

THE ECOLOGICAL AND ECONOMIC METHODS OF ANALYSIS OF THE URBAN TRANSPORT SYSTEM

Baigabulova Zh.^a, Suleimenova K.^b, Bekmagambetova J.^c, Suleimenov I.^d

^aInstitute of Geography, Pushkin str. 99, Almaty, 050000 Kazakhstan – zhannaecokz@yahoo.com

^bState University of Nice, Sophia-Antipolis, France

^cKazakh Academy of transport and communication, Schevchenko 97, 050012 Almaty, Kazakhstan - j.bekmagambetova@kazatk.kz>

^dKazakh National University named al-Farabi, Almaty, Kazakhstan – esenych@yandex.ru

KEY WORDS: Environmental Monitoring, Geographical Information Science Optimization, Transport Planning, Urban Transport

ABSTRACT:

In this work it is shown, that for the macroscopic description of transport streams within the city transport network of a megapolis it is possible to develop the theoretical device - the analogue of thermodynamics. The basis of this method is the concept of the transport temperature accepted in this work and it is an analogue of the appropriate thermodynamic laws. «The thermodynamics of a megapolis» allows to establish a direct connection between ecological and economic parameters.

KURZFASSUNG:

In dieser Arbeit wird gezeigt, dass es für die makroskopische Beschreibung von Verkehrsflüssen im Stadtverkehrsnetz einer Großstadt möglich ist, ein theoretisches Apparat als eine Analogie der Thermodynamik zu entwickeln. Als Grundlage dieser Methode dient derin der Arbeit angewandete Begriff der „Transporttemperatur“, die eine Analogie der entsprechenden Thermodynamischen Einheit ist. „The thermodynamics of a megapolis“ lässt eine direkte Verbindung zwischen ökologischen und ökonomischen Parametern festzustellen.

1. INTRODUCTION

There is a very difficult and critical problem of environmental pollution by exhaust fumes of transport in the largest world cities. In particular this problem is typical for countries with a developing economy, where funds allocated to solution of ecological problems are extremely limited.

In this work we showed the necessity of a complex approach, which takes into account ecological factors as well as transport and economical ones in solution of air pollution problems in such cities like Moscow, St-Petersburg, Almaty. There also exist objective criterions for developing economic measures for regulating transport streams (in particular calculation of transport taxes), which will permit to determine concrete numerical description of financial instruments and ensure regulation of transport roads capacity. To prove the method's efficiency here is considered the question of preventing the occurrence of traffic congestions, using a set of measures; these measures optimize traffic regulation and financial factors at the same time.

The basis for developing approach is a new scientific trend – city's physics (Suleimenov I., Baigabulova Zh., Suleimenova K., Bekmagambetova J, Dyachenko E. Fundamental of city's physics. (in Russian) Almaty. 2007).

For its consecutive development we have to find out the Maxwell's distribution analogue which describes automobiles allocation by speed within the city transport network.

2. THE ANALOGUES OF MAXWELL'S AND BOLTZMAN'S DISTRIBUTION

Taking the easiest case the task of the statistical description of an intricate system can be set as follows. There is a macroscopic set of particles; each of them can be in one of the conditions adequating to discrete levels of energy. For example, it can be atoms or molecules with high temperature when the appreciable part of electrons is being transformed into the excited state (fig. 1). It is required to determine, what probability is that the separate particle is in a condition with the definite energy \mathcal{E} . At such target setting energy can be considered as a variable one, therefore, having specified the function $p(\mathcal{E})$, we will completely describe the system considered. In static physics there is known the solution of such problem. Viz., the function $p(\mathcal{E})$ is described by Boltzman's distribution. Nominally it is possible to draw an analogy between the considered transport task and the diagram submitted on fig. 1. For this purpose it is enough to quantize an interval of change of speeds of the automobile; and refer the automobile possessing the proper speed to one of "levels".

However, to prove applicability of Boltzman's distribution to the nonequilibrium problems, we have to give its conclusion, not resorting to the concept of the thermodynamic balance. Besides in Boltzman's distribution appears such parameter, as thermodynamic temperature. Search of its analogue for transport problems is a nontrivial task. We will demonstrate that it is possible to do it in the most common case. For the generality we will be speaking about "particles" and «resource levels», not concretizing their nature. The last term has some in common features with the used one in (Wilson A. J. Entropic methods of modelling of complex systems. (in Russian)

Moscow, 1976.) where the concept «a stream of the generalized resource» is used.

The number of particles in the system in question will be considered to be macroscopically large but finite. We will put the letter of some alphabet in accordance to each of conditions, assuming that the number of energy levels in the system is finite. Then to every allowable condition in the system considered there will be the certain word containing so many letters as many particles are in the system.

The rearrangement of letters in this word does not change a picture of particles distribution by resource levels. Therefore the probability of realization of a condition with prescribed distribution of particles by resource levels, when it is shown how many atoms, automobiles or other objects are at each of levels, can be found by the same method which is used deducing the Shannon's formula.

Let's consider, that frequency of occurrence of the definite symbol A_k in the messages made of letters of an alphabet which can be put in conformity to resource levels of the system considered, is V_k . It means, that quite volumetric message containing plenty of symbols N , with high accuracy contains $V_k N$ of A_k symbols.

According to the general approach of the theory of the information, for searching of its quantity contained in such word (text), it is required to determine the full number of variants, from which the words containing set number of letters can be made. It is a well-known problem from theory of combinations (Prohorov U.V., Rozanov U.A. Theory of probability. (in Russian) Moscow, 1987) about allocation of various elements N on r cells. Each allocation can be described by combination $(i_1, i_2, i_3, \dots, i_r)$, where i_k - is a number of a cell in which the subject with k number gets.

Let's consider allocations where the number of elements got into a cell with number k is exactly N_k , and each of N elements is in any of the cells:

$$N_1 + N_2 + N_3 + \dots + N_r = N \quad (1)$$

Then the full number of such allocations is (2):

$$G = \frac{N!}{N_1! N_2! \dots N_r!} \quad (2)$$

Amount of information as the measure of vagueness removed by reception of any concrete word (text), can be found by taking the logarithm of numbers of variants (2): In view of the ratio (1) from which the condition $p_1 + p_2 + \dots + p_r = 1$ follows, and using Stirling's formula, we receive:

$$\begin{aligned} \ln G &\approx -N p_1 \ln p_1 - N p_2 \ln p_2 - \dots \\ &\dots - N p_r \ln p_r = -N \sum_i p_i \ln p_i \end{aligned} \quad (3)$$

The expression (3), as it was expected, will coincide by the form with Shannon's formula.

We will show now that on the basis of (3) it is possible to receive Boltzman's distribution for any of the systems adequating to the diagram of fig. 1. We will remind that according to the basic concepts of statistical physics, the equilibrium system appears in the most probable condition, i.e. in which it can be realized by maximum big number of ways. The logarithm of number of ways, by which such condition can be realized, represents the entropy of the system.

We have to emphasize that "balance" here should not necessarily be interpreted in classical (thermodynamic) meaning. Moreover, for searching of the most probable condition there is no need to use this concept.

The decision of a considered problem is coming up to search of such set of numbers (N_1, N_2, \dots, N_r) or such quantities (p_1, p_2, \dots, p_r) for which the size $\ln G$ has the maximal value.

However, in consideration of physical-chemical, or transport-communication systems there are additional restrictions. Namely the physical-chemical system possesses the definite energy, and finding of its characteristics (including thermodynamic) as a function of energy is of interest. In the same way, an integrated stream of the generalized resource is fixed in transport-communication tasks (cumulative costs on transportations, the charge of fuel, etc.).

Consequently, the function $\ln G$ has to be researched on a **conditional** extremum. In other words, it is necessary to find such sequence of quantities (p_1, p_2, \dots, p_r) which will deliver a maximum to $\ln G$ function provided that full energy of the system or a full transport stream is fixed.

Let's make a start from the analogy with problems of statistical physics. Full energy of system is quite definitely connected to how particles on energy levels are allocated. More precisely this size coincides with result of the calculation of average to multiplier, equal to the full number of particles in the system:

$$E = \sum_k N_k \varepsilon_k = N \sum_k p_k \varepsilon_k \quad (4)$$

To find a conditional extremum the method of Lagrangian undetermined coefficients is used. According to this method, it is necessary to generate the sum:

$$\ln G + \lambda E + \lambda_1 = -N \sum_k p_k \ln p_k + \lambda N \sum_k p_k \varepsilon_k + \lambda_1 \sum_k p_k, \quad (5)$$

into which undetermined coefficients λ and λ_1 enter, and then to investigate it for an extremum. Thus the coefficient λ is determined from the condition (4), and the coefficient λ_1 - from the condition of normalization.

If we differentiate (5) by p_k , we receive (the full number of particles in the system is supposed to be constant):

$$\frac{\partial}{\partial p_k} (\ln G + \lambda E) = -N \ln p_k - N + \lambda N \varepsilon_k + \lambda_1 = 0 \quad (6)$$

As a result we have (to constant coefficient):

$$p_k = a \exp \lambda \varepsilon_k \quad (7)$$

The coefficient a can be found from a condition of normalization, the coefficient λ can be found from a condition of preservation of full energy (4). Within the context of statistical physics it is proved, that this multiplier has return temperature meaning

$$\lambda = -\frac{1}{T} \quad (8)$$

Thus, almost the same arguments which reduce to the Shannon's formula in the theory of information, also lead to Boltzman's formula, which describes distribution of particles on energy levels:

$$p_i = a \exp\left(-\frac{\varepsilon_i}{T}\right) \quad (9)$$

The formula (9) shows, that if the temperature of system is given, then concrete values of probabilities by which separate power conditions are realized are also known. In particular, using (9) it is possible to calculate concrete value of entropy for equilibrium, i.e. the most probable (in macroscopical sense) condition of system if in the formula (3) by p_i understand probability of a presence of atom in a separate energy condition:

$$\ln G = -N \sum_i p_i \ln p_i \quad (10)$$

The conclusion of Boltzman's distribution from Shannon's formula which has been given above is trivial at first sight. However it is necessary to accentuate that till present day Boltzman's distribution in manuals on statistical physics was deduced by absolutely other ways where the concept of the thermodynamic balance was actively used. In fact there was stated above the precise analysis of the system containing particles, allocated in the system of energy levels without using the conceptions of thermodynamic balance.

Rejection of the concepts of thermodynamic balance use is extremely important for the further analysis because the temperature in the above mentioned conclusion appears differently than in classical thermodynamics. In particular, there might exist a very big amount of quantities being analogues of temperature. For example, it is possible to talk about transport temperature of automobiles distribution in a megapolis.

For every level on figure 1 we can give the value of movement speed of an automobile. Summing all above arguments up we can easily see that the number of ways in which the condition

$\{p_i\}_0^N$ can be realized is described exactly by the formula (10) ($\{p_i\}_0^N$ means a condition in which the probability of a being

of a particle at a resource level with number i is p_i). Indeed, for every resource level we can put the letter of an alphabet in accordance, and then calculate number of "words" where frequency of occurrence of letters is described by distribution $\{p_i\}_0^N$. It reduces to the formula (10), and we accentuate once again it is absolutely not important what "resource" and "levels" we are talking about. In particular it can also be levels of speed.

The specificity of the task is concerned with to what exactly parameter is maintained constant (is fixed of task conditions to be more precise). For the thermodynamic closed system obviously it is the full number of particles and full energy. For urban transport environment (item 3) as the mentioned parameters we should choose the daily average fuel consumption by a megapolis in general and the full number of automobiles there. At the same time, the fuel consumption can be measured by energy units.

Hence, the analogue of the formula (5), on which basis can be entered the concept of thermodynamic temperature, can be written as:

$$\ln G + \lambda W + \lambda_1 = -N \sum_k p_k \ln p_k + \lambda N \sum_k p_k w(v_k) + \lambda_1 \sum_k p_k \quad (11)$$

where W is an integrated energy consumption of automobiles fuel by megapolis in a time unit (which has capacity dimension); G as before is the number of ways by which the condition

$\{p_i\}_0^N$ can be realized; λ and λ_1 are Lagrangian undetermined coefficients, their meaning will be determined below; $w(v_k)$ is the fuel consumption (in a unit time) by an automobile moving at a speed of v_i .

Automobiles allocation by speed levels can be found by differentiating (11) on each of value p_k and equating result to zero; it leads to the next function of automobiles allocation by speed:

$$p_k = a \exp \lambda w(v_k) \quad (12)$$

where, as before, the constant a is transformed Lagrangian undetermined coefficient; λ_1 can be found from a normalization condition; and the undetermined coefficient λ is determined from a condition

$$\sum_k w(v_k) = W \quad (13)$$

Correlations (12) and (13) completely solve the task given: for every speed level it is possible to indicate the probability of detection on it one of the N automobiles.

Certainly, the conclusion above stated requires the correction. In particular, the real number of automobiles on roads essentially varies within 24 hours, as well as combined fuel consumption by a megapolis. However we can note, the correctness of the used ideas in a case when combined fuel consumption by a megapolis is not constant can be proved by the same means, as a correctness of use of the thermodynamic temperature concept in quasi-equilibrium systems where characteristics slowly change in the course of time.

3. THE ANALOGUES OF THERMODYNAMIC POTENTIALS FOR TRANSPORT SYSTEMS

The question of choice of a parameter which is supposed to be fixed searching of the most probable condition of a system is more than important. It follows from analogies with tasks of statistical physics: full energy of system, i.e. the parameter which characterizes it, is in common thermodynamic potential itself, and besides on its basis other thermodynamic potentials are constructed (Gibbs' potential, enthalpy, etc.).

In other words, the question of choice of values, considered as constants while searching of the most probable distribution, actually is a question of analogues of thermodynamic potentials.

Let's consider firstly physical meaning of "the analogue of thermodynamic temperature", making a start from consideration of a model problem assuming that connection between the average fuel consumption by an automobile W and in its average speed is given by expression:

$$w = \zeta k v^2 \quad (14)$$

where k is friction characteristic, ζ is cumulative efficiency of the transformation of chemical energy of fuel into mechanical one.

Passing from discrete functional of entropy (10) to continuous

$$\sigma = -N \int p(w) \ln p(w) dw \quad (15)$$

and considering the formula (14) and restrictions

$$\int p(w) w dw = 1, N \int p(w) w dw = W \quad (16)$$

we receive functional

$$\begin{aligned} \sigma + \lambda W + \lambda_1 = & -N \int p(w) \ln p(w) dw + \\ & + \lambda N \int w p(w) dw + \lambda_1 \int p(w) dw \end{aligned} \quad (17)$$

Searching of the extremum (17) gives distribution of automobiles $p(w)$ by spent capacity w

$$p(w) = A \exp\left(-\frac{w}{\tau}\right) \quad (18)$$

where A is a normalizing multiplier, and the size τ which apparently has dimension of capacity now can be considered as transport temperature.

From the distribution (18) by the capacity spent by automobiles, we can turn to their distribution by speed. The number of automobiles in an interval of capacity dw is non other than $p(w)dw$. The transition to calculation of automobiles number in a differentially small interval of speeds dv is given by the obvious formula:

$$\begin{aligned} p(w)dw &= A \exp\left(-\frac{w}{\tau}\right) dw = \\ &= A \exp\left(-\frac{w(v)}{\tau}\right) \left(\frac{dw}{dv}\right) dv \end{aligned} \quad (19)$$

or for the elementary case (14)

$$p(v) = n \frac{\zeta k}{2\tau} v \exp\left(-\frac{\zeta k v^2}{\tau}\right) \quad (20)$$

The normalizing multiplier in (20) is obviously calculated. Thus, the distribution of automobiles by speed provided that in the capacity of an analogue of "energy" as thermodynamic potential is used cumulative capacity of the motor fuel spent by automobiles in a city, is given by the distribution similar to Maxwell's. His typical form is shown on figure 1.

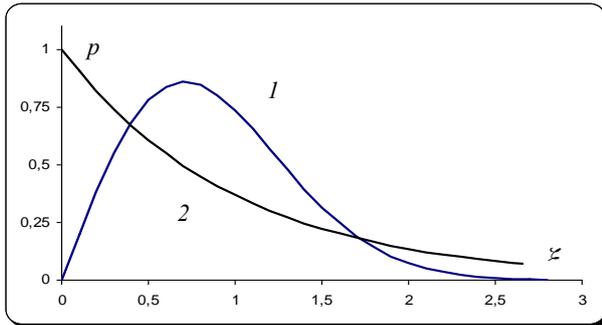


Figure. 1. A typical form of distribution of automobiles by speed by using cumulative capacity of spent fuel as analogue of thermodynamic potential (curve 1) and full stream of automobiles on a given route (curve 2).

We will accentuate that if we will fix not the analogue of "energy" considered above but any other size, the form of automobiles distribution by speed will be another. For example, if as the analogue of energy of a system as thermodynamic potential we will use stream size on a given route, i.e. consider the integral below as fixed:

$$N \int p(v) v dv = J \quad (21)$$

then definition of an extremum of functional as in (17) results in the distribution which has form of falling down exponent; its typical form also is shown for comparison on fig. 1.

Such behavior of the curve of automobiles distribution by speed is not realized in practice, what confirms expediency of use of cumulative capacity of spent motor fuel as analogue of the thermodynamic potential. For the benefit of the conclusion made there also exist economic arguments. Indeed the cumulative capacity of automobile engines in econometrics is still one of the main parameters of a market saturation of a country by this category of goods (automobiles); capacity parameters were used and are used to characterize the level of production as a whole, etc. Therefore it is represented quite justified to describe integrated intensity of transportations inside a year also through the capacity of spent for it fuel.

Thus, the analogue of Maxwell's distribution can be received proceeding from entropy (it is possible to tell also - probabilistic reasons. The most important notion for the usage of this distribution apparently is the concept of transport temperature first time entered in this work. It is possible to show that as well

as in statistical physics the concept given expresses a condition of balance between two contacting systems.

4. OPPORTUNITIES OF USAGE OF ANALOGUES OF THERMODYNAMIC CHARACTERISTICS

It is possible to show that the definition of transport temperature offered in this work allows in turn to consider analogues of phase transitions gas - liquid (free movement of automobiles - densely loaded highway) and liquid - solid body (the loaded highway - a transport congestion) correctly.

In these conditions a task on prevention of occurrence of congestions can be formulated quantitatively, through the maintenance of high enough value of the transport temperature which will not allow to be realized the phase transition to «solid body».

It is also possible to show, that for the unloaded movement (analogue of ideal gas) the transport temperature is directly proportional to average thermal speed of movement of automobiles (that reflects direct analogy to the molecular-kinetic theory of gases). For a case of the raised traffic density on roads this communication becomes more complex, but nevertheless analogues of thermodynamic functional can be described in a closed form.

Making a start from the functional of cumulative charge of fuel considered as analogue of thermodynamic functional of energy it is also possible to show that macroscopical behavior of automobiles on a route is also described by analogue of such thermodynamic size as pressure. This size allows to estimate quantitatively "pressure" of transport streams upon crossroads and other central points of a city transport network. It is convenient to carry out the optimization, for example, of frequency of traffic lights changing through regulation of transport pressure in separate central points of a highway.

Thus the description of distribution of transport streams within a city transport network can be carried out by minimization of functional of capacity (minimization of functional of fuel cumulative charge) as the description of physical and chemical systems through minimization of appropriate thermodynamic value is made.

This conclusion allows to establish a direct connection between economic parameters and a condition of environment in large cities (on automobiles, for example, in Almaty city it is about 90 % of all emissions of polluting substances in an atmosphere). On a basis of functional of the cumulative charge of fuel it is possible to suggest complex ecological and economic criteria for searching optimum number of automobiles within a city transport network. These criterions at the same time take into account such factors as a rise in prices on goods of the daily demand caused by increase of transport charges as a result of increase in road taxes, decrease in life quality as a consequence of environment deterioration, etc. Quantitative estimations of influence of environment deterioration on integrated parameters of life quality can be made on the basis of the techniques offered in [2].

As a whole, the analysis of functional of cumulative charge of fuel in the long term allows to find a technique of calculation of optimum level of the transport tax taking into account distribution by form of used vehicles.

And at last, there becomes a basis for searching of a theoretical limit for number of automobiles within a city transport network; after this limit phase transition to solid body becomes unavoidable. On this basis it is offered calculation of size of transport taxation at which the quantity of automobiles in a city becomes below theoretical limit indicated.

5. CONCLUSIONS

Thus, the concept of the transport temperature considered as analogue of the appropriate thermodynamic quantity can be determined correctly. At such approach functional of cumulative charge of fuel is analogue of thermodynamic functional of energy; it allows to define correctly analogues of all other thermodynamic values, for example pressure, free energy, etc. Thus in this work it is shown that for the

macroscopical description of transport streams within a city transport network it is possible to develop the theoretical device completely similar to thermodynamics.

BIBLIOGRAPHY

1. Suleimenov I., Baigabulova Zh., Suleimenova K., Bekmagambetova J, Dyachenko E. Fundamental of city's physics. (in Russian) Almaty. 2007.
2. Wilson A. J. Entropic methods of modelling of complex systems. (in Russian) Moscow, 1976.
3. Prohorov U.V., Rozanov U.A. Theory of probability. (in Russian) Moscow, 198