ROBUST PHASE CORRELATION BASED FEATURE MATCHINIG FOR IMAGE CO-REGISTRATION AND DEM GENERATION

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ABSTRACT:

This paper presents a robust phase correlation based sub-pixel feature matching technique and its application in motion flow estimation, pixel-to-pixel image-co-registration and DEM generation. We propose to use a phase fringe filter and a highly robust technique in the direct Fourier-based phase correlation algorithm for translational shift estimation in sub-pixel accuracy. Noting the problem that local phase correlation based feature matching may fail in areas either featureless or with significant spectral differences between an image pair, a direct frequency based motion estimation assessment technique and a novel motion flow refinement scheme are designed to improve the unreliable local motion estimates around these areas. With the robust phase correlation based local matching algorithm, we are able to derive accurate pixel-to-pixel image co-registration and disparity mapping for DEM generation in most synthetic and real images from different sensor platforms or different spectral bands.

1. INTRODUCTION

Image registration and feature matching is a crucial step in the analysis and fusion of information among similar images. Numerous methods and techniques [Zitova and Flusser, 2003] have been proposed for different applications in remote sensing, medical imaging and computer vision. In recent years, phase correlation feature matching method has been a popular choice in the global or local image registration due to its remarkable accuracy and its robustness to uniform variations of illumination and signal noise in images [Foroosh *et al.*, 2002].

The original phase correlation method [Kuglin, and Hines, 1975] is known to identify integer pixel displacement between image pairs. However, most practical tasks require sub-pixel image registration. Several Fourier domain methods [Stone *et al.*, 2001; Hoge, 2003; Liu and Yan, 2006] and closely related spatial domain variations [Foroosh *et al.*, 2002] have been proposed for estimating the translational shift with sub-pixel accuracy. Stone *et al.* [Stone *et al.*, 2001] investigated the effects of aliasing on the shift estimation and proposed a direct Fourier-based algorithm for sub-pixel image registration, in which, the translational parameter is directly estimated in Fourier domain through a least-squares fitting (LSF) to a 2D phase difference data set. This direct Fourier-based method is much faster than traditional interpolation based techniques [Tian and Huhns, 1986] for sub-pixel registration. Foroosh *et al.* [Foroosh *et al.*, 2002] claimed that Stone's approach is rather inaccurate since it often requires unwrapping the noisy 2D phase difference data and then fitting the unwrapped data. Alternatively, they extended the original phase correlation method [Kuglin, and Hines, 1975] to sub-pixel accuracy through analytic expressions for the phase correlation of down-sampled images. Hoge [Hoge, 2003] demonstrated that the translational shift between two images can be obtained by finding the rank-one approximation of the phase correlation matrix through the singular value decomposition (SVD) method. Then, the sub-pixel estimates of vertical and horizontal shifts can be derived independently from the left and right singular vectors. The accuracy of the translational shift estimation through the SVD based rank-one approximation of phase correlation matrix is much higher than Foroosh *et al.*'s method [Foroosh *et al.*, 2002]. However, the computation complexity of the SVD operation of a large size matrix is very high.

Motivated by the strengths and limitations of these existing phase correlation methods for sub-pixel translational motion estimation and registration, this paper presents a robust phase correlation technique achieving reliable sub-pixel image feature matching and registration at frequency domain. We propose to use a phase fringe filter [Wang *et al.* 2001] and Quick Maximum Density Power Estimator (QMDPE) robust techniques [Wang and Suter, 2004] in the direct Fourier-based phase correlation algorithm. We first apply the phase fringe filter to reduce the noise in the phase correlation difference matrix, and thus make the 2D unwrapping reliable. We then use the highly robust QMDPE technique to obtain the best fitting estimation of 2D unwrapped phase plane. The bench mark tests indicate that robust phase correlation technique has very good performance for sub-pixel feature matching in different window size or across different spectral bands.

Phase correlation method has been applied locally for motion flow estimation, pixel-to-pixel image co-registration and disparity mapping for stereo matching [Fleet, 1994; Balci and Foroosh, 2005]. However, its degraded performance around featureless and low correlation areas is well recognised, which is also the challenge to most of the existing motion estimation methods. Noting this problem, we first propose to use robust statics based inliers scale estimation for a quality assessment on the phase correlation based motion estimation directly in Fourier domain, and a newly proposed median shift propagation (MSP) technique [Liu and Yan 2008] is then exploited to refine the low quality motion estimates around image areas either featureless or subject to significant spectral changes where phase correlation fails. With the robust phase correlation local feature matching and motion flow refinement, we are able to

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achieve pixel-to-pixel image co-registration and disparity estimation for DEM (Digital Elevation Model) generation at sub-pixel accuracy.

2. ROBUST PHASE CORRELATION BASED GLOBAL FEATURE MATCHING

2.1 Basics of Phase Correlation

Phase correlation provides straight-forward estimation of rigid translational motion between two images, which is based on the well-known Fourier shift property: a shift in the spatial domain of two images results in a linear phase difference in the frequency domain of the Fourier Transforms (FT). Given two 2D functions $g(x,y)$ and $h(x,y)$ representing two images related by a simple translational shift a in horizontal and b in vertical directions, and the corresponding Fourier Transforms are denoted $G(u,v)$ and $H(u,v)$. Thus,

$$H(u,v) = G(u,v)\exp\{-i(au+bv)\} \tag{1}$$

The phase correlation is defined as the normalised cross power spectrum between G and H, which is a matrix:

$$Q(u,v) = \frac{G(u,v)H(u,v)^*}{\left|G(u,v)H(u,v)^*\right|} = \exp\{-i(au+bv)\} \tag{2}$$

If $G(u,v)$ and $H(u,v)$ are continuous functions, then the inversed Fourier Transform (IFT) of $Q(u,v)$ is a delta function. The function peak identifies the integer magnitude of the shift between the pair of images [Kuglin, and Hines, 1975]. To achieve the translation estimation at sub-pixel accuracy based on the delta function of the IFT of phase correlation matrix $Q(u,v)$, a common approach is to oversample images $g(x,y)$ and $h(x,y)$ to sub-pixel level before the FT of phase correlation operations. This however will increase the computing load dramatically. Recognised the drawback, many researchers looked for a direct solution in frequency domain based on the phase correlation matrix defined in (2). As the magnitude of $Q(u,v)$ is normalised to 1, the only variable in (2) is the phase difference defined by $au+bv$, where a and b are the horizontal and vertical magnitudes of the image shift between $g(x,y)$ and $h(x,y)$. If we can solve a and b accurately based on the phase correlation matrix $Q(u,v)$, then the non-integer translation estimation at sub-pixel accuracy can be achieved without applying IFT. Such direct frequency domain approaches [Stone *et al.*, 2001; Hoge, 2003] has been proved more accurate and faster than that based on the delta function method.

The phase difference angle $c= au+bv$ in (2) is simply a planar surface through the origin in u-v coordinates defined by coefficients a and b. Thus a complicated problem of complex numbers in frequency domain becomes a simple issue of finding the best 2D fitting of the phase difference angle data in $Q(u,v)$ to a plane of phase difference in the coordinates of u and v. The phase shift angle c is 2π wrapped in the direction defined by a and b. Any 2D fitting technique for c is not possible without a 2D unwrapping. However, 2D unwrapping on the phase angle data in the $Q(u,v)$ is often unreliable and results in failure of finding a and b correctly [Foroosh *et al.*, 2002; Hoge, 2003]. This is largely because of the noisier data of $Q(u,v)$. To improve the 2D fitting method, the key issues are: to reduce the data noise before unwrapping and to refine the fitting technique.

2.2 Robust Method for Sub-pixel Shift Estimation

As the phase angle data in the $Q(u,v)$ is 2π wrapped, ordinary smoothing filters cannot be applied directly to reduce the noise of such discontinuous periodical data. We implemented a phase fringe filtering technique [5] into the 2D fitting method as below:

1. Denote $\theta(u,v)$ as the phase angle at position u,v in the phase correlation matrix $Q(u,v)$.
2. The $\sin\theta$ and $\cos\theta$ are continuous functions of $\theta(u,v)$, a smoothing filter can therefore be applied to these functions.
3. Derive the filtered phase angle $\overline{\theta(u,v)}$ from smoothing filtered $\sin\theta$ and $\cos\theta$: $\tan\overline{\theta} = \frac{\overline{\sin\theta}}{\overline{\cos\theta}}$.

The window size of the smoothing filter used must be small in comparison to the half wavelength of $\sin\theta$ and $\cos\theta$. For reducing the aliasing error and edge effects in the direct Fourier based method, high frequency components of the phase correlation matrix should be masked out, and only the lower frequency part is kept for the 2D fitting operation [Stone *et al.*, 2001; Hoge, 2003].

In stead of using LSF, a highly robust fitting technique QMDPE [Wang and Suter, 2004] is finally applied to find the best fitting estimates of the unwrapped phase angle data, which often is contaminated by the incorrectly unwrapped data and contain multi-structure mode. The benefit of using the QMDPE robust estimator is that the best estimate of the translational shifts a and b in (2) can be obtained from the noisy phase difference data set.

We use a planar surface model through the origin in u-v coordinates, to fit the unwrapped phase angle data set. Let φ_i be the unwrapped phase angle at point (u_i, v_i). The residual of the fitted plane model of the point (u_i, v_i) can be written as

$$x(u_i, v_i) = \varphi_i - (au_i + bv_i)\ . \tag{3}$$

The translational shifts a and b are estimated using the robust estimator QMDPE, which can tolerate more than 80% of outliers. There are two assumptions when using QMDPE. The first one is the inliers have a Gaussian distribution. The second one is the inliers are a relative majority in the multi-structure data. This robust estimator employs the mean shift procedure [Comaniciu and Meer, 2002] to find the local maximum density power $\psi_{\max}$ [Wang and Suter, 2004], i.e., $\max_J \psi_J$, where J is the index of sub-samples. The density power is defined as:

$$\psi_{DP} = \frac{\hat{f}(X_c)}{\exp(X_c)} \tag{4}$$

where X_c is the position of the local maximum density $f(X_c)$ in the signed residual space. The probability density can be estimated by:

$$\hat{f}(x) = \frac{1}{nh}\sum_{i=1}^{n} K\left(\frac{x - X_i}{h}\right) \tag{5}$$

where n is the number of the data points X_i, h is the band-width. 1-D Epanechnikov kernel [6] is employed:

$$K_e(x) = \begin{cases} \frac{3}{4}(1-x^2) & \text{if } x^2 < 1 \\ 0 & \text{otherwise} \end{cases} \tag{6}$$

A variable bandwidth method is used. The optimal bandwidth $\hat{h}$ can be chosen as:

$$0 < \hat{h} < \left[\frac{243R(K)}{35U_2(K)^2 n} \right]^{1/5} s \tag{7}$$

where $R(K) = \int_{-1}^{1} K^2(\zeta) d\zeta$ and $u_2(K) = \int_{-1}^{1} \zeta^2 K(\zeta) d\zeta$, and $s = 1.4826\ med_i \left| x_i \right|$. So, the bandwidth is variable with the different sub-sample, which is randomly chosen.

Given a data set of n observations, the algorithm repeatedly draw *m* sub-samples (each with *p* different observations, and in a planar surface model *p*=3). For each sub-sample, indexed by *J*, $1 \le J \le m$, the translational shifts *a* and *b* are computed. Therefore, the signed residuals of all other observations are calculated, and the local maximum density in the signed residual space is determined through a mean shift procedure. The corresponding density power ψ_J defined in (4) is also computed. The maximum of ψ_J is found and its corresponding translational motion parameters are the best estimates of the robust method.

In order to improve the statistical efficiency, a weighted least-squares procedure [Bab-Hadiashar and Suter, 1998] is used to obtain more accurate estimation of the translational shifts. A robust standard deviation σ is computed as

$$\sigma = 1.4826(1 + \frac{5}{n-p})\sqrt{\text{med } x(u,v)^2}\ . \tag{9}$$

σ is used to determine a weight $w(u,v)$ for each observation, where

$$w(u,v) = \begin{cases} 1 & \text{if} \left| x(u,v)/\sigma \right| < T \\ 0 & \text{if} \left| x(u,v)/\sigma \right| \ge T \end{cases} . \tag{10}$$

T is a threshold of statistical inliers of the robust fitting estimation. The set of points with weight 1 is identified as inliers, while the set of points with weight 0 represents the outliers. The weighted least-squares method is then used to obtain the translational motion parameters like [Barron *et al.*, 1994] by minimising

$$r(u,v) = \sum_R w_i(\varphi_i - (au_i + bv_i))^2\ , \tag{11}$$

where $w_i = w(u_i, v_i)$ is a weight at the point (u_i, v_i), and R denotes the whole set of the unwrapped phase angle data.

2.3 Rotation and Scale Change

As described in [Reddy and Chatterji, 1996], if the frequency domain is presented in polar co-ordinates, then the rotation will be a shift on the axis corresponding to the angle. Therefore, a rotation angle can be derived by phase correlation based shift estimation in polar co-ordinates. Similarly, a change in scale can be determined based on a phase shift in the frequency domain presented in logarithmic coordinate units.

It should be pointed out that the proposed robust 2D fitting phase correlation technique has some limitation in the estimation of large shift, rotation and scale change duo to the fringe filtering operation used in the technique. Generally speaking, the robust phase correlation technique is able to achieve sub-pixel frame registration with rotation angle less 10 degree.

2.4 Bench Mark Test

An example in Figure 1 shows the effectiveness of the robust phase correlation based translational shift estimation for sub-pixel image registration across different spectral bands. A image pair (512×512) of two different spectral bands, bands 1 (blue) and 5 (short wave infrared), extracted from a 30 m resolution Landsat-7 ETM+ scene is shown in Figure 1(a) and (b) respectively. The correlation between the two bands is 0.69. One of the images is artificially shifted horizontally by 13.33 pixels to the right and vertically by 10.00 pixels up in relation to the other. As shown in Figure 1(c), the phase correlation matrix data become quite noisy because of the low correlation between the two images. Both of the Least-Square Fitting (LSF) and QMDPE algorithms failed in the first attempt without filtering the noise phase angle data. Then the phase fringe filter with filter size 5×5 pixels was applied, which has improved the phase correlation data significantly as illustrated in Figure 1(d). Figure 1(e) is the 3D view of the central part of the filtered phase difference matrix. Although some errors exist in the corresponding unwrapped phase angle data shown in Figure 1(f), QMDPE robust fitting method obtained very good shift estimates. However, LSF cannot find the good fitting estimates in this case. The experimental results are shown in Table1, which indicate that the QMDPE 2D fitting algorithm is able to achieve the sub-pixel image registration across different spectral bands.

Another example is for testing the accuracy of the robust phase correlation based feature matching in different window size. We artificially shift the image shown in Figure 1(a) by 0.25 pixels (left) in horizontal and 0.75 pixels (up) in vertical directions to generate the other one for the test. The robust phase correlation based technique is applied to central part of the image pair with different window size (from 512 to 16). The results of the translational shift estimation with different window size are shown in Table 2. These results indicate that the robust phase correlation technique is reliable for sub-pixel feature matching in different window size, even in a window size as small as 32×32. The best accuracy of feature matching is (*x*: 0.00051, *y*: 0.00015) pixel at window size 512×512, and the worst one is (*x*: 0.06251, *y*: 0.20104) pixel at window size 16×16. Generally speaking, the accuracy of phase correlation based feature matching will drop if it is applied locally within a small window compared with that in global feature matching.

True Shift	LSF	QMDPE
x: 13.3333	13.2466	13.2420
y: -10.00	- 9.3308	- 9.9346

Table 1. Translational shift estimates with fringe filter size 5×5.

Window Size	*x*	*y*
512×512	-0.24949	-0.74985
256×256	-0.25202	-0.74722
128×128	-0.25235	-0.74636
64×64	-0.25473	-0.73862
32×32	-0.26009	-0.67830
16×16	-0.31251	-0.54896
True Shift	-0.25	-0.75

Table 2. Translational shift estimates through robust QMDPE based technique with different window size.

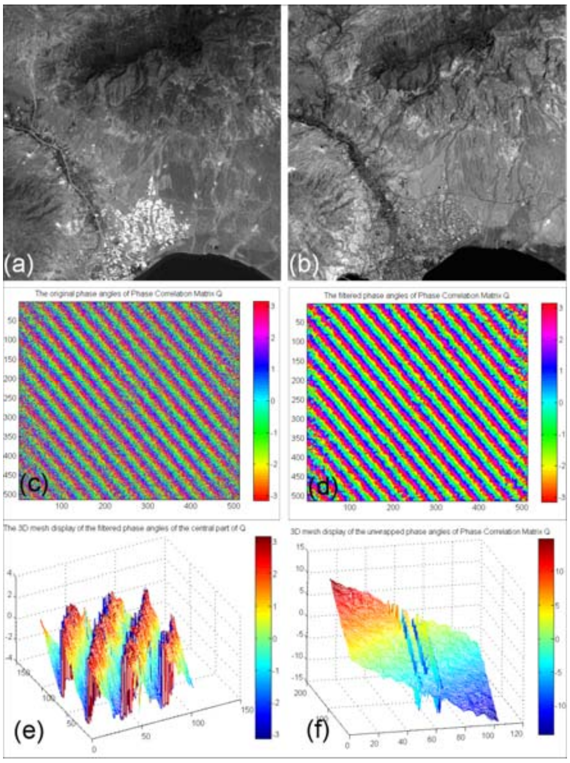

Figure 1: (a) Landsat-7 ETM+ bands 1 image, and (b) Landsat-7 ETM+ bands 5 image. (c) The phase difference matrix. (d) The filtered phase difference matrix (filter size=5). (e) 3D view of the central part of the wrapped phase difference matrix. (f) 3D view of the central part of the unwrapped phase difference matrix.

3. ROBUST PHASED CORRELATION BASED LOCAL FEATURE MATCHING

Phase correlation feature matching techniques are often applied locally for motion flow estimation in a similar image pair. Accurate motion flow estimation is most important step for high quality of pixel-to-to-pixel image co-registration and DEM generation. If the raster scan is applied on a rectified stereo image pair, in which the local motion is mainly along image scan lines, a motion flow field or disparity map of stereo matching can then be derived. Here, the scanning window size is crucial for the quality of phase correlation based local feature matching. If the window size is too small, then the number of data points will be insufficient to achieve accurate measurement of the extracted feature shift. If the scanning window size is too large it may include multiple motions, especially around depth discontinuity areas for stereo matching. Our tests indicate that the scanning window size with 32×32 generally achieves best performance in the small motion cases (less than 6 pixels), while the window size with 64×64 is best tradeoff in most large motion cases.

3.1 Phase Correlation Pixel-to-Pixel Image Co-registration

Obviously, if we know the relative shift of every pixel between two similar images, we can thus co-register the two images pixel by pixel based on the computed motion flow field. This pixel-to-pixel image co-registration that is fundamentally different from those "rubber warping" image co-registration techniques. To the best, the warping co-registration techniques assure the accuracy only at the grid of the image deformation model. In addition, the registration is not achieved at pixel-to-pixel level. The registration error within a scene can vary from place to place depending on the relative deformation between the images. While the robust phase correlation based pixel-to-pixel co-registration can achieve sub-pixel registration accuracy at every image point. Figure 2 illustrates the basic scheme of robust phase correlation based pixel-to-pixel image co-registration method. It should be noted that the *Input* image should first be roughly oriented to the *Reference* image by standard routines of image shift, rotation and scale change before the robust phase correlation based motion flow estimation and image co-registration. The image rotation and minor scale change may introduce geometric errors as the results of interpolation and re-sampling. However, this type of errors will be largely eliminated by the later steps of the pixel-to-pixel image co-registration based on the precise measurements of the shift between every corresponding pixel. In the last step, a bilinear interpolation backwards reconstruction technique is used for pixel-to-pixel co-registration duo to its good performance in computing efficiency and accuracy.

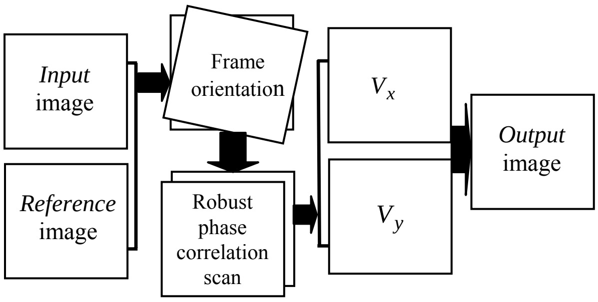

Figure 2: Basic scheme of robust phase correlation based pixel-to-pixel image co-registration.

3.2 Quality Assessment of Robust Motion Estimation and Refinement in Featureless and Low Correlation Areas

Local phase correlation based motion estimation may fail in featureless areas or in areas with significant spectral differences between the image pair duo to very low correlation. Here, we propose to use a ratio of outliers to inliers derived from the robust QMDPE 2D fitting in frequency domain for quality assessment on the phase correlation based local motion estimates. This direct frequency domain based phase correlation quality assessment method is more efficient than our previous NCC based method for quality assessment on motion estimates and its corresponding image co-registration in image domain [Liu and Yan, 2008]. The inliers and outliers of the robust fitting estimation can be derived from Equation (9) and (10). In each window, the robust fitting technique QMDPE is applied to find the best fitting estimates of the unwrapped phase angle data. If the ratio of the outliers to the inliers of the best fitting estimation of the plane model exceeds a certain threshold, the corresponding motion estimate is supposed to be unreliable.

It is very common to mask off the unreliable estimates in motion flow estimation techniques (Barron *et al.*, 1994). However, for image co-registration, the task is to build an *Output* image from the *Input* image data based on the *Reference* image geometry. There should not be any gaps left in the co-registered *Output* image. In order to find the right sample positions in the *Input* image so as to generate a correctly co-registered *Output* image, we must fill the gaps in the estimated motion flow field with some reasonable shift data.

A simple and effective solution, the median shift propagation (MSP) filter, was proposed for improving the quality of motion flow estimation around featureless or low correlation areas in our previous work [Liu and Yan, 2008]. The main idea of MSP is to fill the small gaps of the computed motion flow field with the median of the velocities of the pixels with reliable motion estimation in a scanning window. The key difference between the MSP and the ordinary median filtering is that instead of always applying the median filter to the original flow field, the filter is applied to the velocity data that are modified by the last filtering action. In such a way, the motion flow field are updated continuously during the filtering process, and the feature shifts are thus self-propagate from high quality data to fill the velocity gaps represented as unreliable motion estimates. It is worth mentioning that the filled data tend to become constant if the gap is large or the propagation distance is long, but these are much better estimations than the random variations of low correlation motion flow data.

4. EXPERIMENTAL RESULTS

Based on the robust phase correlation techniques presented in this paper, we have developed a standalone C++ software package PCIAS with a friendly graphic user interface (GUI) and powerful image fusion and analysis functions such as global image registration, motion flow estimation, pixel-to-pixel image registration, and disparity mapping for DEM generation with sub-pixel accuracy. A series of images from different sensor platforms or with different spectral bands have been exploited to examine the accuracy and robustness of the proposed phase correlation based techniques. An example of the experimental results is presented here.

Figure 3 (a) and 3(b) are the small sub-scenes (700×700) of a large stereo pair of SPOT-5 PAN images (12014×12019) with 5 m resolution, which is taken on different dates (9 September 2003 and 26 June 2004) from adjacent orbital path with view angles of 1.978° and 1.572° respectively from nadir tilting in opposite directions. This stereo pair has been chosen for the tests of pixel-to-pixel image co-registration and disparity map estimation for DEM generation.

Through the robust phase correlation global image registration, the *Input* image (Figure 3 (b)) was roughly registered to the *Reference* one (Figure 3(a)) with frame rotation 0.6529 ° (clockwise), and frame shift with 6.1748 (to right) and 0.8025 (down) pixels. The robust phase correlation scan is then carried out to generate the motion flow field (disparity map) after image orientation operations. The initial result of motion flow estimation without the MSP refinement is shown in Figure 3(c) with some small messy areas in the scene, in which the robust phase correlation method failed to get good fitting estimates. Figure 3(d) shows the low quality motion flow data in Figure 3(c) are masked off with the proposed robust inliers scale estimation method, and then refilled with the MSP. The refined result shown in Figure 3(e) presents a smooth motion flow field, and the corresponding disparity map is shown in Figure 3(f). The RGB colour combination of the *Input* and the co-registered *Output* image is shown in Figure 3(h). For comparison, the RGB colour combination of the *Input* and *Reference* images before pixel-to-pixel co-registration is shown in Figure 3(g). The crystal sharpness of Figure 3(h) indicates very high quality of co-registration in every part of the image while the colour patches reveal the differences between the two images, which prove up the capacity of the method for change detection. Finally, a 3D perspective view of the *Reference* image reconstructed from the estimated disparity map (DEM) is shown in Figure 3(i). It demonstrates that fine details and depth discontinuity can be quite effectively recovered with the robust phase correlation based techniques.

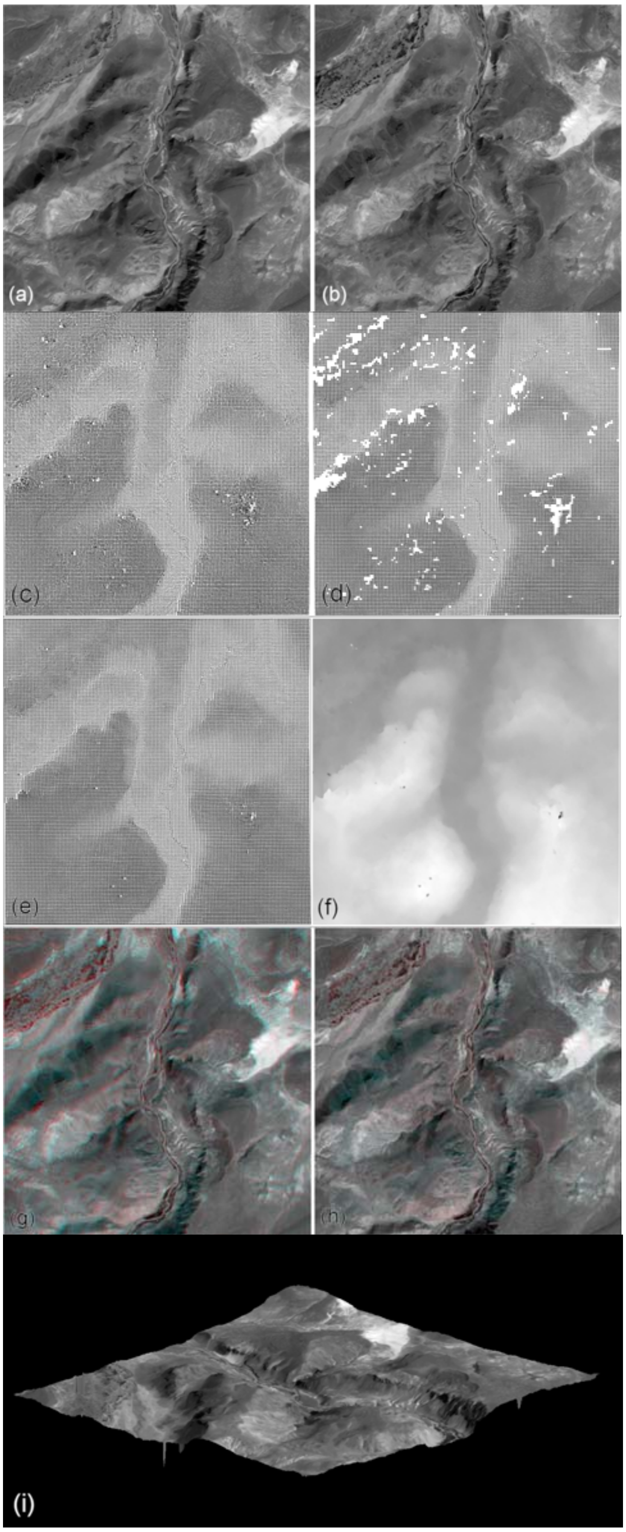

Figure 3: (a) *Reference*: SPOT-5 HRG-2 PAN image taken on 9 September 2003. (b) *Input*: SPOT-5 HRG-1 PAN image taken

on 26 June 2004. (c) Initial motion flow. (d) The low quality motion estimates have been masked off. (e) Refined motion flow field via the MSP. (f) The disparity map corresponding to (e). (g) RGB colour combination of the *Input* and *Reference* images before pixel-to-pixel co-registration. (h) RGB colour combination of the *Input* and the pixel-to-pixel co-registered *Output* image. (i) DEM from the estimated disparity map.

Using the developed PCIAS software in windows XP system, the disparity map estimation and pixel-to-pixel co-registration from this 700×700 image pair with 32×32 scanning window was accomplished around 17 minutes on a Dell desktop with 2.38 MHz Core 2 processor and 4 GB RAM.

5. CONCLUSIONS

This paper presented a robust phase correlation based sub-pixel feature matching technique and its application in motion flow estimation, pixel-to-pixel image co-registration, and DEM generation. In particular, a median shift propagation (MSP) technique has been introduced to refine the unreliable motion estimation in image areas either featureless or subject to significant spectral changes.

Our experimental results have demonstrated that the robust phase correlation based technique is able to achieve sub-pixel accuracy and good performance in global and local feature matching, motion flow estimation and disparity mapping for DEM generation on most synthetic and real images from different sensor platforms or different spectral bands. The strengths of the proposed techniques are its algorithm simplicity, its robustness to illumination change and its good performance in featureless or low correlation areas.

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