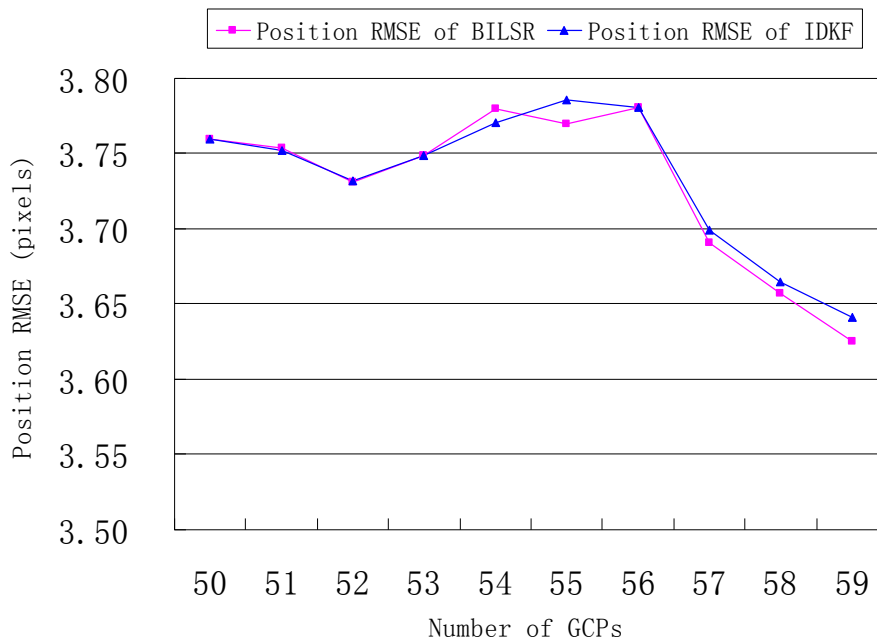


Figure 5 Compare of Point Position Error between BILSR and IDKF

From the test we can see, both the BILSR and IDKF can improve the RPC accuracy with little amount, for RPC is mainly influenced by the distribution of GCP, and it is very hard to improve the accuracy solely based on these two mathematical methods. Table 3 also shows that not all adding GCP could result in accuracy improvement, indicating the uncertainty of these methods. Distribution of auxiliary GCP cannot be tested for the fitting of terrain. Increase of GCP number cannot ensure the accuracy improvement.

6. CONCLUSIONS

Relativity among coefficients when calculating RPC results in the ill-condition and instability of matrix, which can be solved by many approaches. Regularization method is used to ensure the stability of the calculation, during which the selection of regularization parameters is the key to this issue. BILSR and IDKF can improve RPC by mathematical means; they do not modify the model, but resolve the new RPC via calculation. The covariance matrix of parameters must be provided in IDKF. There are little improvement of accuracy within 1 pixel, so both of them are not recommended in calculating of RPC.

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