## **ROBUST BUILDING OUTLINE REPRESENTATION AND EXTRACTION**

Cui Shiyong<sup>a, \*</sup>, Yan Qin<sup>a</sup>, Liu Zhengjun<sup>a</sup>, Li Min<sup>a, b</sup>

 <sup>a</sup> Key Laboratory of Mapping from Space of State Bureau of Surveying and Mapping Chinese Academy of Surveying and Mapping, Beijing, 100039, China
 <sup>b</sup> Surveying and Mapping College, Shandong University of Science and Technology, Oingdao, 266510, China

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## ABSTRACT:

This paper presents an efficient and robust strategy for building description and extraction from high resolution aerial image. This approach is different from traditional techniques in shape formation. Traditionally, extraction methods usually rest purely on edges or regions. However, this proposed approach integrates edges and regions in an excellent manner. Firstly, region grow method in RGB space is used to obtain the approximate shape of the building. Secondly, Hough transformation is used to derive the two perpendicular line sets corresponding to building edges. Thirdly, the intersectional nodes can be utilized to construct a structural graph of building after analysis of each line segments. Finally, cycle detection algorithm is implemented to obtain cycles in the graph. With this cycles, the exact building boundary can be easily attained.

# 1. INTRODUCTION AND BACKGROUND

Man-made object recognition is one of the crucial research issues for many years in the field of remote sensing and photogrammetry. As a result, various strategies and methodologies have been brought forward to address the task of building detection and extraction. Most of these methodologies about building extraction can be classified into two categories (Shivong Cui, 2008): edge-driven approaches and region-driven approaches. Methods of first category are mainly bottom-up edge-driven approaches, and the emphasis is laid on the elimination of spurious low-level features and the construction of relationship between extracted features. Traditionally, after extraction of linear features, an attempt is made to link or group the line segments corresponding to the buildings to obtain building boundaries. In a region-driven strategy, the source image is initially segmented entirely into different regions. After feature extraction of each region, various methodologies, including artificial neural network, support vector machine, machine learning strategies and other traditional schemes in pattern recognition, are applied to classify the feature vectors.

#### 1.1 Previous Work

Edge-driven methods, including perceptual grouping and contour tracing, are investigated most extensively and deeply. Morhan and Nevatia (1989) use perceptual organization to detect and describe building in aerial images. They first detect linear features, which are then grouped into parallels. Parallel collation with aligned endpoints triggers the formation of a U structure. Two U structures trigger the formation of a rectangle hypothesis. The BABE system (D. McKeown, 1990) performs perceptual organization of lines and orthogonal corners into chains and rectangles to form building hypotheses that are later verified using shadow information. Lin and Nevatia (1998) derive the analytic geometric relationship between building margin lines and building shadows according to general illumination model to analyze the relationship of different ground features. Sohn and Dowman (2001) extract the polygon of building based on Fourier Transform and Binary Space Partitioning tree. Zheltov (2001) adopts the linear extraction method proposed by Burns (2001) and combines with the rectangular building model to extract building information from aerial images.

Region-driven methods are also widely used approaches in building extraction. Fua and Hanson (1987) segment the scene into regions, find edges lying on-region boundaries, and then see if there is evidence of geometric structure among these edges to classify the region as a man-made object. Lee (2003) proposes an approach using classification results of Iknos multispectral images to provide approximate location and shape for candidate building. Their fine extraction is then carried out in the corresponding panchromatic image through segmentation and squaring. Baaze and Schäpe (2000) put forward a new method that combines the multi-scale region segmentation with object-oriented decision tree classifier to classify the target features by investigating the spectral, texture and context information. In the VISION (A. Hanson, R. E. Riseman, 1978) system region segmentation is the primary techniques used and the regions are classified by their shape and spectral properties. In (T. Matsuyama, V. Hwang, 1985) a region segment method is used and the relationship between such objects as roads and houses is used to improve detection. Segl and Kaufmann (2001) combine supervised shape classification with unsupervised image segmentation for detection of small buildings in suburban areas.

While so many techniques have been applied towards this task, there are still many challenges in detection and description of building in aerial image, such as complexity of scene, low contrast and structural variety. Most of these edge-driven approaches usually start from an edge map followed by a line extraction process. However the lines including the building-

<sup>\*</sup> Corresponding author. gisyong@126.com

correspondence lines and other spurious lines are randomly distributed and fragmented in the scene. This is the primary difficult of the edge-driven approach. A mass of endeavours (T. Matsuyama, V. Hwang, 1985, A. Huertas, R. Nevatia, 1988, R. Mohan, R. Nevatia, 1989, Venkateswar V., Chellapa R, 1986, Irvin R, McKeown D, 1989) have been made to link or group the line segments corresponding to buildings to obtain desired building boundaries. In a region-driven building extraction strategy, it also confronts with the problem of fragmentation as the region building occupies may be not fully homogenous, which discounts the robustness. There are also a number of strategies presented in the literature which combine edge finding and image segmentation, which offer a better solution to approaches based merely on edges or regions. However, this category of methods usually can not fully utilize the complementary advantages of individual approach. Therefore, most of these methods can not extract building robustly and effectively in the presence of noise and interfering structure. In addition, it is even more impossible to extract tiny structure of the building.

This proposed scheme mainly consists of Hough transform and a graph search algorithm. Graph representation has been used by several researchers in building extraction. In (Christopher. O. J., Frank Stolle, 1994), feature relationship is represented by a weighted feature relation graph; the maximally weighted independent set of cycles in the relation graph is perceived as the final set of roof boundaries. However, the construction is still a grouping strategy. Kim (1999) utilize graph to describe the relationship of the lines, then extraction of the cost function smallest is implemented. However, the graph construction is still based on a complex grouping process. Croitoru (2004) also use graph approach to express building structure. A search algorithm is also used to extract certain node sequence corresponding to building model. However, that was a model based approach which is confined to certain type of building. Inspired by this approach (A. Croitoru, Y. Doytsher, 2004), we believe cycle in graph is an elegant way to express building boundaries. There are several issues which depend on finding a certain cycle (E. T. Dixon, S. E. Goodman, 1976, M. M. Syslo, 1979), a subset of cycles (E. Hubicka, M. M. Syslo, 1975), and all cycles of a graph (P. Mateti; N. Deo., 1976) which can be solved using the cycle space method.

## 1.2 Proposed Method

In this paper, this scheme integrates robustness of Hough transformation and the facility of graph-based shape representation in an excellent manner. The main idea is to utilize the building corner to construct a building structural graph. A graph search algorithm is then applied to detect the potential building polygon from the building structural graph, nodes of which are intersections of the building dominant line sets.

The remaining sections of this paper are organized as follows: Section 2 demonstrates the robustness of the Hough transform and the graph application in shape representation. Section 3 details the extraction scheme. In section 4 experiment results and discussion are presented. In the last section, conclusion of this extraction strategy will be given.

## 2. METHODOLOGY

Outlines of most building roofs in urban area and industrial

centre are rectangular or combination of several rectangles. Therefore the extracted building polygon should be rectangular. In this sense, Hough transformation is an effective strategy to extract perpendicular and parallel lines which comprised the frame of the building. To strengthen the efficiency, the extraction scheme should incorporate region information of the building. This paper proves that integration of edges and region information in a graph based approach is effective and robust.

# 2.1 Parallel and perpendicular lines detection by Hough transformation

The commonly used methodologies to extract straight line are based on various edge detection technique followed by a rasterto-vector or chain code process. Then edge chains are segmented into straight line segment. Although such algorithms are straightforward for line extraction, they are of low robustness in the presence of noise or occlusions which are common in aerial images. This may lead to numerous spurious lines, which raise computational complexity to derive parallel and perpendicular lines in image. Therefore, we need a robust line detection algorithm. Alternatively, Hough transformation (R .O. Duda, P. E.Hart, 1972), which is based on the accumulation of evidence about the existence of lines in accumulator space ( $\rho$ ,  $\theta$ ), provide a better solution to this problem. This method is based on the dual property of the pointto-curve, and involves transforming each of the pixels into a parameter space in which each point represent a straight line in the image. Two distinct advantages result from the way in which parallel and perpendicular lines are transformed into accumulator space, can be used to detect the two line sets. Parallel lines in image are mapped into peaks aligning vertically in Hough space. Perpendicular lines in image are transformed into peaks, distance between which along horizontal axis in Hough space is about 90 degree. Consequently, detection of parallel lines in image necessitates the retrieval of all the peaks with same  $\theta$  in accumulator space. Parallel lines with this orientation can be identified by detecting the peaks surpassing a threshold value which is about 30 percent of the maximum value in Hough space. Lines which are perpendicular to a set of parallel lines can be detected in same way along the column with horizontal coordinate  $\theta_2 = \theta_1 \pm \pi/2$  ( $\theta_1$  is assumed to be the horizontal coordinate of these parallel lines). In this way, the dominant line sets of the building can be extracted effectively and robustly.

## 2.2 Region Collection

Based only on the lines, there are two disadvantages which lead to the difficulty in extraction building. The first is that, if Hough transform is applied on the edge map of the source image, there may be many interfering lines which can be seen in Figure.2 (d). Another is that building region information can not be incorporated into the extraction process. However, this issue can be addressed by applying edge detection on the image, pixels of which is inside the building, instead of the source image. In this way, not only spurious lines can be reduced but also region information can be integrated into extraction. Collection of the pixels inside the building is usually carried out by image segmentation. Nevertheless, region growing may be superior to image segmentation in certain cases because region growing is a local operation with low computational complexity. On the contrary, image segmentation is a global operation with high computational complexity. If the focusing area is small, the distinction can be neglected.

Through the lines retrieved by Hough transformation and the regions obtained by image segmentation or region grow algorithm, the outline of the building is more prone to be extracted. This simple but effective approach is of great advantageous as it needs not the complex feature extraction and the grouping process.

## 2.3 Graph in Shape Representation

Building structure can be represented by an undirected graph, vertex of which represents a building corner, and similarly each edge of the graph represents a building edge connected by two corners. If all the intersections of the dominant line sets comprised building outline is used to construct an undirected graph, it will be expensive to process. Accordingly, elimination of certain edges which are not related with building is indispensable. It is sensible to assume that there should be certain difference in terms of the grey value between two sides of the building edges. The average grey value of the two rectangles at two sides of the edge is used to remove false edges. If the difference of the average grey value exceeds a threshold, the edge can be safely removed. According to this criterion, certain building-unrelated edges will be eliminated. After the filter of false edges, the remaining edges are used to construct the building structural graph.

#### 2.4 Cycle Detection

There has been extensive work done about sequential algorithms for this issue. The most commonly used method is backtrack algorithm. The first step is to find all the back edge in the graph using depth-first search. Each back edge identifies a fundamental cycle in cycle vector space of the graph. The second step is to determine all cycles by listing all combination of basis cycles of a graph using a cycle vector space method. The dimension of the space is  $\mu = e - n + 1$ , where *e* the number of the edges is and *n* is the number of the vertices. However, this category of backtrack algorithm may not be suitable for undirected graph. We adopt another scheme which may be more suitable for undirected graph. There are mainly three steps:

*Step 1:* to detect a spinning tree of the graph. On approach is firstly removing all the edges of the graph. Then each edge of the graph is put back to the graph gradually. If the graph contain cycles after the return of certain edges, this certain edges is assumed to be a chord. In this step, we can derive all the chords of the graph. The result of this step may not be sole. However, this does not influence the detection of all the cycles.

*Step 2:* to find fundamental cycles of the graph. Each chord derived in previous step corresponds to a fundamental cycle. After the restore of each chord, the spinning tree must contain a unique cycle. Consequently, if there are K chords, there will be K fundamental cycles. Each cycle contain a chord which does not exist in all the other K-1 cycles. These cycles constitute a series of fundamental cycles.

*Step 3:* to list all the feasible union of the fundamental cycles. We enumerate all the feasible combination of  $1, 2 \dots K-1$  fundamental cycles. There may be certain case that the closed

path is composed of two or more cycles. We exclude all this kind the closed path.

If the graph consists of several connected components, depthfirst search is utilized to compute each connected component of the graph. The idea is to run depth-first search on the graph and mark all vertices in the same DFS tree as belonging to the same connected component. Then this scheme is applied to each connected component.

### 3. EXTRACTION SCHEME

Based on the method depicted in previous section, a robust extraction scheme is designed. The main idea of this approach is to construct building structural graph and to detect the building polygon from the graph. From the user's point of view, he/she just need to select the seed pixel inside the building and then the scheme is carried out automatically. There are mainly three steps in this scheme.

#### 3.1 Determination of building frame

This step consists of two procedures. The first is region grow which can be substitute with image segmentation. Region grow algorithm is carried out to determine the approximate shape of the building. In this procedure, a threshold is needed to collect the pixels inside the building. The second procedure is to extract the building dominant line sets by Hough transformation. To extract the parallel and the perpendicular line sets, the following steps is carried out.

- (1) Find the maximal alignment and its corresponding  $\theta_1$  value.
- (2) Find a perpendicular vertical alignment and its corresponding  $\theta_2$  value.
- (3) Extract the peaks above the threshold  $T_0$  from each vertical alignment ( $\theta_1, \theta_2$ )

#### 3.2 Graph construction

After previous step, the outline frame of the building has already been extracted precisely and robustly. The shape of the building can be defined by a node sequence. Therefore, the objective of this step is to construct a structural graph using the building- related node.

Most of the building corners are included among the intersections of the two line sets extracted in previous step. But there are also many intersections which are not building corners. Accordingly, elimination of certain edges and nodes which are not related with building is indispensable. It is rational to assume that there should be certain difference of the grey value between two sides of the building edges. The average grey value of the two rectangles in two sides of the edge is computed. If the difference exceeds a threshold  $T_1$ , this edge is supposed to be not building edges. According to this criterion, certain building-

unrelated edges will be eliminated. The remaining intersections and the edges connected two node are used to construct the graph.

## 3.3 Cycle Detection

Once the graph has been constructed, the issue is transferred to extraction of certain cycle representing the building boundary from the structural undirected graph. The cycle detection which consists of three steps is implemented to list all the cycles in the graph. The area of each polygon corresponding to cycles is computed. The polygon with maximal area is considered to be the building boundary.

## 4. EXPERIMENT AND EVALUATION

The proposed approach is implemented in the following experiment using high resolution aerial image. The following experiment is carried out under the VC++6.0 environment using real world imagery. In addition to these experiments, complexity analysis is also discussed in this section.

#### 4.1 Experiment

The experimental image shown in Figure 1 (a) is about 203 by 198 pixels and the spatial resolution is about 0.6 meter. The scene is comprised of one salient building which has two subordinate swings.

The two line extraction schemes described in Section 2.1 are illustrated in Figure.1. Edges shown in Figure.1 (b) are detected by canny edge detector with variance 0.4 for Gaussian smooth. Then all the edge pixels are linked into edge chains. The Douglas-Peuker Algorithm (Douglas, D., T. Peuker, 1973) with threshold 2.5 is used to split each chain code to segments which are approximately straight. Finally, the least square fitting is used to obtain straight lines presented in Figure.1 (c) from each segmented chain code. Straight line in Figure.1 (d) is detected by applying Hough transform on the image in Figure.1 (b).

The conclusion from Figure.1 is that the lines extracted by Hough transformation are regular and there are many interfering lines extracted by chain code algorithm which increase computational complexity. However, there are still many straight lines among lines extracted by Hough transformation which are not building edges.

The next step is to integrate region information into the extraction process to decrease lines which are irrelevant to building. This strategy is demonstrated in Figure.3.

Black pixels in Figure.2 (b) is result of region grow after mean shift segmentation in Figure.2 (a). In this step, the threshold value is 10. This threshold is crucial to control the approximate shape of the building. Red lines in Figure.2 (b) are the dominant line sets obtained by Hough transformation. The threshold used in line extraction control the line density. If it is larger, fewer lines will be extracted, and on the contrary, more lines will be extracted. This parameter determines whether tiny structure of the building can be extracted.

Image in Figure.1 (a) is segmented by mean shift based approach recommended in (Dorin Comaniciu, Peter Meer, 2002). In the process of segmentation, suitable segment scale is critical because if the scale is over large or small the building shape may be not fragmented.



Figure.1 Comparison of line extraction result: (a) example of building; (b) edges by canny detector;
(c) Straight lines extracted by chain code split; (d) straight lines extracted by Hough transform.



Figure.2 (a) segmented image using mean shift segmentation algorithm; (b) lines extracted using Hough transformation; (c) green line segments are related with building; (d) extraction result

Green line segments in Figure.2 (c) are ones related with building edges. The width of rectangle at two sides of line segments is 10 and the threshold of the grey value difference is 10. The width of the rectangle and the threshold value is most important to the final extraction result as it determines whether there are cycles in graph. These parameters are determined by the specific scene.



Figure.3 the corresponding structural graph

The structural graph corresponding to the building is presented in Figure.3. Cycle in red colour is the one with maximal area and is accordingly the building boundary shown in Figure.2 (d). The searched cycles in structural graph and the corresponding area are listed in table. 1.

Table.1 all the cycles in graph and the area of each cycle

ID	Sequential vertices of each cycles	Area
1	1 2 3 4 5 11 17 23 22 21 27 26 20 14 13 12 6 7 1	10612.7609
2	1 2 3 4 5 11 17 23 22 21 27 26 25 19 13 12 6 7 1	11022.4115
3	1 2 3 4 10 16 22 21 27 26 20 14 13 12 6 7 1	10036.4388
4	1 2 3 4 10 16 22 21 27 26 25 19 13 12 6 7 1	10446.0894
5	1 2 8 7 1	112.4531
6	2 3 4 5 11 17 23 22 21 27 26 20 14 13 12 6 7 8 2	10500.3078
7	2 3 4 5 11 17 23 22 21 27 26 25 19 13 12 6 7 8 2	10909.9584
8	2 3 4 10 16 22 21 27 26 20 14 13 12 6 7 8 2	9923.9857
9	2 3 4 10 16 22 21 27 26 25 19 13 12 6 7 8 2	10333.6363
10	22 23 17 11 5 4 10 16 22	576.3221
11	26 25 19 13 14 20 26	409.6506

## 4.2 Discussion and Evaluation

In the extraction process, there are two crucial steps which are region growing and Hough transformation. And in most cases region grow method can be substitute with image segmentation. Another key point is that Hough transformation is sensitive to certain parameters, such as the threshold described in section 2 and the width of the rectangle used in the identification of the building-related line segments. The threshold determines the number of lines in the two line sets. It is determined by shortest lines to be detected. This parameter can be adjusted to extract some short lines. This kind of short lines is critical to extract tiny and complex structures which can be seen in Figure.4. The tiny structure in the Figure.4 (a) and complex structure in Figure.4 (b) are not likely to be extracted by traditional boundary approximation. The width of the rectangle should be determined by specific scene. And another threshold value used in the determination of line segments is determined by approximate shape of the building and the shape of the building. If this threshold is not suitable, some line segments may be omitted, which may lead to no paths in the graph closed.

The computational complexity of this strategy focuses on building region collection, graph construction and cycle detection. As region grow is carried out at a local area, it runs with a very low computation complexity relatively, especially when the image is very large. If image segmentation is used to identify the building region, we propose to apply it at a local scope around the building. The complexity of the graph construction is determined by the number lines extracted by Hough transformation. Most buildings are not enormous complex. Accordingly, line number of the building is not very tremendous. In the case of complex structural graph, the cycles detection process may become time consuming as the total number cycles in a planar graph can even be exponential in its vertices (Dogrus"oz; M. Krishnamoorthy, 1995). In most cases, the search process takes one second more or less.



Figure.4 tiny structure can be extracted by fine control of the parameters

## 5. CONCLUSION AND FUTURE WORK

This paper presents a simple but effective approach to tackle the task of building description and extraction. The experiment proves that this scheme can extract building robustly and exactly, especially for the plain right angle building.

There are several evident advantages of this scheme. Firstly, it integrates the robustness of the Hough transform and the graph search algorithm. It improves the quality of line extraction and avoids the complex feature grouping process and boundary approximation. Secondly, this approach is not restricted by the shape of the building as long as the building is composed of orthogonal corners. Finally, this scheme can finely control the extracted shape of the building. Tiny structure (such as the case in Figure. 4) of the building can also be extracted by adjustment of some parameter.

However, there are still some defects of this approach needed to be perfected in the future research. The key step of this proposed method is building region collection. If this step is failed, the follow steps will not be able to extract the exact shape of the building. Another defect of this approach is that it can not deal with roofs which are not planar. In future research, this approach should be extended to this kind of roofs.

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