AN NOVEL THEORY TO SEGMENT ISO-SURFACE IN MARINE GIS 3D DATA

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ABSTRACT:

Segmentation of 3D data (some time 4D data) is a very challenging problem in applications exploiting Marine GIS data. To tackle this problem, this paper proposes a topological approach based on the Digital Morse theory which is a kind of Discrete Morse theory to high dimension Grid points. The essence of the approach concerns detecting critical points in the High dimension Data, which represent parts of the topology changing. Because less or more some prior information could be got, our approach is quite robust against noise. Experimental results demonstrate the validity of our method

1. INTRODUCTIO

In the marine GIS, there are lot three dimensions Volume Data from ocean surveys of water temperature, salinity, and contami nants. The space distributing features of these data contains the key environment information of the sea from which they came.

One of the most important ways to analysis the space distributing features of 3D Data is to compute iso-surface of these data, and then visualize them. There are many method to compute or segment iso-surface in 3D date, the most popular one is matching cube, but in many cases it will commit errors, this is related to a typical problem in mathematics involves attempting to understand the topology, or large-scale structure, of an object with limited information. This kind of problem also occurs in mathematical physics, dynamic systems and mechanical engineering. Morse theory is a generalization of calculus of variations, which draws the relationship between the stationary points of a smooth real-valued function on a manifold and the global topology of the manifold. Morse theory consists of two parts: one is the critical point theory and another is the application in calculus of variations.

Dr. J.L.Cox and Dr. D.B.Karron from City University of New York developed a Digital Morse Theory, which expands the fundamental insight of Morse theory to the critical point and criticality graph theory in discrete set. With this powerful theory we can easily recognize iso-surface and analysis the geometry and topology of high dimensional data set.

Here I gave a comprehensive introduction to Digital Morse theory in this report and showed a few simple successful applications in Marine 3D Data iso-surface segmentation. This report is intended, as far as possible, to give an exact insight into digital Morse theory to the readers, who are interested in the theory. Fore I believe it is a powerful tool to analysis high dimension temporal –space data in complicated GIS system such as Marine GIS

2. MORSE THOERY

Traditional Morse Theory begins with this insight: Let f be a C_2 continuous function defined on a compact, smoothly differentiable manifold M . A Morse function has the following properties: Each critical point of f is an isolated point, and at each critical point the Hessian (matrix of second order partials) is nonsingular. In other words each criticality is a single isolated point and is a true local maximum, minimum or saddle point (there are no points of inflection). Then the topology changes of the level sets of f occur only at the critical values and are completely characterized by the number of negative Eigen values of the Hessian at each critical point, which determines the number of linearly independent down directions, and thus whether it is a maximum, a minimum, or determines the type of saddle.

Morse theory can be thought of as a generalization of the classical theory of critical points (maxima, minima and saddle points) of smooth functions on Euclidean spaces. Morse theory states that for a generic function defined on a closed compact manifold (e.g. a closed surface)) the nature of its critical points determines the topology of the manifold. Morse functions are generic functions for which all the critical points are nondegenerate (the Hessian matrix of the function at the critical point is non singular).

For a Morse function, the critical points determine the homology groups of the manifold, that is a sets of points for which the function is less than a given value x . Moreover these sets can fully describe the topology of the manifold. The way the manifold is embedded in the 3D space can be coded

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using the Reeb graph which is a skeleton graph that encodes the evolution and the arrangement of the homology groups.
Reeb graph represents the configuration of critical points and their relationship and provides a way to understand the intrinsic topological structure of a shape. Consequently, the Reeb graph has been used in many applications such as shape matching, shape coding and surface description and compression.

3. INTRODUTION OF DIGITAL MORSE THEORY

Digital Morse theory applies this insight to discrete setting, where it cannot be assumed that the function is Morse in the aforementioned sense. This is because the data may contain clusters of identically valued readings, and thus, in general, not all critical points can be assumed isolated. Since in discrete setting data readings (function δ) can be extended to a continuous function f in unaccountably many ways, we have to make some base assumptions, and these assumptions are consistent with the majority of the current practice and literature on so-called volumetric or density data.

3.1 Relaxing Morse conditions with combinatorial insight

Definition 1: For simplicity, we assume that our data readings have been mapped onto the n-dimensional integer lattice in Euclidean space, that is, are given by a real-valued function δ defined on Z^n. We form unit hyper cubes in the natural way, i.e., a line segment (edge) connects points $p, q \in Z^n$ iff their Euclidean distance is 1. Thus the data readings are given at the hypercube vertices (or lattice points). We shall assume, without loss of generality, that δ is non-negative. We further assume that $\delta > 0$ on a finite subset of Z^n.

Definition 2: We will say that a continuous real-valued function f interpolates δ if the domain of f is Euclidean n-space, E^n, f is non-zero on a closed and bounded subset of E^n, and for all $p \in Z^n$, $f(p) = \delta(p)$.

Definition 3: We denote by $X_f^{\geq\tau}$, the set of $p \in E^n$ such that $f(p) \geq \tau$, for f that interpolates δ. Similarly, $X_\delta^{\geq\tau}$ denotes the set $p \in Z^n$ such that $\delta(p) \geq \tau$. Clearly, if f interpolates δ then $X_\delta^{\geq\tau} \subseteq X_f^{\geq\tau}$.

Definition 4: We denote the topological boundary of a set X by $B(X)$.

A first assumption made by most researchers' interpolation methods is that the underlying function is continuous. We shall reduce questions of the topology of the boundary components of $X_f^{\geq\tau}$ to basic combinatorial questions. Much will be made of connected sets of data readings in Z^n. The reason for this is simple. Each component O of $X_f^{\geq\tau}$ contains a subset of $X_\delta^{\geq\tau}$.

We will regard this set as connected, that is, a set of data readings is connected if and only if the set is contained in the same level set component. As we shall argue, a reasonable set of assumptions on the interpolation method will allow us to reverse this implication. In other words, if we determine the connectivity of $X_\delta^{\geq\tau}$, this will determine the topology of the components of $X_f^{\geq\tau}$.

This is the basic idea behind the digital topology program: that objects are defined by discrete data reading connectivity. While we could use a discrete topology on $X_\delta^{\geq\tau}$ to obtain our results, for generality we prefer to examine the standard point set topology of $X_f^{\geq\tau}$, for a reasonable class of interpolation functions f.

We will restrict the class of interpolation functions f by axioms on the structure of $B(X_f^{\geq\tau})$, for each τ. Since, in point of fact, most algorithms in the literature are for interpolating the components of $B(X_f^{\geq\tau})$ for specific τ, without actually specifying f on all of E^n, this makes sense. For ease of exposition, we define with respect to each real number τ.

Definition 5: A point $p \in Z^n$ is High if $p \in X_\delta^{\geq\tau}$, that is, if $\delta(p) \geq \tau$, and is Low if $\delta(p) \leq \tau$.

We shall assume that for each τ not in the (finite) range of δ (τ not equal to a data reading), $X_f^{\geq\tau}$ consists of a finite collection $O_i(\tau)$, $i = 1, \ldots, k$, of path connected, full-dimensional components, and that the boundary $B(O_i(\tau))$, of each component consists of a finite set of closed, bounded, and oriented manifolds. This is consistent with the literature and goals of the imaging community.

Most methods, in the absence of further information, construct $X_f^{\geq\tau}$ with the simplest topology consistent with the data, that is, they don't introduce extraneous holes and handles in the objects, in the sense that every component of the level sets or their complement contains at least 1 data reading (integral point).

We will assume that the manifolds of $B(X_f^{>\tau})$ intersect our hyper cubes in simple ways, so that any intersection with a hypercube edge is at a point, with a hypercube face is a one-dimensional set and, in general, the intersection of $B(X_f^{>\tau})$ with a d-dimensional hypercube is a d-1-dimensional set (see axiom 2 below). This is a reasonable non-degeneracy assumption, as it merely means that at an iso-value τ not precisely equal to a data reading, the plateau regions of $B(X_f^{>\tau})$ never overlap a cube face.

Most methods interpolate a single boundary surface crossing point on a hypercube edge if and only if the endpoint readings are High and Low. Even if one employs a method, for example, that creates a level set boundary that snakes back and forth across the hypercube edge connecting two Highs, the two Highs will most surely be part of the same component of $X_f^{\geq\tau}$, and thus no topological generality is lost by assuming that the entire hypercube edge is contained in $X_f^{\geq\tau}$.

Since edge adjacent Highs are assumed part of the same component, one may be tempted to define connectivity of $X_\delta^{\geq\tau}$ by the transitive closure of the hypercube edge adjacency. Unfortunately, as observed in the seminal work in digital topology, edge adjacency alone leads to asymmetry in the sense that the complementary components will not be edge-connected (in 3-dimensions they will be 14-connected rather than 6-connected).

We will explain below why we feel that 6-connectivity for Highs is not the right choice. However, for any (local) connectivity one chooses, we can develop the same results: algorithms for identifying criticalities and constructing a criticality graph. This is because our axioms completely specify the topology of the boundary manifolds of the level sets, subject to the connectivity rule that one chooses.

3.2 edge-connectivity is insufficient

Definition 6 We call the interpolated boundary intersection points of $B(X_f^{>\tau})$ with cube edges, ``hit points''.

Definition 7 When a hypercube face contains diagonally opposite Highs and diagonally opposite Lows we call it a 4-hit face, since by the above assumptions there is an intersection point with $B(X_f^{>\tau})$ on each edge (see figure 4).

Definition 8 Choosing the diagonally opposed Highs in a 4-hit face as adjacent means that we will regard then as path connected through the cube face F, that is part of the same component of $F \cap X_f^{\geq\tau}$. In this case a pair of hits on edges that share a common Low vertex will be connected by a boundary curve within the face. Thus we call this choice ``Knit Low''. Similarly, the choice that makes the Highs nonadjacent (and thus the Lows adjacent) is termed ``Knit High''.

These are the only two choices for face F, since if there is a path $\pi \subset X_f^{\geq\tau} \cap F$ between the two Highs then there cannot be a path between the two Lows in $X_f^{\leq\tau} \cap F$, as it would have to cross π (and conversely). In two dimensions these will be our only two choices. As we shall see we will make a similar choice for 3 and higher dimensions. In this case we will regard them as adjacent if we determine that they are path connected through the interior of a cube sharing face F, however, as we shall see when we discuss critical 4-hit faces, no topological generality will be lost if we assume the path is through F

Proposition 3.1 If f interpolates δ then $X_{-f}^{\geq-\tau} = X_f^{\leq\tau}$.

3.3 correct choice for diagonal adjacency

So we must choose an adjacency rule that makes $X_{-f}^{>-\tau} = X_f^{<\tau}$.

Now the sets $X_f^{\geq\tau}$ obviously satisfy monotonicity as τ is decreased, in the sense that once a point becomes a member of the set it remains a member. This is obviously true as $f(p) \geq c$ implies $f(p) > \tau$ for $\tau < c$. This implies that if two Highs of a 4-hit face F are adjacent for a given threshold c, then they must then be adjacent for all values $\tau < c$. For 3 dimensions and higher we have to decide the maximum iso-value c for which there is a path through the interior of a hypercube sharing F, between the two Highs. The different methods one can use to interpolate c effect the values of certain types of criticalities but will not change the essential character of our results. The disambiguation value will only be important if there is no path between the two vertices that passes through any other High vertex within the hyper cubes that share the face, at the disambiguation value.

3.3.1 Interpolating the disambiguation value:

Definition 9 Disambiguation Rule: In 2 dimensions we bilinearly interpolate the disambiguation value as follows: For each 4-hit face we linearly interpolate δ across each edge. Now interpolate the position of the point p, called the disambiguation point, such that both the vertical and horizontal lines (with respect to the two coordinate directions on the face) that pass through this point intersect identical values on the opposite edges. Interpolate the value C of this point, called the disambiguation value, and we extend the domain of δ to include p (we call this the extended δ).

In 3 dimensions we use trilinear interpolation to determine the maximum value c at which the diagonally opposite Highs are path connected through the interior of either cube that shares the face F. In higher dimensions we similarly use multi-linear interpolation to choose the maximum value c for which the Highs are connected through the interior of any hypercube sharing F. As in 2 dimensions, we interpolate an interior point p with value c. The interior point p so interpolated is called the disambiguation point and is associated with the face F. For specific τ, if $\tau > c$ with respect to face F, regard the Lows as adjacent (knit High), else regard the Highs as adjacent (knit Low), with respect to F.

3.3.2 Critical 4-hit faces defined.

Definition 10 We define a 4-hit face as critical, if for all $T > c$, the disambiguation value, the diagonally opposite Highs are not path connected within any n-dimensional hypercube that shares the 4-hit face.

3.4 The Disambiguates Marching Cube and it's solution

Marching cube theory assumes a discrete 3D image that maps a value $V(x, y, z) \in R$ to each grid point $(x, y, z) \in Z^3$. The image V can also be considered as a density function on a subset of Z^3. The Marching-Cubes (MC for short) algorithm was first introduced by Lorensen and Cline to extract a triangulated surface from V corresponding to an iso-density value. The first application of this work was the visualization of iso-density surfaces in medical imaging. We first consider cubic cells of coordinate (x, y, z) whose vertices are placed on the 8 input samples $(x+i, y+j, z+k)$ of the volume data, with i, j, $k \in \{0,1\}$; The triangulated iso-surface given by the Marching-Cubes algorithm is locally computed according to the way of the surface intersects each cell of V using a look-up table with 14 possible configurations (see figure 1). The coordinates of the MC vertices along an edge of a cell is given by an interpolation process between the values of V and the chosen iso-level.

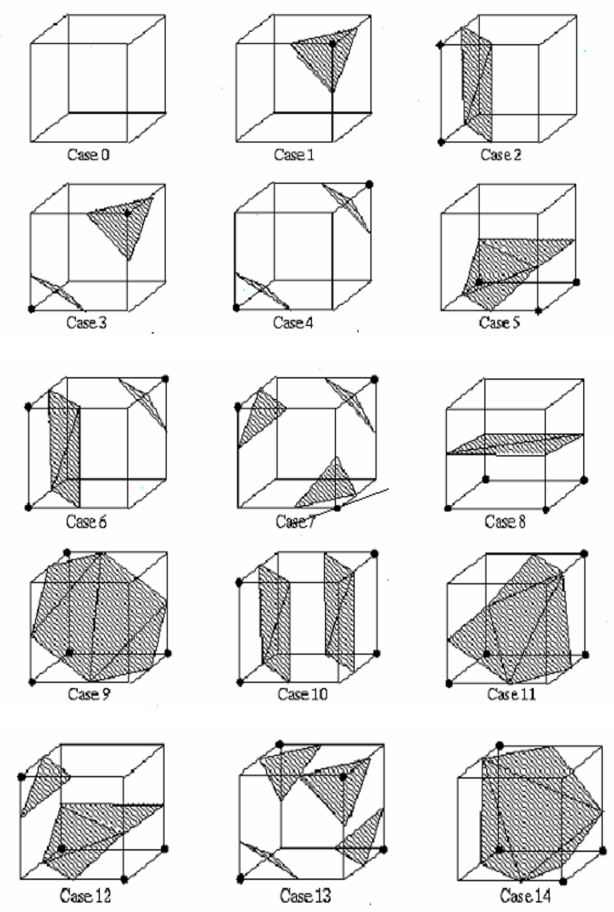

Figure 1

Note that some of original Lorensen and Cline's configurations may lead to ambiguities in the reconstruction and thus construct surfaces with holes. As showed in Figure 2, the when connecting left cube with right cube, there is a hole in construct surface. For the right face of the right cube (the left face of the right cube) is ambiguous in classic Marching Cube.

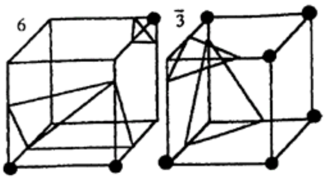

Figure 2

With the method of correct choice for diagonal adjacency in Digital Morse theory, we can solve this problem. As shown in Figure 3, for each 4-hit face we linearly interpolate δ across each edge and interpolate the position of the point p, p is called the disambiguation point and is associated with the face F. For specific τ, if $\tau > c$ with respect to face F, regard the Lows as adjacent (knit High), else regard the Highs as adjacent (knit Low), with respect to F.

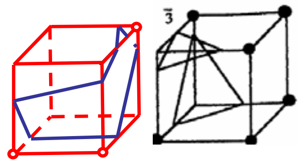

Figure 3

4. EXPERIMENTS AND CONCLUSION

Digital Morse Theory provides a novel way to manipulate images in terms of n-dimensional criticality defined objects and assemblages of objects, instead of individual pixels in an image plane. Our preliminary experience with using DMT has enabled the rapid segmentation of salinity and temperature distribution of sea, as depicted in figure 4 and figure 5, the result was accepted by marine scientists.

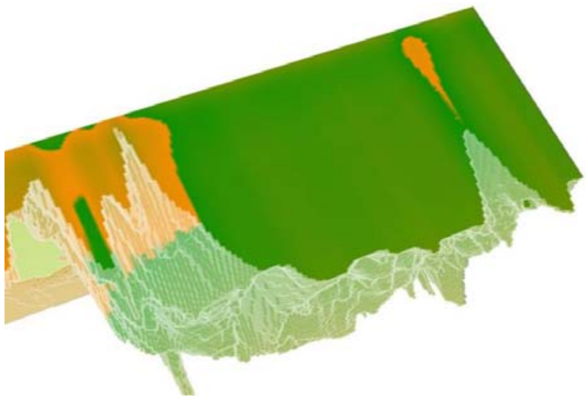

Figure 4 salinity

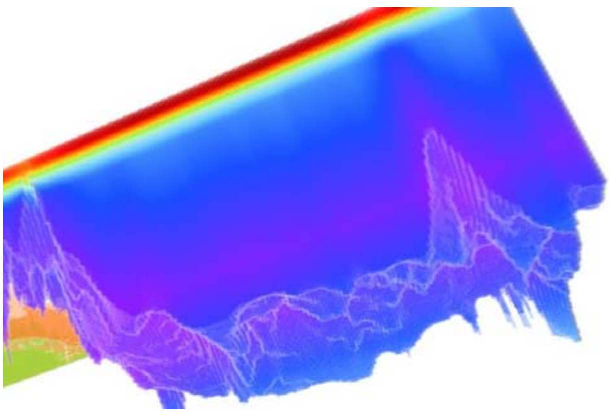

Figure 5 temperature

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