A MATHEMATICAL MODEL FOR EVENT DETECTION IN SPATIOTEMPORAL CITY ENVIRONMENT

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ABSTRACT:

All entities in an urban environment such as buildings, parks, roads etc. are changing more or less over time. A seamless documentation of these changes in a spatiotemporal city model may require extraordinarily large storage capacity and a high computing intensity if the stored changes are visualized. This paper presents a mathematical model for defining and capturing the relative significance of events from various changes. The model is established on the basis of the inherent dependence that exists between the spatial and temporal scales or resolutions. Experiments show that changes can be evaluated and differentiated using their values of event significance on different occasions (with a certain temporal and spatial resolution).

1. INTRODUCTION

"This being so," asked the Earl of the River, "may I take heaven and earth as the standard for what is large, and the tip of a downy hair as the standard for what is small?"

"No," said the Overlord of the North Sea. "Things are limitless in their capacities, incessant in their occurrences, inconstant in their portions, uncertain in their beginning and ending. For this reason, great knowledge observes things at a relative distance; hence it does not belittle what is small or make much of what is big, knowing that their capacities are limitless."

Chuang Chou, ca. 300 B.C.

Chuang Chou (around 300 B.C.) studied the problem about how to observe the world around us. As his words already indicated that change exists in a relative space; whether it is significant or not (small or large) depends on the current environment in which the change is observed.

In order to distinguish or extract the significant change from numerous changes during the evolution of object(s), the notion of event is introduced. In the case of a city model, events may be embedded in changes of locations, shapes, sizes, textures and semantic attributes of the objects such as 3D buildings. However, it is sometimes difficult to define the change of states, especially for the changes which happen evenly. The problems we face are: (i) how to determine the start and end point of a change and (ii) which change can be counted as an event? Is there a criterion of degree or quantity to judge whether a change is an event or not?

This paper is dedicated to a mathematical model that tries to clarify these problems, particularly the events in geometries. The idea is rooted in the inherent relationships between the temporal granularity and spatial granularity that are used to determine a certain spatiotemporal environment where events occur. In fact, the mutual dependence between spatial and temporal resolution can be obviously observed in our everyday life:

- With a temporal resolution of year or decades we tend to notice large geometrical changes of buildings, for example, construct or destruct of a whole building or a group of buildings.
- With a temporal resolution of day or week we are more interested in smaller and local geometrical changes of building or building parts such as façade, wall elements, etc.
- With a temporal resolution of day or hour we may even be able to distinguish trivial changes such as the installation of a window or painting a wall with new color.

(Liu, 2007) discussed similar phenomenon in his work about distributed management of global mass remote sensing image data. In line with the citation of Chuang Chou in the beginning of this paper, it can be stated that there are some inherent regularities in the relations between temporal and spatial resolution when we observe the world around us.

In other words, whether a change could be viewed as an event or not depends on the amplitude of the change, duration of change, currently used temporal and spatial resolution. In our approach we apply a reasonable combination of these four parameters as a criterion to define events in geometric entities and have termed this combination as **Event Significance** (ES).

In our work geometric changes are categorized in 3D and 2D which, for instance, respectively correspond to the evolution of a building and the decrease of meadow as the result of

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urbanization. The changes in 2D and 3D are differently handled in the mathematic model and for different levels of granularities.

The paper is organized as follows. In Section 2 we present the related work. In Section 3 we introduce and describe the mathematic model for event detection in a multi-resolution spatiotemporal city environment. In Section 4 we deploy the mathematic model, not only for mobile objects, but also for stationary objects. Finally, Section 5 concludes the paper and outlines future research directions.

2. RELATED WORK

Many existing 2D techniques based on multi-temporal, multi-spectral, multi-sensor data have demonstrated a potential to detect, identify, map and monitor geometric changes (Coppin et al. 2004). In almost all of the previous work, the images are stored in a database whenever a change is detected from them in a time sequence. Whether the change is significant enough or not is disregarded. This inevitably leads to the storage of redundant information especially in a snapshot data model, because for every new change a new layer (all of the information, changed or not changed, is stored in the layer) is required (Nadi & Delavar, 2003).

To solve this problem the notion of event is introduced as a means to describe how the world may change (Stratulat et al., 2001), just like setting mile stones to mark the significant changes. Similarly, an event is defined as the change from one state to the next, if the world is viewed as a series of states or “snapshots” (Lansky, 1986).

In the previous work (Fan et al. 2008; Fan & Meng, 2008) within the frame of the ongoing project “integrating time-dependent features into 3D city model” not every change is regarded as an event, rather only the significant change counts. Consequently, changes that are not significant are neglected, which means a reduction of required storage capacity.

Nevertheless, it remains a difficult task to detect event from a series of changes because whether a change can be treated as an event depends on the spatiotemporal context in which the change occur, and this context is scale-dependent in both spatial and temporal sense (MacEachren & Kraak, 2001). In other words, a change can be regarded as event which is meaningful only in a certain range of spatiotemporal scales.

In fact, ecologists deal with processes that occur at a variety of temporal and spatial scales (Turner et al. 1989). Scientists in this field have been aware of linkage between spatial and temporal scales already over a long time (Gibson et al. 2000). And they tried to define and constrain intervals of spatiotemporal scales in order to analyze geospatial events more adequately (Dickson, 1988;, Allen and Hoekstra, 1990, Meyer et al. 1992 etc.). However, the conception is restrained to the macro-spatiotemporal scales.

In this work, we establish a mathematical model which is defined by an equation for calculating the significance degree of a change along with two intervals that constrain the spatiotemporal occasions in which the change can count as an event.

Additionally, the mathematic model can be also employed for selecting events and determining the interpolation interval during the dynamic representation of a 3D city model. Thus the interpolation does not have to be conducted at every point of time, although the involved entity changes all the time in the queried time interval.

3. THE MATHEMATIC MODEL

The mathematical model takes the inherent dependence of geo-spatial change on spatial and temporal scales into account. It consists of two parts: Event Significance that indicates the activity of the referred change and two intervals for valid spatiotemporal scales for constraining the spatiotemporal occasions where the change counts as an event.

3.1 Event significance

As mentioned in the Section 1, **Event Significance** is used to define and capture an event among various changes in geometries.

Definition 1: Event Significance is the importance and necessity that a geometrical or textural change is captured and defined as an event.

In our approach an Event Significance is indicated by P_e, which is defined as follows:

$$P_e = \begin{cases} \dfrac{\frac{\Delta_s}{R_s}}{\left\lceil \frac{\Delta_t}{R_t} \right\rceil} & \text{for } \Delta_t > \lambda \cdot R_t \\ \\ \dfrac{\frac{\Delta_s}{R_s}}{\left\lceil \frac{\Delta_t}{R_t} \right\rceil \cdot 1000} & \text{for } \Delta_t \le \lambda \cdot R_t \end{cases} \qquad (1)$$

where Δ_s is the measure of the change of interest, in case of the geometrical change Δ_S is the changed length in the changing direction.
R_s is the currently used spatial resolution.
Δ_t is the real duration of the change.
R_t is the currently used temporal resolution.

For the dynamic representation of an event, we suppose to visualize the change by using more than one increase (or decrease) in the main moving direction, so that the user can notice the dynamic process of the change. Therefore, a factor of λ ($\lambda > 1$ and $\lambda \in \mathrm{N}$) is added in Equation (1). In case that the duration of the referred change is longer than $\lambda \cdot R_t$, P_e will be calculated. Otherwise, the change will be regarded as insignificant since it appears instantaneously. For this reason, a factor of 1000 is added in the denominator, in order to belittle the value of P_e.
In general, the Equation (1) could be abstracted as:

$$P_e = \frac{F_S}{F_T} \qquad (2)$$

Where $F_S = \frac{\Delta_s}{R_s}$ is a spatial factor and $F_T = \left\lceil \frac{\Delta_t}{R_t} \right\rceil$ is a temporal factor.

The spatial factor in numerator of equation 1 and 2 is the measured length in the moving direction regardless of the spatial dimensions (2D or 3D) of the referred change. The temporal factor in the denominator does not make difference between 3D and 2D either. Therefore, P_e is essentially the ratio between the magnitude a change reaches and the needed time to reach this magnitude, i.e. the speed of the referred change. From this point of view, the mathematic model reflects the two issues that are most important for sensing the changes objects: the magnitude and the speed.

In our research work, so far, we use changed length that is measured in the main changing direction as the quantity of Δ_s. For instance, Δ_s is trajectory for a moving object; in case of 2D area change, Δ_s is then the distance between the front line at the beginning of change and the front line at the end of the change; and Δ_s is the increase or decrease in the height in case that geometric change of 3D building is referred.

3.2 Derivation of intervals for valid spatiotemporal scales

In line with common sense knowledge and the cognitive aspect, an event should be so represented that the viewer can experience it at once – in a single glance (Peuquet, 2002). The most appropriate space to view an event is therefore the tabletop spatiotemporal scales of the event. For further decrease of the spatial and temporal scale, the event can still be visible but becomes less apparent, until it can no longer be perceived any more.

- Deriving the interval of the valid spatial scale R_s

The interval of the valid spatial scale indicates the range of the linear continuum in geometric space, in which the geometric change of the event can be sensed without much effort. In our approach we set the tabletop scale as the left boundary of the R_s. The right boundary of R_{sn} can be deduced using

$$R_{sn} = \max\left\{ R_{si} \middle| T_{si} \le \frac{\Delta_s}{\lambda \cdot R_{si}}, i = 1,2,...,n \right\} \quad (3)$$

whereby: Δ_s represents the length of the changed part in the main changing direction. And T_{si} is the minimum length which is still visible at the scale of R_{si}.

- Deriving the interval of the valid temporal scale

Actually, an event could be represented either slowly or fast at every valid spatial scale. For example, at the table-top scale the trajectory can be represented as slowly as extending the line with the minimum length which is just visible at every time point. In contrast, it can be represented as fast as appearing instantaneously. These two time scales form the interval of the valid temporal scale at the table-top spatial scale. With the decrease of the spatial scale inside the R_s the minimum visible length becomes larger and larger. In sequence, the left boundary of the interval of the valid temporal scale becomes larger as well, while the right boundary remains. That means that the interval of the valid temporal scale at a large spatial scale contains that at a smaller spatial scale. Then all the intervals of the valid temporal scale form an inverse rectangle (Figure 1) corresponding to the valid spatial scales.

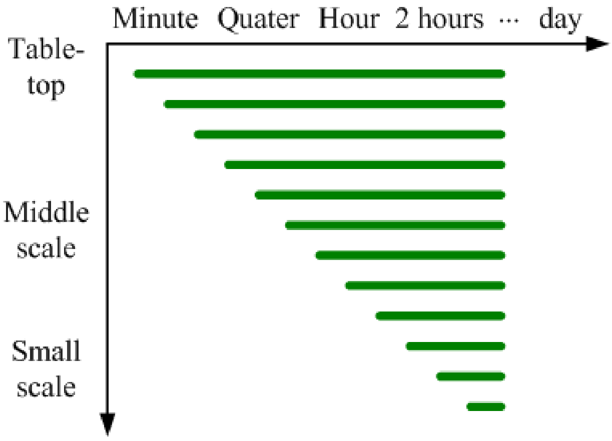

Fig.1. The interval of temporal scale is spatial scale dependent. And range of the interval decreases with the decrease of the spatial scale, however with the same right boundary.

In the presented approach, the interval of the valid temporal scale will be calculated at the tabletop spatial scale, the rest will be calculated according to the currently employed spatial scale when the event is visualized instead of deriving it in a pre-process. Otherwise, the calculation of the temporal interval will lead to much requirement of the storage, since for each spatial scale an interval of temporal scale is needed. Therefore, in the event structure the interval of temporal scale will be given only for the tabletop spatial scale. The temporal interval for the increased spatial scale can be represented by using an implicit interval $R_{ti} = [R_{ta}, R_{te}]$ which will be dynamically conducted during the visualization, whereby

$$R_{ta} = \left\lceil \frac{\Delta_t}{\Delta_s / T_{si}} \right\rceil \quad (4)$$

$$R_{te} = \left\lceil \frac{\Delta_t}{\lambda} \right\rceil \quad (5)$$

Note: the factor of λ in Equation (3) and (5) has the same meaning as that in the equation (1), since we want to represent the dynamic motion of the event, not the instantaneously appearing or disappearing effect.

4. DEPLOYMENTS OF THE MATHEMATIC MODEL

In this section the deployments of the mathematic model will be described using three example applications respectively. In all these examples, we just set $\lambda = 5$ for the dynamic visualization, and the $T_{si} = 5[\text{mm}]$ as the minimum length on the screen which can still be visible at certain spatial scale.

4.1 Management and visualization of trajectories of moving object

In this example an event called "Going to the Kaufhof after work" is presented. The event took place in the afternoon of 14th October, 2009, as two colleagues of the Department of Cartography at the Teschnische Universität München went to buy something in Kaufhof directly from their working place. The data were captured using a GARMIN GPS Navigator. Figure 2 shows the entire trajectory, where the start point and the end point are marked with green and red balloons respectively. The symbolized tram stations serve as switch points.

Fig.2. The trajectory of the event "Going to the Kaufhof after work".

According to the transportation modes, the event can be composed of four smaller events: (i) Go from the university to the tram station of Pinakotheken by walk (Figure 3), (ii) Wait for the tram at the station of Pinakotheken (Figure 4), (iii) Go to the tram station of Karlsplatz by tram and (iv) Go from the station of Karlsplatz to the entrance of Kaufhof by walk. The measured data for every event are shown in Table 1.

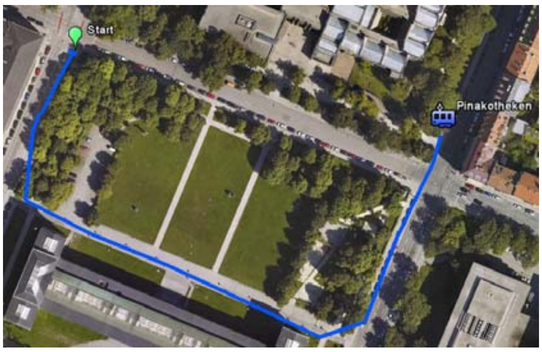

Fig.3. Trajectory of "Go from the university to the tram station of Pinakotheken by feet". The tabletop scale is larger than that of the mother event.

Fig.4. Trajectory of "Wait for the tram at the station of Pinakotheken". It has the largest tabletop map scale with the smallest scope of the trajectory.

Table 1. The trajectories of the events

ID	Length of trajectory	Average speed	Duration
i	390 m	4 km/h	6 minutes
ii	80 m	0.4 km/h	9 minutes
iii	1520 m	20 km/h	4.5 minutes
iv	80 m	4 km/h	1.5 minutes
Total	2070 m	6 km/h	21 minutes

Note: As shown in Figure 4, the second event "wait for the tram at the station of Pinakotheken" does not mean that the person just stood there to wait for the tram, instead they slowly moved around the station.

The intervals of their valid spatial scales and the corresponding intervals of the valid temporal scales at the tabletop level can be derived using the equation introduced in Section 3. And the derived intervals are listed in Table 2.

Table 2. The valid spatiotemporal intervals for the events

Event ID	Interval of valid spatial scale	The corresponding interval of valid temporal scale
i	[3000, 13000]	[15, 70] in [s]
ii	[1000, 3000]	[30, 100] in [s]
iii	[10000, 50000]	[10, 60] in [s]
iv	[1000, 3000]	[5, 20] in [s]
total	[10000, 70000]	[30, 250] in [s]

Comparing the spatiotemporal intervals in Table 2, we can find that there is no intersection among the four child events. This means that we can not find an appropriate spatiotemporal environment in which all these four changes can count as events. For this reason, some trajectories have to be neglected if events are queried at a certain spatiotemporal scale. Otherwise, the spatiotemporal scale should be adapted for the current event if all trajectories are treated as events and represented in sequence. This requires then the model of visualization with dynamic spatiotemporal scales.

If these four events have to be represented at the same time, The values of their Event Significance will be calculated respectively for all environments of their valid spatiotemporal scales (Figure 5). The maximal value of Event Significance denotes the most significant change (the third event in the example) in the event. The corresponding spatiotemporal scales will be selected for the current visualization. Then the spatiotemporal scale has to be adapted to the next maxima of Event Significance values. The process terminates when all the events are visualized.

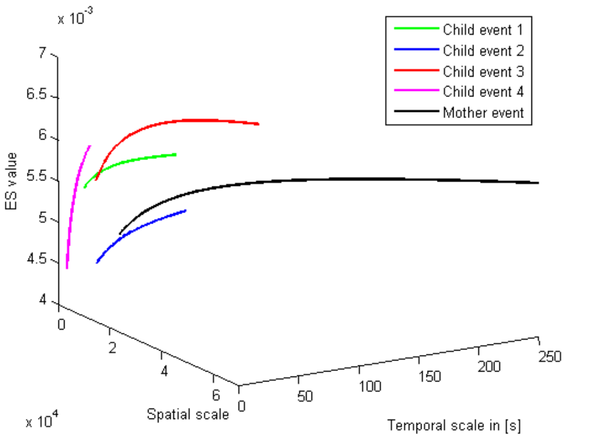

Fig.5. Event Significance values of events in relation with spatiotemporal scales

4.2 For detecting the change in the land use

The images in Figure 6 were downloaded from Google Earth. And the timestamps indicate when the satellite images were captured. The areas of the meadows and the changed parts were roughly estimated according to the measurements in Google Earth.

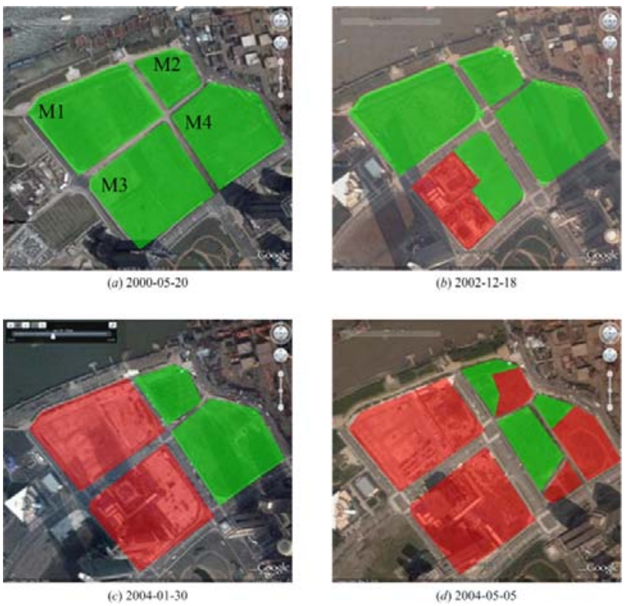

Fig. 6. A sample area that illustrates the change of four parcels of meadow caused by the construction in a corner of Pudong, Shanghai from 2000 to 2004.

The meadows are numbered with M1 to M4 respectively (Figure 6a). On a part of M3, a few buildings were constructed within 180 days during the time window of [year 2000 year 2004] (Figure 6b). The remaining part of M3 was constructed within 60 days and the whole M1 started to be constructed at the same time, and the work lasted 370 days (Figure 6c). Figure 6d records the change of M2 and M4. Part of M2 was changed to free field preparing for new construction within three days while two parts of M4 were covered by rubble within 20 days.

So far we have the fundamental information about the changes of the four meadows (See Figure 3).

Table 3. the fundamental information about the changes of the four meadows.

	Area changed in [m²]	Duration in days	Average speed [m²/day]
M1	45500	370	123
M2	4000	3	1333
M3 (1st part)	28000	180	156
M3 (2nd part)	23000	60	383
M4	21000	20	1050

According to the changes of the meadows in Table 3, their corresponding intervals of valid spatiotemporal scales can be derived (Table 4).

Table 4. the valid spatiotemporal intervals of the changes in the example.

	Interval of valid spatial scale	Interval of valid temporal scale
M1	[1000 10000]	[week double months]
M2	[500 2500]	[5hours 15hours]
M3 1st part	[1000 5000]	[week month]
M3 2nd part	[1000 5000]	[3days week]
M4	[1000 5000]	[day 4days]

The intervals in Table 4 denote that there is an intersection of the intervals of valid spatial scale. But there is no temporal scale which is appropriate for describing the changes in all the four meadows.

The values of Event Significance can be calculated regardless their intervals of valid spatiotemporal scales.

Table 5. The ES values of the four changes at different spatiotemporal scales

	M1	M2	M3 1st part	M3 2nd part	M4
$R_S = 2000$ $R_t =$ half day	0.0002	0.0033	0.0002	0.0005	0.0015
$R_S = 2000$ $R_t =$ day	0.0003	0	0.0003	0.0009	0.0030
$R_S = 2000$ $R_t =$ week	0.0024	0	0.0024	0.0064	0
$R_S = 5000$ $R_t =$ 2weeks	0.0019	0	0.0019	0	0

Comparing the values in the above table, it is obvious that whether a change is significant or not is scale-dependent. A significant change at a spatiotemporal scale may become insignificant if the temporal and/or spatial resolution increase. On the other hand, a change (M2) could be more significant than another one (M4) at a certain spatiotemporal scale if it has a higher changing speed than another one, although its changed magnitude may be smaller than that of another one. However, at a lower temporal resolution and/or a lower spatial resolution, we observe the opposite relationship between their Event Significance, because on these occasions the changes appear to be instantaneous; or they change unremarkable, as the changed parts look small in a small spatial scale.

4.3 Interpolation and selection during 3D visualization

An example is given here to explain and show how this mathematical model works for the determination of time interval of interpolations: there is a building with a square footprint of 40 x 40 m². It is 50 m high. The construction of this building lasted one and half years (Δ_t = 1.5 years). Then the interval of the valid spatial scales could be [500 2000], and the interval of the valid temporal scales could be [month quarter].

If this building should be visualized within the time of its construction, its geometries should be interpolated, so that the effect of a continuous change for the dynamic visualization could be achieved. Assume that the spatial resolution is 1:1000 ($R_s = 1000$), the temporal resolution is one month (one month corresponds one second in the visualization environment. $R_t = \text{month}$). Then the process of the construction can be visualized for exactly 18 seconds. This means, the building can be increased from the ground in the height in 18 steps. In other words, the height of the building will be calculated using interpolation of $h_n = n \cdot \frac{50}{18}$ [m] , whereby n increases from beginning to end of visualization in [s]. (the height of the building is increased with the speed of around 2.78m/s).

For the same spatial environment, the time period of the interpolation should be four seconds, if the temporal resolution used while visualizing is changed to a week ($R_t = \text{week}$), because the increase height at every second is too small to be noticed. In this case, the visualization appears, however, discontinuously, since one has to wait four second to realize a noticeable change. This reflects the importance of constraining spatiotemporal interval for changes. In this example, a continuous animation of the change is impossible, because the used temporal resolution does not fall in the interval of the valid temporal scales. Therefore, this change will be not recommended for a dynamical visualization at the spatiotemporal scales of $R_s = 1000$ and $R_t = \text{week}$. In some cases where the change has to be visualized at such a spatiotemporal scale, the interpolation will be done once every four seconds, instead once every second. In this way, the cost of computation is reduced obviously.

If we use year ($R_t = \text{year}$) as temporal resolution instead, while keeping the same spatial resolution, the time required for visualization will be reduced to one and half seconds. In this case, the change appears to be instantaneous. As a result, this change will not be selected for the dynamical visualization at the spatiotemporal scales of $R_s = 1000$ and $R_t = \text{year}$.

5. CONCLUSION AND FUTURE WORKS

Geospatial changes are scale-dependent in both spatial and temporal sense. A change can be treated as an event only in a certain range of spatiotemporal scales.

This paper presented a mathematic model for detecting event in spatiotemporal urban environment. The model is composed of two parts. The first part combines spatial and temporal parameters to calculate the degree of the Event Significance of a change. The second part consists of two intervals: interval of valid spatial scale and interval of valid temporal scale in order to constrain the spatiotemporal scopes where the referred change can be regarded as an event. In the space outside of the valid spatiotemporal scales the change will be regarded as insignificant, because it appears to be instantaneous; or it changes subtly, as the changed parts look small at certain spatiotemporal scales.

Aiming to represent the process of change dynamically, the largest valid temporal scale should be smaller than the time duration of the change. In the current research we propose to visualize the change by representing it λ ($\lambda > 1$ and $\lambda \in \mathrm{N}$) times increase (or decrease) in the main moving direction. In this paper we empirically set $\lambda = 5$, because we think that the changed geometry should be represented by at least five increase or decrease in the main moving direction, so that one can remark (or realize) the motion. In the future, this factor will be investigated according to the visual perception.

At the time being our mathematical model is restricted to describe changes which have an apparent main moving direction. Therefore, it can hardly handle a change in which the geometry expands or contracts averagely in many directions, for example flooding in the city. In this case, we may use changed area or volume as the magnitude measure in the mathematical model.

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