# CURVELET APPROACH FOR SAR IMAGE DENOISING, STRUCTURE ENHANCEMENT, AND CHANGE DETECTION

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KEY WORDS: SAR, Imagery, Structure, Extraction, Change Detection, Method, Urban

#### ABSTRACT:

In this paper we present an alternative method for SAR image denoising, structure enhancement, and change detection based on the curvelet transform. Curvelets can be denoted as a two dimensional further development of the well-known wavelets. The original image is decomposed into linear ridge-like structures, that appear in different scales (longer or shorter structures), directions (orientation of the structure) and locations. The influence of these single components on the original image is weighted by the corresponding coefficients. By means of these coefficients one has direct access to the linear structures present in the image. To suppress noise in a given SAR image weak structures indicated by low coefficients can be suppressed by setting the corresponding coefficients to zero. To enhance structures only coefficients in the scale of interest are preserved and all others are set to zero. Two same-sized images assumed even a change detection can be done in the curvelet coefficient domain. The curvelet coefficients of both images are differentiated and manipulated in order to enhance strong and to suppress small scale (pixel-wise) changes. After the inverse curvelet transform the resulting image contains only those structures, that have been chosen via the coefficient manipulation. Our approach is applied to TerraSAR-X High Resolution Spotlight images of the city of Munich. The curvelet transform turns out to be a powerful tool for image enhancement in fine-structured areas, whereas it fails in originally homogeneous areas like grassland. In the change detection context this method is very sensitive towards changes in structures instead of single pixel or large area changes. Therefore, for purely urban structures or construction sites this method provides excellent and robust results. While this approach runs without any interaction of an operator, the interpretation of the detected changes requires still much knowledge about the underlying objects.

# 1 INTRODUCTION

Nowadays spaceborne SAR data is easily available. Thanks to the high resolution of up to one meter (TerraSAR-X) it is suitable for urban applications, e.g. urban growth modeling as well as for damage mapping in conjunction with (natural) disasters. A main problem for SAR image interpretation apart from the geometrical aspect is the high noise level caused by the combination of deterministic (speckle effect) and random noise. The reduction of noise, e.g. by the multi-looking approach, often goes along with a loss of resolution. While structure preserving filters do not enhance fine-structured areas, smoothening filters even blur the structures apparent in SAR data over urban areas. So resolution and structure preserving filter algorithms are still a topic of research. In this context alternative image representations like wavelets have been applied. While wavelets are used to separate point singularities (Candès and Donoho, 1999), second generation wavelets, e.g. curvelets, are more suitable for the extraction of two dimensional features, as they are able to describe image discontinuities along a smooth line (an edge) with a minimum number of coefficients (Candès and Donoho, 1999). The elementary components are the so-called ridgelets - due to their appearance like a ridge - that can have different scales (equivalent to their length), directions and positions in the image. This enables a selection of two dimensional features to be suppressed (assumed noise) or to be emphasized (structure) by manipulating the corresponding coefficient of each ridgelet. In the following a short overview to related work especially to the development of curvelets is given. Then, the curvelet representation is roughly explained and three applications are presented: image denoising, structure enhancement and change detection over the city center of Munich (imaged by TerraSAR-X in the high resolution spotlight mode and VV polarization). So this paper shows the potential of the curvelet transform for SAR image analysis.

## 2 RELATED WORK

The curvelet transform used in this approach has originally been developed by (Candès and Donoho, 1999) to describe an object with edges with a minimal number of coefficients in the continuous space. Much research work was done to examine the behaviour of curvelets (Candès and Donoho, 2002a, Candès and Demanet, 2002b, Candès and Guo, 2002), to transfer the definitions from the continuous to the discrete space (Candès and Donoho, 2003a, Candès and Donoho, 2003b) and to accelerate the computing time (Candès et al., 2005) so that digital image processing becomes feasible. Many applications in different scientific fields have been published so far, e.g. in geo- and astrophysics, that are summarized on the curvelet homepage (Demanet, 2007).

Denoising of SAR images to simplify image analysis has also been a research topic during the last years where many approaches have been published. (Ali et al., 2007) proposed a combination of a wavelet based multi-scale representation and some filters to improve the results obtained by the "standard" filtering techniques like the Lee-filter. A bayesian-based method using "a trous" filter in the wavelet domain has been proposed by (Moghaddam et al., 2004). Because of the properties of the wavelet transform, originally developed for one dimensional data, these two methods are able to smooth regions and to suppress point-like noise, but they do not take into account the two dimensional nature of images. The advantage of second generation wavelets for despeckling has been examined by (Gleich et al., 2008) for the bandelet and the contourlet transform. The application of curvelets on optical and ultrasound images respectively in the medical context has been published by (Ma et al., 2007). The only publication on the use of curvelets in the remote sensing context by (Sveinsson and Benediktsson, 2007) presents a denoising technique with a

combination of wavelets and curvelets. A total variation based segmentation algorithm divides the image in structured regions, that are subsequently denoised by a curvelet-based method, and homogeneous regions, denoised by a wavelet approach. For large scenes with different land cover types, this method seems to be very promising. As we concentrate on urban applications in this paper, we use a purely curvelet-based approach.

Change detection in SAR images being a very difficult task has often been discussed in literature. An overview to principal SAR change detection methods, their advantages as well as their disadvantages can be found in (Polidori et al., 1995). Some more specialized methods are touched in the following. The approach of (Balz, 2004) uses a high resolution elevation model (e.g. acquired by airborne laserscanning) to simulate a SAR image which is subsequently compared to the real SAR data. The quality of the results is naturally highly dependent on the resolution of the digital elevation model and its co-registration to the SAR image. This nontrivial co-registration constraints this approach to small scale exemplary applications. Another idea starting with the fusion of several SAR images of different incidence angles to a "superresolution" image is presented by (Marcos et al., 2006) and (Romero et al., 2006). Man-made objects, i.e. geometrical particularities that are not captured by the digital terrain model used for the orthorectification of the SAR image, are classified by their diverse appearance in the single orthorectified images due to the different acquisition geometries. So, seasonal changes in natural surroundings can easily be distinguished from changes in built-up areas. One disadvantage is the large number of different SAR images of the same area needed to generate the "superresolution" image. (Wright et al., 2005) exploits the coherence (phase information) of two SAR images, which implies a relatively short repeat-pass time to avoid additional incoherence caused by natural surfaces. (Derrode et al., 2003) and (Bouyahia et al., 2008) adopt a hidden and a sliding hidden Markov chain model respectively to select areas with changes in reflectivity even from images with different incidence angles. Although this method allows to process very large images and does not need additional parameter tuning, except the window size, according to the authors still a lot of research work has to be done to improve the preliminary results.

## 3 CURVELET REPRESENTATION

The curvelet representation consists of three components according to (Candès and Donoho, 1999):

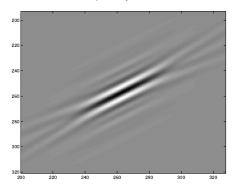
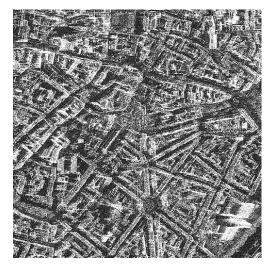


Figure 1: Ridgelet in spatial domain (Candès et al., 2005)

**Ridgelets** These two dimensional waveforms are the basic elements of the curvelet representation. In the spatial domain, they appear like a ridge or a needle (see Fig. 1); in the curvelet domain their contribution to the original image is



(a) Spatial domain



(b) Curvelet coefficients

Figure 2: City center of Munich, imaged by TerraSAR-X, High Resolution Spotlight mode, Polarisation VV, Spatially Enhanced Multi Look Ground Range Detected product

measured by a coefficient. The magnitudes of the ridgelets extracted from Fig. 2(a) are depicted in Fig. 2(b) by gray-values. Bright pixels mark high magnitudes. In contrast to wavelets, curvelets are additionally defined by their orientation in the two dimensional space (Ying et al., 2005). Hence, this is a method of image analysis suitable for image features with discontinuities across straight lines.

Multiscale ridgelets As the decomposition into ridgelets is dependent on the scale, a pyramid of windowed ridgelets is used, renormalized and transported to a wide range of scales and locations. For example, a ridgelet on the finest scale (N4-neighborhood) can only be horizontally or vertically oriented, i.e. two different orientations, while a ridgelet on the next coarser scale has already twice as much, i.e. four different orientations. Consequently, the resolution in orientation increases with coarser ridgelet scales. The number of directions is given by the formula  $2^{subband}$ . For redundancy reduction a wavelet decomposition is commonly used on the finest scale, where only horizontal and vertical directions are discriminable anyway (Candès et al., 2005). The different scales appear in Fig. 2(b) as single rings, whereas the outer rings show the finer scales. The gaps between the rings are just for visualization.

Bandpass Filtering Before the computation of the ridgelets can be done, the original image has to be separated out into a series of disjoint scales. This is done by a Laplacian pyramid which implies a high redundancy in the order of multiplying the original data volume by the factor 16 (Donoho and Duncan, 2000). The interesting thing for images with edges is, that most of these coefficients can be set to zero without loosing any structures. So, data volume reduction gets possible although the initial increase.

If one compares the original SAR image (Fig. 2(a)) to the coefficients' magnitudes (Fig. 2(b)) it is recognizable that the main axes of the city center (a cross slightly rotated clockwise to the vertical and the horizontal direction respectively) correspond in their direction with accumulations of brighter points, i.e. with higher coefficients, in the illustration of the curvelet representation. Now, the idea is to manipulate these coefficients to accent certain structures by preserving the related coefficients or to suppress certain structures by removing the related coefficients before the inverse curvelet transform is done to get the enhanced image in the spatial domain.

#### 4 IMAGE ENHANCEMENT

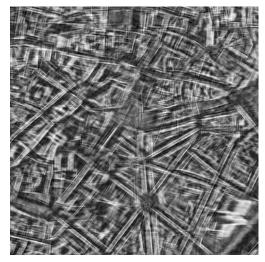
The first application presented here is image enhancement by simple noise suppression and structure extraction respectively.

#### 4.1 Image denoising

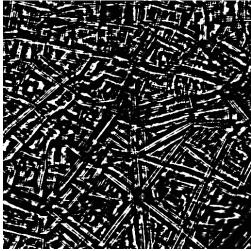
Noise is commonly associated with insignificant curvelet coefficients, therefore a thresholding can set minor coefficients to zero. One problem is that the number of coefficients preserved also corresponds to the complexity of the scene, i.e. if the number of coefficients preserved is defined as constant in advance the complexity of all scenes is seen as equal. By contrast if a magnitude threshold is chosen to exclude minor coefficients, the complexity of the scenes may vary. But in this case the mean magnitude of the coefficients, which is correlated with the contrast in the original image, is misleadingly seen as constant. So, only structures of a certain contrast would be extracted. Fig. 3(a) shows an example where a magnitude threshold of 0.1 was applied, i.e. all lower coefficients were set to zero. It is obvious that the main structures are enhanced, but also many artifacts are produced, that constrain the interpretation. Hence, the determination of a suitable threshold is a difficult task.

# 4.2 Structure enhancement

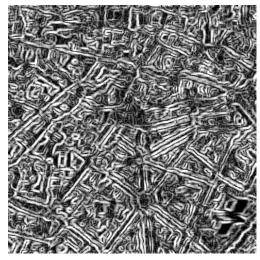
Another possibility is to access the structures via their belonging scale. The finest structures are gray value differences in a N4-neighborhood. As this scale probably only contains noise, all coefficients of this scale are set to zero. The coarsest scale influences the brightness of the image and should be kept unchanged. The scales in-between gather the remaining structures according to their length. So, it is possible to choose only those structures of a certain length to be kept and to suppress all other structures by setting the corresponding coefficients to zero. For example in Fig. 3(b) only the structures of a length from 3 to 300 m are preserved to extract structures that presumably belong to buildings. One can perceive that the main structures of the original image (Fig. 2(a)) are strengthened and all clutter is removed. At first glance the Touzi edge extractor (Fig. 3(c)) and the curvelet approach provide similar results. The lines extracted by the Touzi operator (Touzi et al., 1988) are smoother and closed, but also many lines inside the building blocks are displayed. The important difference between the two approaches is that the curvelet



(a) Reconstructed "denoised" image



(b) Structure reconstruction by curvelets



(c) Touzi edge extractor (r=4)

Figure 3: Denoising and structure extraction of Fig. 2(a)

approach only enhances the existing structures while the Touzi extractor traces discontinuities in-between dark and bright structures. Hence, a single linear bright feature on a dark background is strengthened by the curvelet approach, but it is split into two edges by the Touzi extractor.

#### 5 CHANGE DETECTION

As mentioned before SAR images are highly affected by noise. Although the influence of the deterministic speckle effect should be exactly the same under the same conditions, it is impossible to assure exactly the same conditions over a longer period of time. So, if two SAR images are differentiated pixel by pixel the result is expected to appear very noisy. Alternatively this differentiation can be calculated in the curvelet coefficient domain. If the input images are co-registered and same-sized, the images share also the same combination of curvelet coefficients. Before the difference image is transformed back to the spatial domain, the coefficient differences can be either denoised following Section 4 or weighted quadratically. In the latter case each coefficient is multiplied by its own magnitude to suppress low and to strengthen high coefficients. Additionally the influences of the different scales are equalized by the factor  $2^{subband}$  (cf. Section 3). As the resulting image contains positive as well as negative values, the positive values showing regions that brightened up are coded in green and the negative values showing regions that darkened are coded in red. For TerraSAR-X data the geolocation of the detected data product turned out to be sufficient for the change detection, so that no further co-registration was necessary.

A disadvantage of this method might be its high demand on memory. The curvelet representation itself is very redundant increasing the data volume of an image by the factor 16. Although most coefficients are nearly zero or set to zero during the image enhancement process (cf. Section 4), but they have to be processed during the differentiation as well. If more than three images are compared the difference matrix including all relative differences between the input images inflates. But the increasing number of coefficients goes along with an increasing flexibility in approximating linear features in the input image. Tests with other second order wavelets proved that critically sub-sampled approaches do not provide comparable results. To get an impression of the processing time: The example in Section 5.2 including three input images of 2091x1113 pixels are processed with a Matlab implementation and require seven minutes on a Solaris workstation.

In the following two examples over the city of Munich are presented. The first one deals with short time changes in the well-known fairground "Theresienwiese", the second one surveys construction activities near the central station over the period of one year. The processed data sets are acquired by TerraSAR-X in the High Resolution Spotlight mode and delivered as Multi Look Ground Range Detected product.

### 5.1 Short time changes

The two images of the fairground "Theresienwiese" (Fig. 4(d)) have been acquired in December 2008 and January 2009. Being processed as spatially enhanced product they have a pixel spacing of 0.5 m on ground. Because of the relatively short time lag, the reflectivity of the surrounding is expected to be the same, so all changes should be man-made. Comparing visually the two input images (Fig. 4(a) and 4(b)) one can remark a brighter area in the upper middle of Fig. 4(a) that darkened in the second image (Fig. 4(b)). Especially on the streets inside the fairground many single pixel changes are obvious. For urban applications single pixel changes do only disturb the interpretation as one is more interested in changes happened to structures like streets or buildings. So, these single pixel changes have to be excluded. Spatial averaging would help to find large areas with high changes, but fine linear structures would be smeared over and probably get lost. The curvelet approach is able to preserve the structures while single pixel changes are suppressed. In Fig. 4(c) there

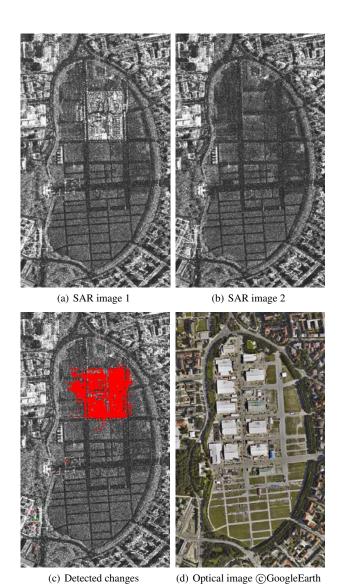


Figure 4: Change detection in the fairground "Theresienwiese" (1: 05.12.2008, 2: 18.01.2009)

is one red region in the upper middle of the image, that accords with the visual interpretation. These changes refer to the "Winter-Tollwood" festival that took place during the first acquisition. The pavilions caused a much higher reflectivity than the bare soil during the second acquisition. Additionally there are some small red and green structures at the bottom left of Fig. 4(c) that were not visible before. Those refer to buses and cars on a parking lot. The slightly darkened region in the middle right of Fig. 4(a) and 4(b) respectively is not marked as change because it does not contain any structure. In summary, the change image shows nearly no disturbances as all small scale changes are excluded. The curvelet approach is very sensitive towards structures (e.g. buses) and very robust towards slight large scale changes caused by environmental influences.

# 5.2 Long time changes

For damage mapping after natural disasters it is only seldom possible to access up-to-date reference data, as most events cannot be predicted yet. So, seasonal changes in the surrounding of the regions of interest have to be taken into account. The three images of the railway station "Donnersberger Brücke" acquired in March 2008 (Fig. 5(b)), September 2008 (Fig. 5(c)), and March 2009 (Fig. 5(d)) are used to map the construction progress inside the

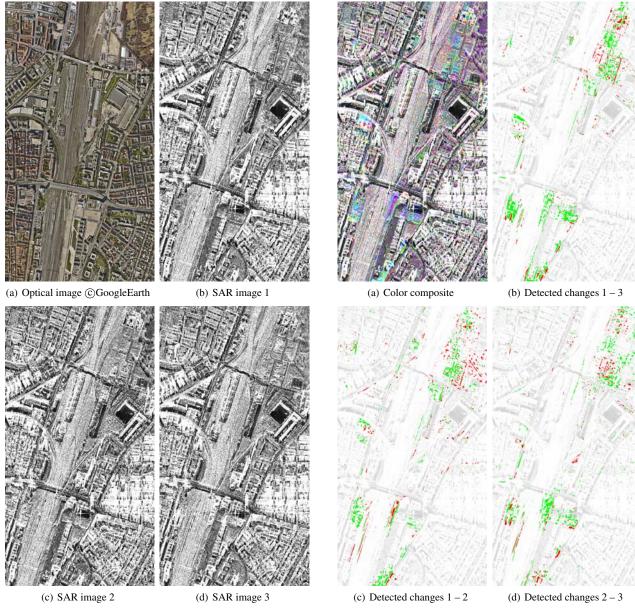


Figure 5: Construction site near "Donnersberger Brücke" (1: 30.03.2008, 2: 22.09.2008, 3: 17.03.2009)

construction sites along the railway tracks where new residential and office buildings are planned. As radiometrically enhanced products they share a pixel spacing of 1.25 m on ground. The color composite (Fig. 6(a), 1:R, 2:G, 3:B) shows many colored regions, that help to identify the construction sites. But it is still impossible to interpret these changes. Fig. 6(b) indicates the detected changes by the curvelet approach. Many green structures stand for an increase in reflectivity over the period of one year. A higher reflectivity refers to new objects, e.g. walls or houses while the darkened regions (in red) usually refer to strong scatterers that have disappeared, e.g. scaffoldings. At the bottom left there are sequences of green and red lines which can be interpreted as new buildings. One the one hand a new risen building causes a higher reflectivity (green), on the other hand it also causes new radar shadows (red). Some long green or red lines can be perceived in the middle of the image that refer to trains in the railway depots. Having a look at Fig. 6(c) and 6(d) much more small structures especially at the top right appear. Most of these are marked in red in Fig. 6(c) and in green in Fig. 6(d), so that they compensate each other over the whole year (Fig. 6(b)).

Figure 6: Change detection (cf. Fig. 5)

These changes are mainly found in the "Hirschgarten" park (see Fig. 5(a) at the top right) comparing the images acquired in spring with those acquired in fall. As these changes are restricted to natural surroundings, they supposedly refer to seasonal changes in the reflectivity by the tree's growth. The blank branches in March cause a much higher reflectivity in the co-polarized channel than the leaves in September. Again the curvelet approach produces a change image with no single pixel disturbances. Changes in the underlying structures are emphasized. Unfortunately it is a difficult task to distinguish man-made changes from seasonal changes in the natural surrounding without a high resolution land cover mask.

## 6 CONCLUSION

A new approach for SAR image enhancement and change detection based on the curvelet transform has been proposed and applied to TerraSAR-X data of the city center of Munich. As input data any amplitude image can be used, for change detection two equally sized and co-registered images are necessary. Radar

inherent noise is reduced and underlying structures are enhanced depending on their length, their orientation or their intensity.

In the image enhancement context this approach is most suitable for fine-structured areas, e.g. city centers. The main problem lies in the determination of thresholds for suppression and emphasis of structures. The determination of the threshold and the number of coefficients respectively is still experiential and highly dependent on the image content. If the scenes are reconstructed by a fix number of coefficients, the complexity of the scene is restricted. As the image description by the curvelet coefficients is purely based on structures, by omitting coefficients originally smooth areas are often affected by artifacts. At the moment the quadratic weighting of the single curvelet coefficients seems to be the best solution for fully automatic processing chains.

The change detection approach provides excellent results in urban areas. The great advantage over pixel based methods is the sensitivity towards changes in structures and the possibility to predefine the scale and the strength of changes to be mapped. Problems occur in natural surroundings like forested areas, where the status of the foliage has an important seasonal impact on the backscattering behavior. Not to mention the weather conditions, snow cover with different moistures can highly modify the appearance in a SAR image. In consequence of that the interpretation of the detected changes is very challenging. Although the change images contain clear structures without any disturbances, it is nearly impossible to distinguish man-made from natural, e.g. seasonal, changes, without a priori knowledge about the land cover.

As the present results proved that two single polarized SAR images can be used to indicate changes happened to the imaged area, but they do not provide the information needed to interpret these changes, our future research will try to include other data sources into the processing chain. To discriminate natural cover from man-made objects, a coherence layer, that exploits the phase information of the input images could be helpful. Polarimetric layers could facilitate the interpretation by attaching information about the scattering types to the detected changes. Apart from remote sensing data it is quite conceivable to introduce a priori knowledge by overlaying the change layer with land cover classifications from optical data sources as well as with cadastral data sets.

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