VARIANTS TO COMPUTE VARIANCE INFORMATION FOR MASS DATA

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ABSTRACT:

Besides the development of automatic algorithms for the registration of terrestrial laser scans, the derivation and supply of quality information is of increasing interest when collecting scan data. Quality information in terms of variances of scanned data is appropriate for multiple use as it indicates the quality of the scan result. Moreover, it is a prerequisite to use scans for many engineering tasks such as deformation analysis, etc.
From the point of view of stochastic modelling, the uncertainty measures of scanned points are not homogeneous. Their values depend on the properties of the scanner itself, on referencing, on the scanning geometry and on additional environmental portions (e. g. atmospheric conditions). All these observations influence the uncertainty of a scanned point. Therefore, variance propagation becomes very expensive, as it takes place in a very high dimensional observation space. This paper presents three alternative approaches to propagate variance information. Two versions of the unscented transformation are discussed and in addition an incremental approach is developed to reduce computational efforts. All approaches were applied to a scan of a mobile-mapping system considering uncertainties of scanning and georeferencing. The transformations consist of extensive and highly nonlinear functions. The investigations reveal the potential of the approaches regarding computational efforts and numerical results. Finally, the benefits of variance information are discussed regarding acquisition and interpretation of laser scans.

1 INTRODUCTION

The increasing data acquisition rate and the more precise measurements of current laser scanner systems allow to use this technology for many applications. As a first result, a scan is represented by the coordinates of the scanned points. This leads to a three-dimensional documentation of a scene. Moreover, the point clouds can be used to estimate parameters of a model, etc. Concerning the kinematic case when monitoring an object, a scan can be used to analyse processes like deformations. The quality of all derived information is based on the quality of the scan. To describe the geometrical characteristics, additional information has to be considered to achieve a meaningful statement concerning uncertainty.

The stochastic properties of the observations of a scan are influenced by the specifications of the used laser scanner, the referencing of the scanned points and the geometry and structure of the particular environment. The stochastic model of the observations is only valid for a single scanned point. Normally, the law of variance propagation is used to calculate the variance-covariance matrix (vcm) of the parameters. Due to the high acquisition rate, computing the resulting vcm is very expensive. This paper discusses some alternatives how to compute variance information for mass data like terrestrial laser scans. Hence, the variance information of laser scans is a basis to quantify the uncertainty of points and derived information.

Regarding the specifications of a laser scanner, many investigations have been done for calibrating and detecting systematic effects, e. g. (Schulz, 2007, Reshetyuk, 2009). They quantify the uncertainties for the observed elements in a special environment. So, different sensors become comparable. In addition, the uncertainties can be used to set up the vcm of the observations for the polar elements. Besides internal geometrical characteristics describing the polar elements of a scanner system, physical and environmental effects have impact on the vcm. (Lichti et al., 2005) emphasize the meaning of the beam width for the angular uncertainty in horizontal and vertical directions and involve it in the variance budget of the observations. Similar to photogrammetric methods, some studies determine systematic effects on-the-job (Dorninger et al., 2008). Basically, such correction parameters can be seen as observations and they can also be integrated into the vcm.

Laser scans can be referenced directly or indirectly. (Pfeifer and Briese, 2007) give an overview of the current state of art. In the indirect case, identical (artificial) points are introduced supplying the transformation parameters to an upper system. Scans from more than two stations can be referenced via bundle adjustment. Transformation parameters and the respective uncertainties are results of the adjustment and they can be considered for the variance propagation. Conversely, if a level of tolerable uncertainty for the scanned points is given previously, simulated scan stations can be used to calculate the propagated variances. So variance propagations is a condition to find an optimal configuration for the scan process and it provides the potential to reduce costs.

Direct (geo-)referencing uses additional sensors to provide the transformation parameters automatically (Vennegeerts et al., 2009). Integration of the scanner into a multi-sensor system requires the synchronisation of all involved sensors. This synchronisation can also be regarded as uncertain. In this case it has to be considered in the stochastic model. The uncertainty of the solutions of referencing sensors is not constant. This is especially true, when GNSS is included. The solutions are primarily dependent on the current constellation of available satellites. In the kinematic case states are typically filtered concerning the vcm of the solutions of the referencing sensors. Especially for moving platforms on a vehicle (e.g. mobile-mapping systems) these solutions are qualitatively very different when the time-dependent GNSS signal is shadowed. This has to be distinguished from airborne laser scanning (ALS), where referencing is based on continuously available GNSS positions.

Overall, uncertainties are quantified for the scanner system, for referencing and for the scanned environment. Integrating such influences also increases the dimension of observations and the functional terms become more lengthy. The aim of this paper is to compare algorithms and numerical results for different alternatives of variance propagation, considering variances and covariances of an high-dimensional observation space. Quantifying the vcm of a scanned point is done here one-way, but the transformation of uncertainties is a basis especially if more influences like scanning geometry, surface roughness, etc. are involved.

The paper is organised as follows: Section 2 introduces the mathematical model for the basic geometrical transformation of the scanned points. In section 3 three variants of variance propagation for mass data are presented. These approaches were applied to a mobile-mapping system (Section 4). Section 5 draws the conclusions and ends with an outlook for further studies.

2 MATHEMATICAL MODEL

To transform the scanned coordinates to an external coordinate system we start with a common approach existing of least n observations

$$\boldsymbol{l} = [(x,\, y,\, z)_p \Psi\Theta\Phi\, (d,\, \alpha,\, \theta)_s]^T. \tag{1}$$

Converting the local polar coordinates of a scan system $(d,\, \alpha,\, \theta)_s$ to a cartesian coordinate system leads to

$$\boldsymbol{x}_s = \begin{bmatrix} d \cdot \sin\theta \sin\alpha \\ d \cdot \sin\theta \cos\alpha \\ -d \cos\theta \end{bmatrix}. \tag{2}$$

The Euler angles Ψ, Θ, Φ and the position $(x,\, y,\, z)_p$ enable a transformation to an external system $\boldsymbol{x}_e$

$$\boldsymbol{x}_e = \boldsymbol{R}_s^e \cdot \boldsymbol{x}_s + \boldsymbol{x}_p. \tag{3}$$

where

$\boldsymbol{R}_s^e$ rotation matrix from scanner system to reference system, e. g. via Euler angle Ψ, Θ, Φ (DIN 9300)

$\boldsymbol{x}_p = (x,\, y,\, z)_p$ = position of a platform (static/mobile) in cartesian coordinates.

The strategies to acquire a terrestrial laser scan vary by the assumptions made. For a static scan, e.g., one supposes that the referencing elements $\boldsymbol{R}_s^e$ and $\boldsymbol{x}_p$ are constant during the scan process. In contrast, reference parameters are variable for all scanned points, if a moving platform is used.

For all observations we introduce a vcm $\boldsymbol{\Sigma}_{ll}$, providing variances and covariances. For the scanner itself the manufactures' instructions and additional investigations provide the variances of the scan parameters. Regarding the case of static scans, the vcm is a result of the (bundle) adjustment. For a moving platform, the vcm of the state and referencing parameters, respectively, can be estimated with a KALMAN filter.

3 ALTERNATIVES OF VARIANCE PROPAGATION

A common approach to obtain variances for parameters is the law of variance propagation which uses the first derivative of a function (e.g. (Koch, 1999). According to our model (Sec. 2), the Jacobian

$$\underset{3,n}{\boldsymbol{J}} = \frac{\partial \boldsymbol{x}_e}{\partial \boldsymbol{l}} \tag{4}$$

expresses the linear relation between the referenced coordinates $\boldsymbol{x}_e$ and the observations evaluated at $\boldsymbol{l}$. The corresponding covariance matrix is

$$\boldsymbol{\Sigma}_{\boldsymbol{x}_e \boldsymbol{x}_e} = \boldsymbol{J} \cdot \boldsymbol{\Sigma}_{ll} \cdot \boldsymbol{J}^T. \tag{5}$$

However, computing the Jacobian is very expensive in case of a high-dimensional observation vector as for mobile-mapping systems. Using a series of rotation matrices, an algebraic differentiation of the $n \times 3$ elements of the Jacobian leads to extensive terms. So functional relations are often simplified or the Jacobian is partitioned for a group of observations, where correlations are not considered at all. Rapid numerical solutions of derivatives are delivered by the automatic differentiation (Griewank and Walther, 2008), recently named algorithmic differentiation.

The method is based on elementary operation rules for orded pairs consisting of a value and the derivative. Any vector function can be decomposed into a sequence of this rules which leads to an excat solution for the derivatives at a particular argument. The solutions are within the computational accuracy. The algorithmic differentiation can be used to calculate the Jacobian and derivatives of higher order.

In the following we present three variants to propagate variance information for mass data: (1) the (classical) unscented transformation (UT), (2) the modified unscented transformation and (3) the incremental variance propagation.

3.1 Unscented transformation (classical)

Uncertainty can be expressed through the vcm of the observations. Regarding the GUM (Guide to the Expression of Uncertainties)(GUM, 1995), the probability density function specifies the uncertainty. GUM refers to the uniform, triangular or trapezoidal or the normal distribution function depending on additional information about the input quantities. To compute a combined uncertainty of a number of input quantities, Monte Carlo methods provide a solution via a deterministic approximation of any density function and an arbitrary function (Koch, 2008). The disadvantage is the high computational cost due to the randomly generated density function.

Under the assumption of a symmetrical density function, (Julier and Uhlmann, 1997) introduced so-called sigma points for approximation. The fundamental idea is to approximate the distribution function instead of the (nonlinear) function. This is the difference to the law of variance propagation (Eq. 5), where functions are linearised. Instead of a randomly generated distribution function as applied in Monte Carlo methods, the UT uses a deterministic algorithm and only $2n + 1$ sigma points to represent a n-dimensional multivariate distribution function of the observations

$$\begin{aligned} \boldsymbol{\mathcal{L}}^{(0)} &= \boldsymbol{l} \\ \boldsymbol{\mathcal{L}}^{(i)} &= \boldsymbol{l} + \boldsymbol{a}^{(i)} \\ \boldsymbol{\mathcal{L}}^{(i+n)} &= \boldsymbol{l} - \boldsymbol{a}^{(i)} \end{aligned} \tag{6}$$

where $i = 1...n$. Here the sigma points are denoted with $\boldsymbol{\mathcal{L}}$ to emphasize the relation to the observation vector $\boldsymbol{l}$. The vector

$\boldsymbol{a}^{(i)}$ is the i-row of the matrix $\boldsymbol{A}$, defined using the matrix square root of $(n+\kappa)\boldsymbol{\Sigma}_{ll}$. It can be computed with the Cholesky decomposition

$$\boldsymbol{A}\cdot\boldsymbol{A}^T = (n+\kappa)\boldsymbol{\Sigma}_{ll}. \quad (7)$$

$\kappa \in \Re$ scales higher orders terms for a better approximation of the distribution. (Julier and Uhlmann, 1997) suggest $\kappa = n-3$, when $\boldsymbol{l}$ is Gaussian. Furthermore, weight factors are introduced to emphasize the loading of the observation $\boldsymbol{l}$

$$\begin{array}{rcl} w^{(0)} & = & \kappa/(n+\kappa) \\ w^{(i)} & = & 1/2(n+\kappa) \\ w^{(i+n)} & = & 1/2(n+\kappa) \end{array}. \quad (8)$$

Each sigma point $\boldsymbol{\mathcal{L}}$ has to be transformed to a point $\boldsymbol{\mathcal{X}}$ in the cartesian system according to the function

$$\boldsymbol{\mathcal{X}} = \boldsymbol{f}(\boldsymbol{\mathcal{L}}). \quad (9)$$

The mean of the weighted average of all transformed sigma points

$$\overline{\boldsymbol{x}} = \sum_{i=0}^{2n} w^{(i)}\boldsymbol{\mathcal{X}}^{(i)} \quad (10)$$

represents an unbiased estimation of $\boldsymbol{x}$ with the corresponding covariance matrix

$$\boldsymbol{\Sigma}_{\overline{\boldsymbol{x}}\overline{\boldsymbol{x}}} = \sum_{i=0}^{2n} w^{(i)}(\boldsymbol{\mathcal{X}}^{(i)}-\overline{\boldsymbol{x}})(\boldsymbol{\mathcal{X}}^{(i)}-\overline{\boldsymbol{x}})^T. \quad (11)$$

As mentioned before, $2n+1$ sigma points have to be transformed and the vcm can be estimated without evaluating the Jacobians. Further studies aim to reduce the number of sigma points (Julier, 2003). A recent one (Li et al., 2007) provides a solution with less computational effort. It is discussed in Section 3.2.

3.2 Modified unscented transformation

The progress to develop modifications of the UT is driven by KALMAN filtering for nonlinear systems. To process positions real-time for sensors with a high acquisition rate, many investigations have been done to minimise computational effort. The algorithms used for the variance propagation of the predicted state are general and can be applied to any kind of (nonlinear) transformation.

(Li et al., 2007) present a set of sigma points, which is unbiased for mean and variance and consists of only $n+1$ points. The matrix

$$\underset{n,n+1}{\boldsymbol{D}} = \sqrt{n+1}\left[\underbrace{\frac{1}{\sqrt{j(j+1)}}\cdots\frac{1}{\sqrt{j(j+1)}}}_{j=2\ldots n} - \frac{1}{\sqrt{(j+1)/j}}\ \underbrace{0\ldots 0}_{n-j}\right] \quad (12)$$

(here one of n rows is shown) scales the distances to the observations $\boldsymbol{l}$.

Like in Section 3.1, the vcm is considered applying $\boldsymbol{A}$ (see Eq. 7). In matrix representation this leads to the sigma points

$$\underset{n+1,n}{\boldsymbol{\mathcal{L}}} = \boldsymbol{1}^T \otimes \boldsymbol{l} + \boldsymbol{A}\cdot\boldsymbol{D} \quad (13)$$

where $\otimes$ denotes the Kronecker product. The elements of the vector $\boldsymbol{1} = [1...1]^T$ are equal to 1. The weight factors $\boldsymbol{w}$ are introduced with identical values

$$\boldsymbol{w} = [w^{(i)}\, w^{(i)} ...\, w^{(i)}]^T \text{ with } w^{(i)} = \frac{1}{n+1}\ \forall\, i = 1...n+1 \quad (14)$$

and

$$\boldsymbol{\Lambda} = diag(w^{(i)}) = \frac{1}{n+1}\boldsymbol{I}_{n+1}. \quad (15)$$

After transforming each sigma point

$$\boldsymbol{\mathcal{X}}^{(j)} = \boldsymbol{f}(\boldsymbol{\mathcal{L}}^{(j)}) \quad (16)$$

($\boldsymbol{\mathcal{L}}^{(j)}$ is the j-th row vector of $\boldsymbol{\mathcal{L}}$ for $j = 1...n+1$) the weighted mean and the covariance matrix are given by

$$\underset{n,1}{\overline{\boldsymbol{x}}} = \underset{n,n+1}{\boldsymbol{\mathcal{X}}} \cdot \underset{n+1,1}{\boldsymbol{w}} \quad (17)$$

and

$$\boldsymbol{\Sigma}_{\overline{\boldsymbol{x}}\overline{\boldsymbol{x}}} = \left(\boldsymbol{\mathcal{X}} - \boldsymbol{1}^T\otimes\overline{\boldsymbol{x}}\right)\Lambda\left(\boldsymbol{\mathcal{X}} - \boldsymbol{1}^T\otimes\overline{\boldsymbol{x}}\right)^T \quad (18)$$

(in analogy to Eqs. 10, 11). The two variants of the UT in Section 3.1 and 3.2 propagate variances with an deterministic approximation of the density function. The law of variance propagation (Eq. 5) is using the Jacobian as linearised representation of a function. In case of a highly nonlinear function, computing the Jacobian is expensive. The approach in the following Section 3.3 reduces the computational costs.

3.3 Incremental variance propagation

As a third variant to compute variance information we introduce an incremental one, which is based on the law of variance propagation (Eq. 5) and on extrapolation. Usually the Jacobian $\boldsymbol{J}$ is calculated for each observation $\boldsymbol{l}$, even if transformations of adjacent observations captured by high-resolution sensors are similar. Instead of computing a Jacobian for each observation we evaluate $\boldsymbol{J}$ in a series expansion. As known from Taylor series, a scalar function of one scalar variable $x = f(l)$ can be evaluated at the point $l^{(i)}$ with

$$x \approx f(l^{(i)}) + \frac{\partial f}{\partial l}|_{l^{(i)}} \cdot (l - l^{(i)}) + ... , \quad (19)$$

which is a linear approximation of $f(l)$ near $l^{(i)}$. For an observation vector $\boldsymbol{l}$ in the neighbourhood of $\boldsymbol{l}^{(i)}$ and a vector of functions $\boldsymbol{f}$ the Jacobian

$$\boldsymbol{J} := \frac{\partial \boldsymbol{f}}{\partial \boldsymbol{l}}|_{\boldsymbol{l}} \quad (20)$$

at $\boldsymbol{l}$ can be evaluated in a series, too:

$$\boldsymbol{J}(\boldsymbol{l}) \approx \boldsymbol{J}^{(i)} + \frac{\partial^2 \boldsymbol{f}}{\partial \boldsymbol{l}^2}|_{\boldsymbol{l}^{(i)}} \cdot (\boldsymbol{l} - \boldsymbol{l}^{(i)}) + ... \quad (21)$$

where the second derivative of $\boldsymbol{f}$ is the Hessian

$$\boldsymbol{H} := \frac{\partial^2 \boldsymbol{f}}{\partial \boldsymbol{l}^2}|_{\boldsymbol{l}^{(i)}}. \quad (22)$$

In case of a vector function $\boldsymbol{f}$, consisting of u functions for n observations of $\boldsymbol{l}$, the Hessian is a three dimensional matrix

$$\dim(\boldsymbol{H}) = u \times n \times n \quad (23)$$

and the approximated Jacobian at $\boldsymbol{l}$, evaluated at $\boldsymbol{l}^{(i)}$ is

$$\underset{u,n}{\boldsymbol{J}}(\underset{n,1}{\boldsymbol{l}}) = \underset{u,n}{\boldsymbol{J}}(\underset{n,1}{\boldsymbol{l}^{(i)}}) + \underset{u,n,n}{\boldsymbol{H}}(\underset{n,1}{\boldsymbol{l}^{(i)}}).*(\underset{n,1}{\boldsymbol{l}} - \underset{n,1}{\boldsymbol{l}^{(i)}}) \quad (24)$$

where the operator $.*$ denotes a "layer-wise" product of the submatrices $\underset{j,n,n}{\boldsymbol{H}}$ for all $j = 1...u$. This Jacobian can be used to propagate the variance regarding Eq. 5.

Note, although terms of higher order are involved, the incremental approach is different from the nonlinear variance propagation

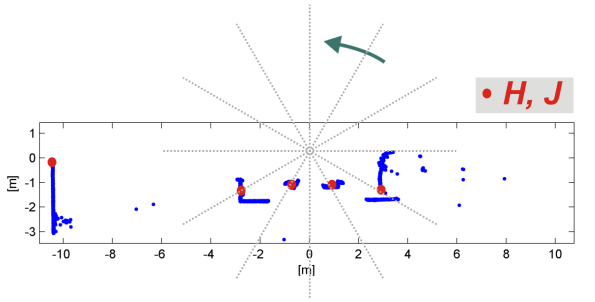

Figure 1: Scanned points in a profile divided in 12 circle segments. At the first point $\boldsymbol{l}^{(i)}$ of a segment relative to the scan direction the Jacobian $\boldsymbol{J}$ and the Hessian $\boldsymbol{H}$ is computed. The Jacobians of the following scanned points (observations $\boldsymbol{l}$) are approximated according to Eq. 24.

(Grafarend and Schaffrin, 1993), where the Hessian is introduced for a better approximation of a function. The incremental variance propagation is a linear propagation with a Jacobian approximated for adjacent observations.

The answer to the question where to set the point $\boldsymbol{l}^{(i)}$ depends on computational effort and precision caused by linearization. In case of sensors acquiring data sequentially, spatial and temporal vicinity should be given. For terrestrial laser scanning we applied a solution referring to the deflection unit. For a scanned profile the points $\boldsymbol{l}^{(i)}$ can be set for k equal circle segments. In Fig. 1 $k =$ 12 segments are introduced. The figure shows points scanned in a profile of a mobile-mapping system. According to the rotation direction of the deflection unit, for the first point in a segment the Jacobian and the Hessian are calculated. The Jacobians for the next points in the segment are evaluated according to Eq. 24. The concept ensures a spatial neighborhood between $\boldsymbol{l}^{(i)}$ and $\boldsymbol{l}$, which is necessary due to the linearization.

4 APPLICATION EXAMPLE: MOBILE-MAPPING SYSTEM

Because of the permanent movement, in mobile-mapping systems each scanned point has to be referenced on its own. This has be taken into account, when setting up the vcm of the observations. Furthermore, referencing via an integration of navigation units (IMU, GNSS, etc.) needs several rotations to transform a scanned point to an earth-fixed geodetic system. Considering rotation and translation parameters as observations leads to an observation vector

$$\boldsymbol{l} = [(xyz)_b\,(\phi\lambda)_b\,(\Psi\Theta\Phi)\,(\Delta xyz)_{s2b}\,(r_{xyz})_{s2b}\,(r_z)_{s2s}\,(d,\theta)_s]^T \tag{25}$$

where

$(xyz)_b$	cartesian position of the body system (ECEF, earth-centered-earth-fixed)
$(\phi\lambda)_b$	longitude, latitude of the body system
$(\Psi\Theta\Phi)$	euler angle (DIN 9300)
$(\Delta xyz)_{s2b}$	translation vector scan-system to body-system (leverarm)
$(r_{xyz})_{s2b}$	rotation angles from scan-system to body-system (boresight)
$(r_z)_{s2s}$	rotation angle around the z-axis (when using a terrestrial 3d-scanner)
$(d,\theta)_s$	polar coordinates of scanned point.

In addition to Eq. 3 a series of transformations is necessary to transfer scanned points $(d,\theta)_s$ to an earth-fixed system $\boldsymbol{x}_e$. Implemented transformations and filters are described in detail in, e.g., (Vennegeerts et al., 2008). They are briefly repeated here to keep the paper self-contained.

$$\boldsymbol{x}_e = \boldsymbol{R}_{n2e}\boldsymbol{R}_{b2n}(\boldsymbol{R}_{s2b}\boldsymbol{R}_{s2s}(d,\theta)_s + \boldsymbol{\Delta x}_{s2b}) + \boldsymbol{x}_b \tag{26}$$

where is

$\boldsymbol{R}_{n2e}$	rotation matrix: navigation- to earth-centered-earth-fixed-system (ECEF)
$\boldsymbol{R}_{b2n}$	rotation matrix: body- to navigation-system
$\boldsymbol{R}_{s2b}$	rotation matrix: scanner- to body-system
$\boldsymbol{R}_{s2s}$	rotation matrix: about the z-axis of scanner (in case of using a terrestrial 3d-scanner)
$\boldsymbol{\Delta x}_{s2b}$	translation vector: scanner- to body-system
$\boldsymbol{x}_b$	position body (navigation unit) in earth-centered-earth-fixed-system.

The calibration parameters of the system $(\Delta xyz)_{s2b}$ and $(r_{xyz})_{s2b}$ are determined by photogrammetrically determined transformation points. Tab. 1 shows the components of the vcm for the calibration of the system including a mounted laser scanner (Z+F Imager 5003). Distance-related values are specified for a typical range around 10 m. The components of the vcm quantified in Tab 1 include no correlations.

	Parameter	σ
System (scanner to body system)	Translation $(\Delta xyz)_{s2b}$ Rotation $(r_{xyz})_{s2b}$	5 mm 0.01 deg
Z+F Imager 5003	Range d Vertical angle θ	1 mm + 30 ppm 0.01 deg

Table 1: Uncertainties for the vcm of the observations

Uncertainties of the referencing parameters - variances and covariances - were taken from the KALMAN filtered states. We applied all three alternatives to propagate variance information of scanned points in the referenced system, consisting of the vcm $\boldsymbol{\Sigma}_{\boldsymbol{xx}e}$. Derivatives needed for the incremental (Sec. 3.3) and standard (Eq. 5) approach were calculated by algorithmic differentiation (Griewank and Walther, 2008).

To compare the computational time, Fig. 2 shows how many points were propagated in one minute. The results were obtained on a Desktop PC (Core 2 Quad at 2.4 GHz, 3 GB RAM, 32-bit). Note, computation time includes variances of all parameters and variances and covariances of the referencing parameters (Eq. 25).

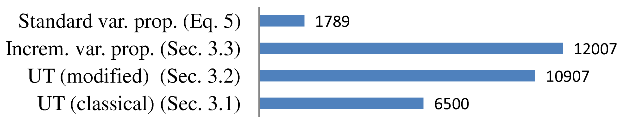

Figure 2: Number of vcms of scanned points propagated in one minute

Therefore, using the modified unscented transformation leads to an improvement in computation time of 68% compared to the classical one. This can be expected approximately, because the modified variants needs $n + 1$ sigma points versus $2n + 1$ points of the classical version. The fastest way to propagate variances for this application is the incremental approach. The computation time is comparable to the modified unscented transformation and about 85% faster than the classical unscented transformation.

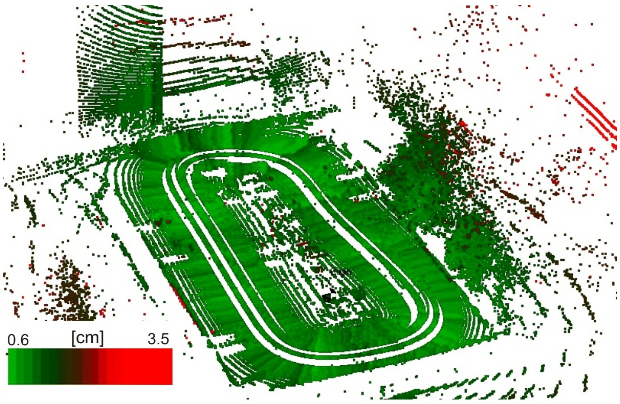

Figure 3: Stochastic point cloud with points colored related to the mean coordinate uncertainty $\bar{\sigma}$ (Eq. 27)

To compare the propagated vcms we calculated a mean coordinate uncertainty

$$\bar{\sigma} = \sqrt{\frac{trace\left(\boldsymbol{\Sigma}_{\boldsymbol{xx}e}\right)}{3}} \qquad (27)$$

for each referenced point. The results of a test drive can be seen in Fig. 3, where points are colored regarding the mean coordinate uncertainty.

The distribution of the uncertainties seems to be quite homogeneous, revealing the high quality of the used IMU: the mean coordinate uncertainty is fairly independent from the scanning range pointing to the low uncertainty of the orientation angles. Obviously, the uncertainty is dominated by the solutions for the kinematic GNSS positions. This part has to be investigated in future studies.

In the following the numerical results of the modified unscented transformation and the incremental approach are compared to the classical unscented transformation. Due to the number of sigma points, the classical unscented transformation can preferably be seen as a kind of reference. A histogram of the differences between the uncertainties (Eq. 27) of the two unscented transformations is shown in Fig. 4.

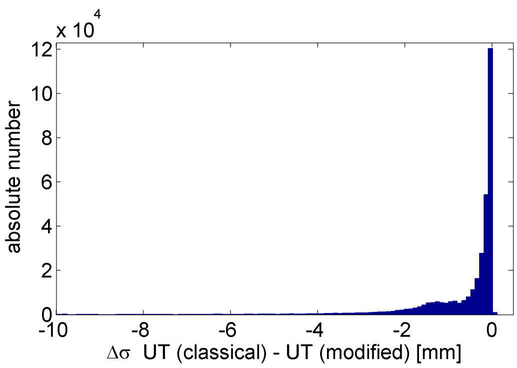

Figure 4: Differences of $\bar{\sigma}$ UT (classical) - UT(modified)

All differences are negative, so that the uncertainties from the modified approach are always greater. For a very few points absolute values are greater than 5 mm, which might be caused by functional discontinuities. The quality of transformation of a distribution and thus the variance propagation depends on how the sigma points represent the distributions. Less sigma points approximate a distribution less precisely. So minimizing the number of sigma points can become crucial for highly nonlinear transformations. Overall, the differences lie within a range of 1-2 mm, which is acceptable for many applications.

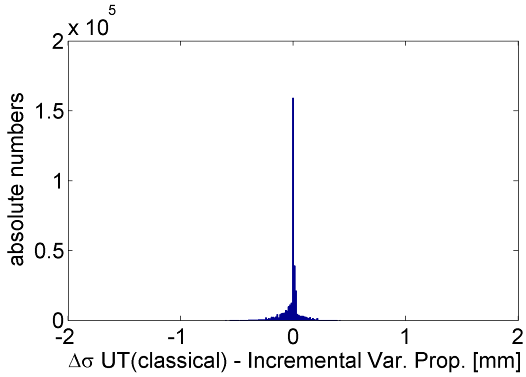

Figure 5: Differences of $\bar{\sigma}$ UT (classical) - Incremental variance propagation

Comparing the results of the incremental approach with the classical unscented transformation leads to Fig. 5. The differences are distributed around zero, while the absolute values are negligible. Regarding the present application, the incremental approach delivers acceptable results spending a fraction of time compared to the standard variance propagation, calculating a Jacobian for each point (Fig. 2). Only 15% of computation time is necessary to compute a Jacobian for one point.

Certainly, in case of the incremental approach, the effort depends on the linearization strategy - here on the number of segments. Generally, in a polar measuring system the uncertainty transverse to the observation direction arises with the distance. Higher order terms like the Hessian show how changes of the observations effect the Jacobian (c.f. Eq. 24). The linearization succeeds as better as smaller the changes of the distances in the segment of a scene in relation to the entire distance are. The higher the resolution of a sensor the more reduces the incremental approach computation effort in relation to a pointwise calculation of the Jacobian.

5 CONCLUSIONS AND OUTLOOK

Variance information of scanned points provides an opportunity to improve processes of collecting and analysing scan data. (Schaer et al., 2007) present some options for airborne laser scanning. In the terrestrial case, this information is useful to increase efficiency of registration and referencing. Regarding scanning from a static platform, the number and locations of the scan stations are often chosen on the condition to scan an object completely with a minimum number of scan stations. In other applications, especially for engineering tasks, the condition is to scan an object with a given maximum tolerable uncertainty and a minimum number of stations. Today, the decisions are made generally based on practical knowledge. If the configuration of the scan station is simulated previously, propagating variances offers options to optimise the scan process. Regarding the registration and adjustment of a set of single scans, the observations are usually weighted equally. A sophisticated loading based on variance information can lead to more precise results.

As mentioned before, in case of direct (geo-)referencing using the GNSS, the quality of the positions varies. This has impact on the geometric uncertainty of the scanned points. Especially in areas which are shadowed from the GNSS signals variance information quantifies the resulting geometric scan quality. This is particularly meaningful for design purposes if, e.g., high-end

IMUs are substituted for low-cost micro-electromechanical systems (MEMS).

Finally, variance information does not only indicate the quality of a scan process. It is an important component for the process of interpretation and analysing. In case of determining parameters of a model and relating stochastics, reliable input quantities are needed to get meaningful estimated parameters.

The short scanning time of current laser scanners is ideally suited to scan objects multi-temporally. These data are often acquired from variable positions and different scanners. To compare the states, the distinct conditions have to be taken into account, if changes are to be evaluated stochastically. The additional information enhances the flexibility to process multi-temporal and multi-scaled laser scans.

Future work aims to develop and investigate approaches of the unscented transformation and modifications of the incremental propagation to minimise the number of points where to calculate the Jacobian and Hessian. Besides the influences of scanner and referencing, further studies should include the scanning geometry. Therefore, an integrated mathematical model has to be set up including geometric parameters such as the angle of incident, etc.

To consider more influences, the observation space has to be enlarged. Thus, efficient variance propagation is necessary to reduce the increasing demands on computation - and to provide a basis for a qualitative processing of laser scanning data.

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