TOTAL LEAST SQUARES REGISTRATION OF IMAGES FOR CHANGE DETECTION

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ABSTRACT:

The first step of detecting changes between at least two temporal images is geometrically registering two images. After the registration had been performed, then the differences of the images are simply found by subtraction techniques. On the other side, registration task is performed in two steps. At first corresponding features from individual datasets are found with a search strategy based on image matching. Secondly, the datasets are geometrically aligned by using the found corresponding point sets. In order to register images there are various registration techniques. One of the effective matching technique is adaptive least squares matching (ALSM) technique, which is based on the minimization of the radiometric differences of the source and target image patches. The minimization of the radiometric differences, namely the errors between source and target patches are performed with the least squares (LS) adjustment technique. ALSM technique assumes that the source image patch is error free while the target is assumed to be erroneous. However, the source patch should also involve errors. When the errors of the source patch are ignored, these ignored errors reside as uncertainty in the model and thus this situation theoretically must affect the estimation results negatively. When TLS is used, both source and target patches are all assumed erroneous and thus the errors of the data matrix are also taken into account so that it removes uncertainties. We give the first results and discussions of our work on TLS matching.

1. INTRODUCTION

Detection of changes of a region by using the images of that region is a task which is used for various applications. From remote sensing (Collins et al., 2000; Wren et al., 1997) to video imaging (Huertas et al., 2000; Zhong et al., 2006); from medical applications (Bosc et al, 2003; Dogan and Altan, 2003) to industrial applications (Fang et al, 2003), change detection is videly being used. One of the important application area is map updating, (Vosselman et al. 2004; Champion et al.; 2009). The update process of a map begins with change detection. A comparison should be made to detect the changes between two or more datasets. Datasets should be aligned geometrically to find the corresponding features to be compared. This correspondence problem is regarded as registration, (Pottmann, 2006). After the registration of the datasets, the temporal changes can easily be found with a comparison technique. If both datasets are images, then the comparison can be performed for example with simple image subtraction techniques, (Radke et al., 2005).

Registration is the process of determining the point by point correspondence between two or more datasets, (Pottmann, 2006). Registration has two main steps. At the first step, corresponding features from individual datasets are found with a search strategy and in the second step, two datasets are geometrically aligned in space with transformation functions whose parameters are estimated with corresponding points, (Besl and McKay, 1992; Kaneko et al., 2003; Goshtasby, 2005; Rusinkiewicz and Levoy, 2001). One of the most frequently used registration technique is iterative closest point transform (ICP), (Besl and McKay, 1992). In different variants of ICP, only the similarity measure metrics, search strategies and transformation function parameters are changed, (Rusinkiewicz and Levoy, 2001). In spite of the ICP algorithm, there are other registration and matching techniques too. For example, Lucas-Kanade registration, (Lucas and Kanade, 1981; adaptive least squares matching, (Gruen, 1985), adaptive least squares surface matching, (Gruen and Akca, 2005) etc.

In this paper, we motivate on the TLS solution of the adaptive least squares matching (ALSM) technique (Gruen, 1985) for registration. ALSM is performed with LS estimation method. This matching model assumes that the source patch is error free while the target patch is erroneous. These ignored errors reside as uncertainty in the model. However, the source patch should also involve errors. To take in to account for those source patch errors, we investigate TLS estimation model. In the following section, we give the mathematical model of ALSM matching briefly. TLS estimation is explained in chapter 3. In chapter 4, we give some discussion for comparison of the results of TLS and LS.

2. ADAPTIVE LEAST SQUARES IMAGE MATCHING

The basic approach of ALSM model assumes that the source and target patches are two dimensional discrete functions expressed by f(x,y) and g(x,y) respectively (Gruen, 1985). The ideal correlation is established if

$$f(x,y) = g(x,y) \tag{1}$$

Because of random errors in both images, equation (1) is inconsistent (Gruen, 1985). But however, template patch

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(template image) is assumed to be error free in the estimation model for simplification. To provide consistency, an error vector e(x,y) is added, resulting in,

$$f(x,y) - e(x,y) = g(x,y) \quad (2)$$

Now the goal is to minimize the error vector e(x,y) which corresponds to radiometric differences of the error free template and erroneous target patches.

Since LS is a linear estimation model, equation (2) is linearized and fully affine image shaping parameters are involved into model. The resulting linear model is obtained as follows (Gruen, 1985).

$$\begin{aligned} f(x,y) - e(x,y) = g^0(x,y) + g_x\, da_{11} + g_x\, x_0\, da_{12} + \\ g_x\, y_0\, da_{21} + g_y\, db_{11} + g_y\, x_0\, db_{12} + \\ g_y\, y_0\, db_{21} + r_s \end{aligned} \quad (3)$$

where, $g_x = \dfrac{\partial g^0(x,y)}{\partial x}$ and $g_y = \dfrac{\partial g^0(x,y)}{\partial y}$

In equation (3), da_{11}, da_{12}, da_{21}, db_{11}, db_{12}, db_{21} are six affine parameters and r_s is radiometric shift parameter. Above equation can be restated by adding a radiometric scale parameter r_t, such that f(x,y)-e(x,y) = r_t g(x,y) + r_s. Now the equation (3) can be written in matrix form as **A x** = **b** – **e** where **A** is data (design) matrix, **x** is unknown vector and **b** is observation vector. The LS solution is obtained as

$$x = (A^T P A)^{-1} A^T P b \quad (4)$$

where **P** is weight matrix.

In the next chapter, we give TLS estimation of the linear model given by equation (3) and in chapter (4) give some comparison results.

3. TOTAL LEAST SQUARES ESTIMATION

In TLS, both the observation vector and the elements of data matrix (design matrix) are assumed to be erroneous, (Golub and Loan, 1980; Akyilmaz, 2007; Huffel and Vandewalle, 1991; Golub and Reinsch, 1970; Golub, 1973). If all of the elements of the data matrix are erroneous, this is called as basic TLS. If some columns of the data matrix are constant and thus error free, then this situation is called as mixed TLS. In the following two subsections these two types are given briefly.

3.1 The Basic TLS Problem

The basic TLS model assumes that all of the elements of the data matrix **A** are erroneous. This situation is mathematically expressed as

$$\boldsymbol{b} + \Delta\boldsymbol{b} = (A + \Delta A)\,x\,,\ rank\,(A) = m < n \quad (5)$$

where, Δ**b** is error vector of observations and Δ**A** is error matrix of data matrix **A**. Both errors are assumed independently and identically distributed with zero mean and with same variance. In this case, the estimation procedure is an optimization problem given by

$$minimize \left\| [A;b] - [\hat{A};\hat{b}] \right\|_F \quad , \quad [\hat{A};\hat{b}] \in R^{n\times(m+1)}$$
$$subject\ to: \boldsymbol{b} + \Delta\boldsymbol{b} = (\boldsymbol{A} + \Delta\boldsymbol{A})\boldsymbol{x} \quad (6)$$

where, m is number of unknowns an n is number of observations. Once a minimizing $[\hat{A};\hat{b}]$ is found then any **x** satisfying $\hat{A}\,x = \hat{b}$ is called TLS solution and $[\Delta\hat{A}\,;\Delta\hat{b}] = [A;b] - [\hat{A};\hat{b}]$ is corresponding TLS correction, (Golub and Loan, 1980; Akyilmaz, 2007; Huffel and Vandewalle, 1991; Golub and Reinsch, 1970; Golub, 1973). In the above equation, $\|\dots\|_F$ denotes the Frobenius norm.

In order to solve the basic TLS problem given by the equation (6), SVD can be used, as pointed in (Golub,1973) and more fully in (Golub and Loan, 1980) and (Huffel and Vandewalle, 1991). For solution of the **Ax** ≈ **b**, we write this functional relation as follows:

$$[A;b]\,[x^T;\,-1]^T \approx 0 \quad (7)$$

Now, SVD of the augmented matrix [**A**; **b**] is computed as follows:

$$[A;b] = U\,\Sigma\,V^T \quad (8)$$

where, **U**= [$\mathbf{U_1}$; $\mathbf{U_2}$], $\mathbf{U_1}$ = [$\mathbf{u_1}$, ... , $\mathbf{u_m}$], $\mathbf{U_2}$ = [$\mathbf{u_{m+1}}$, ... , $\mathbf{u_n}$] and $\mathbf{u_i}$ ∈ R^n, $\mathbf{U^T}$ **U** = $\mathbf{I_n}$. **V** = [$\mathbf{v_1}$, ... , $\mathbf{v_m}$, $\mathbf{v_{m+1}}$], $\mathbf{v_i}$ ∈ R^{m+1}, $\mathbf{V^T}$ **V** = $\mathbf{I_{m+1}}$. **Σ** = diag(σ_1, ... , σ_m, σ_{m+1}) ∈ $R^{n\times(m+1)}$.

In the equation (8), the rank of the matrix [**A**; **b**] is m+1. The rank must be reduced to m. For this purpose, Eckart-Young Minsky theorem is used, (Eckart and Young, 1936). After the rank reduction, the solution of the basic TLS is obtained by,

$$[x^T;-1]^T = \frac{-1}{v_{m+1,m+1}}\, v_{m+1} \quad (9)$$

If $\mathbf{v_{m+1,m+1}} \neq 0$, then $\hat{\mathbf{b}} = \hat{\mathbf{A}}\mathbf{x}$ = -1/ ($\mathbf{v_{m+1,m+1}}$) $\hat{\mathbf{A}}$ [$\mathbf{v_{1,\,m+1}}$, ... , $\mathbf{v_{m,\,m+1}}$ $]^T$ and this belongs to column space of $\hat{\mathbf{A}}$, and hence **x** solves the basic TLS problem, (Huffel and Vandewalle, 1991).

3.2 The Mixed LS-TLS Problem

In the mixed TLS model, one or more columns of the data matrix **A** is/are really error free. For example in matching problem, the column which corresponds to the radiometric shift parameter is constant and thus error free. In such cases, TLS solution must be obtained with the mixed model.

Mixed LS-TLS means a mixture of LS and TLS. Here it can be thought that, fixed or error free columns of the data matrix **A** and their corresponding unknown parameters are just like in the case of LS. On the other hand, erroneous columns of **A** with their corresponding unknown parameters are just in the form of basic TLS. Then the problem may be considered as the mixture of LS and basic TLS. According to this approach, the first step of the formulation begins by separating fixed columns of **A** and their corresponding **x** components from the rest of the erroneous columns of **A** and rows of **x** respectively. This procedure can be defined as follows:

Given a set of n linear equations in m unknown vector **x**, such that: (Golub and Loan, 1980; Huffel and Vandewalle, 1991; Akyilmaz, 2007).

$$A \in R^{n\times m}, b \in R^{n} \text{ and } x \in R^{m} \tag{10}$$

Partition data matrix **A** and **x** vector as follows:

$$\begin{aligned} A &= [A_1;\ A_2], \quad A_1 \in R^{n\times m_1} \text{ and } A_2 \in R^{n\times(m-m_1)} \\ x &= [x_1^T;\ x_2^T]^T, \quad x_1 \in R^{m_1} \text{ and } x_2 \in R^{(m-m_1)} \end{aligned} \tag{11}$$

where m_1 is the number of fixed columns. Then mixed LS-TLS problem seeks to

$$\begin{aligned} & \text{minimize} \left\| [A_2; b] - [\hat{A}_2;\ \hat{b}] \right\|_F = \| [\Delta A_2;\ \Delta b] \|_F , \\ & [A_2; b] \in R^{n\times(m-m_1+1)} \end{aligned} \tag{12}$$

$$\text{subject to: } b + \Delta b = [A_1;\ A_2] \begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \end{bmatrix}$$

once a minimizing $[\hat{\mathbf{A}}_2;\ \hat{\mathbf{b}}]$ is found, then any $\mathbf{x} = [\mathbf{x}_1^T; \mathbf{x}_2^T]^T$ satisfying $\hat{\mathbf{A}}\,\mathbf{x} = \mathbf{A}_1\,\mathbf{x}_1 + \hat{\mathbf{A}}_2\,\mathbf{x}_2 = \hat{\mathbf{b}}$ is called "mixed LS-TLS solution" and $[\Delta\mathbf{A}_2;\ \Delta\mathbf{b}] = [\mathbf{A}_2; \mathbf{b}] - [\hat{\mathbf{A}}_2;\ \hat{\mathbf{b}}]$ is the corresponding mixed LS-TLS correction, (Huffel and Vandewalle, 1991).

Solution of equation (12) begins with computing a QR factorization of the "known" columns and then solving the TLS problem of the reduced dimension. Let matrix **A**=[$\mathbf{A}_1$; $\mathbf{A}_2$] be given whose first m_1 columns $\mathbf{A}_1$ have no error and have full column rank. Then the algorithm is as follows:

Perform m_1 Householder transformations Q on the matrix [**A**; **b**] so that,

$$Q^T [A_1;\ A_2;\ b] = \begin{bmatrix} R_{11} & R_{12} & R_{1b} \\ 0 & R_{22} & R_{2b} \end{bmatrix} \tag{13}$$

where $\mathbf{R}_{11}$ is ($m_1\times m_1$) upper triangular matrix. $\mathbf{R}_{12}$ is ($m_1\times(m-m_1)$) matrix, $\mathbf{R}_{22}$ is ($(n-m_1)\times(n-m_1)$) matrix, $\mathbf{R}_{1b}$ is ($m_1\times 1$) matrix and $\mathbf{R}_{2b}$ is ($(n-m_1)\times 1$) matrix. (Akyilmaz, 2007; Huffel and Vandewalle, 1991).

Now in order to solve mixed LS-TLS problem, at first $\mathbf{x}_2$ is computed by

$$R_{22}\, x_2 = R_{2b} \tag{14}$$

To solve equation (14), the basic TLS solution given by equation (9) is used. Later, $\mathbf{x}_1$ is computed by back substitution of the estimated $\mathbf{x}_2$ in the following equations;

$$R_{11}\, x_1 = R_{1b} - R_{12}\, x_2 \tag{15}$$

and thus the solution is obtained.

4. DISCUSSION

Theoretically, the TLS estimation method gives better results than LS. When the theory of the TLS and its minimizer norms are investigated, it is seen that it gives equivalent results with LS in the worst case and gives better results in all other cases. In this preliminary work, we have investigated the generic TLS model which means that it has a unique solution (Huffel, 1991). But there are situations where TLS has no unique solution. Then non generic solutions are possible (Huffel, 1991). When considering both the ill posed nature and iterative procedure of the image matching problem, it is possible to be face to face non generic cases. So the implementation algorithm of the TLS should cover all possible cases. In this preliminary work we have dealt only with the generic solution as mentioned before.

On the other side, comparison of TLS and LS results in a compact and clear way is also a difficult task. Statistical properties of the LS are well known. But stating the statistical relations between the data errors and the observations errors is still problematic. For these purposes, some approaches based on titration models and error equilibration models are used. At the moment we are studying on error propagation theory for statistical interpretation of the TLS solution given in the previous chapter.

Here, in order to show the results of the TLS and LS matching, we compare the manually measured coordinates of the points to TLS and LS matched coordinates. This approach is not so appropriate for evaluation and comparison but however may give some insight to the readers.

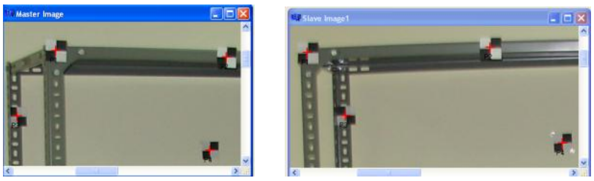

Figure 1. Source image (left), target image (right); with corresponding points

In Figure 1, two sample images with corresponding points are shown. There are significant scale change and rotation between two images. In Figure 2, source patch of point p1 and corresponding matched patches by TLS and LS are shown respectively.

Figure 2. Source patch (left), TLS matched target patch (middle) and LS matched target patch (right)

As seen in Figure 2, TLS result seems to better fit to the source location. In Table 1, coordinates of four matched points are shown. The results of TLS matching are in the first column, the LS results are in the second column of Table 1. Third column shows the manually measured target coordinates. The coordinate values are in the pixel unit.

Pt. #	TLS x, y	LS x, y	Manual x, y
1	458.584, 119.517	458.349, 122.287	459.130, 119.933
2	769.850, 110.739	770.250, 112.050	770.067, 112.067
3	523.408, 226.874	524.342, 226.358	523.735, 226.330
4	892.167, 273.025	892.555, 273.438	892.200, 272.930

Table 1. Matched coordinates.

We assume that the manually measured coordinates are correct. Then the corresponding differences of the coordinate values in the table are the errors of the corresponding coordinates. Their mean squared errors are $\sigma_{xTLS} = \pm 0.337$ pixel, $\sigma_{yTLS} = \pm 0.748599$ pixel, $\sigma_{xLS} = \pm 0.533368$ pixel and $\sigma_{yLS} = \pm 1.204206$ pixel. As seen the results are unacceptable both for TLS and LS. For LS matching, during the matching procedure, an appropriate test procedure to exclude the nondeterminable parameters are required. The same problem is also valid for TLS matching. But however, the given results in Table 1 and our previous results always shown that the TLS results are more realistic. Of course there are many problems to be answered for TLS matching. For example, when data matrix is assumed erroneous, it means that the estimation model will modify the data. In this case, what can be said about the radiometric shift and scale parameters? Should they be included in to model? There are various questions to be answered and these are interest of our future works.

5. CONCLUSION

In this paper, we investigated the TLS estimation method for data registration problems to find the corresponding match points which are necessary for geometrically register the datasets to each other. And we applied TLS to adaptive least squares matching technique. Although there are still many problems to be solved for TLS matching, this preliminary work shows that it is an effective and more realistic approach than LS. In the worst case, TLS gives the equivalent results as LS and in all other cases it is better than LS.

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