DIFFERENTIAL RADAR INTERFEROMETRY AND ITS APPLICATION IN MONITORING UNDERGROUND COAL MINING-INDUCED SUBSIDENCE

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KEY WORDS: DInSAR, PSInSAR, SBAS, IPTA, Mine Subsidence

ABSTRACT:

Ground subsidence due to underground coal mining threatens the safety of surface infrastructure such as buildings, roads, bridges, powerlines and telecommunication cables, and can cause environmental problems such as water pollution, cessation of stream flow, and so on. Monitoring the spatial distribution of surface deformation may be helpful for land-use planning, land reclamation, safety and establishing the extent of legal liability. Traditional monitoring techniques that use levels, total stations and GPS can only measure on a point-by-point basis and hence are comparatively costly and time-consuming (Ge et al., 2007). Differential synthetic aperture radar interferometry, or DInSAR, is a remote sensing technique that can measure earth surface deformation. DInSAR is capable of obtaining dense information related to the deformation across a large area in an efficient and economic manner. In this paper, the principles and basic concepts of traditional DInSAR, as well as some advanced DInSAR techniques as PSInSAR, SBAS and IPTA, are introduced, its application to monitoring underground coal mining-induced subsidence is reviewed, and critical issues such as the limitations of these methods and their likely future development are examined.

1. INTRODUCTION

This paper focuses on mine subsidence measurement based on the use of differential interferometric Synthetic Aperture Radar techniques (DInSAR) that make use of data acquired by space-borne SAR sensors. Airborne DInSAR, which still plays a minor role in deformation applications, is not considered in this paper.

Subsidence-induced fault reactivation may cause moderate to severe damage to concrete foundations, houses, buildings, and underground services, as well as damage to agricultural land through disruption of drainage and alteration of ground gradient (Donnelly et al., 2008). Monitoring and analysing the spatial distribution of the deformed surface may be helpful for land-use planning, land reclamation or remediation.

Monitoring of mine subsidence by means of classical methods such as optical levelling or Global Positioning System (GPS) surveys can reveal deformations with sub-millimetre to sub-centimetre precision for localised areas. However, to provide data on a reasonably dense two-dimensional (2D) benchmark grid are costly and time-consuming for all but the smallest areas. Moreover, a regular re-survey of a mining region may be required, and a retrospective study of a specific deformation event is usually not possible since monitoring networks are often implemented after the major events have occurred.

Synthetic Aperture Radar (SAR) images provide an alternative and complementary method to classical methods to measure ground deformation. Since radar beams scan in the range direction, the movement of the platform in the azimuth direction completes the 2D imaging of the mining region (e.g. Stow & Wright, 1997; Perski & Krawczyk, 2000). By taking multiple observations (subject to geometric constraints) that span the area of interest，differential radar interferometry can measure deformation to high degree of accuracy (better than 1cm) over large spatial extents with high spatial resolution (Massonnet et al., 1993). In particular, differential SAR interferometry (DInSAR) can give a synoptic view of the deformation events projected along the line-of-sight (LOS) over areas of hundreds to thousands of square kilometres (Massonnet and Feigl, 1998).

This paper will discuss the following topics:
a. principles of the standard DInSAR techniques, as well as some advanced ones;
b. analysis of some application cases and their limitations;
c. the availability of DInSAR software tools;
d. the satellite data sources for DInSAR applications; and
e. the key issues related to using DInSAR to monitor mining-induced ground subsidence.

2. DINSAR AND ITS APPLICATIONS

2.1 Principles

The technique of differential SAR interferometry (DInSAR) was introduced by Gabriel et al., (1989). In general, there are three types of differential SAR interferometry: two-pass differential SAR interferometry, three-pass differential SAR interferometry, and four-pass differential SAR interferometry.

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Two-pass DInSAR uses an interferometric image pair and an external digital elevation model (DEM). Among the two single-look complex (SLC) images, one is typically acquired before the surface displacement and the other after the event. The external DEM is converted to a corresponding phase image. This is illustrated in figure 1, where P is a ground point in the two images. The sensor acquires the first SAR image (which is referred as the master image) at time t_1, measuring the phase Φ_M; and then acquires a second SAR image (slave image) some time later at time t_2, measuring the phase Φ_S. Assuming that the surface displacement occurred during this period, the point P is assumed to have moved to P1.
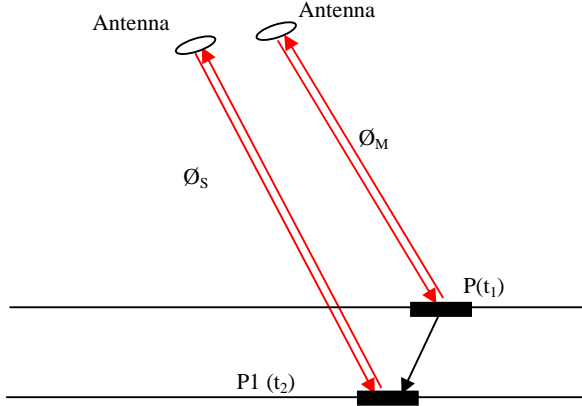

Figure 1. Principle of two-pass DInSAR.

After exploiting the phase difference between Φ_M and Φ_S, one obtains the interferometric phase $\Delta\Phi$. As P moved to P1 between the two image acquisitions, the $\Delta\Phi$ includes:

$$\Delta\Phi = \Phi_{Topo} + \Phi_{Mov} + \Phi_{Atmos} + \Phi_{Noise} \quad (1)$$

where Φ_{Topo} is the topographic phase component; Φ_{Mov} is the terrain change contribution; Φ_{Atmos} is the atmospheric delay contribution; and Φ_{Noise} is the phase noise. Two-pass DInSAR uses an external DEM to simulate the topographic phase Φ_{Topo_Simu}, and then the so-called DInSAR phase $\Delta\Phi_{DInSAR}$ can be computed:

$$\Delta\Phi_{DInSAR} = \Delta\Phi - \Phi_{Topo\_Simu}$$
$$= \Phi_{Mov} + \Phi_{Atmos} + \Phi_{Noise} + \Phi_{Res\_Topo} \quad (2)$$

where Φ_{Res_Topo} represents the residual component due to errors in the simulation of Φ_{Topo}. In order to derive information on the terrain change, Φ_{Mov} has to be separated from the other phase components. Three-pass interferometry can be used without an a priori known DEM, but requires at least three images acquired over the same scene (Zebker et al., 1994). For a general review of SAR interferometry see Rosen et al. (2000).

2.2 Case Study

DInSAR may be applied to all kinds of surface deformation processes: co- and post-seismic deformations (Massonnet et al.1993, 1994; Ge et al. 2008), volcano deformation (Rosen et al. 1996), glacier and ice-sheet dynamics (Goldstein et al. 1993), as well as mine-induced subsidence (e.g. Stow and Wright, 1997; Perski and Krawczyk, 2000; Ge et al., 2001a; 2001b; 2004a; 2004b and 2004c). As mentioned in Ge et al. (2007), DInSAR using images acquired by the Japanese JERS-1, because of their longer wavelength (L-band) is more robust in the presence of vegetated or agricultural ground cover, and also more suitable for areas experiencing a high rate of ground deformation, compared to ERS-1 and ERS-2 (shorter wavelength) DInSAR. However, DInSAR of short wavelength is much more sensitive to mine-induced subsidence. Here two special cases will be discussed.

2.2.1   One day subsidence. This example deals with a surface vegetated underground mine area in New South Wales, Australia, which was reported in Ge et al. (2007). An interesting result was generated by using a Tandem ERS SAR pair. Subsidence with a maximum amplitude of 1cm over a 24 hour period was detected with a resolution of 2mm (see figure 2). This was independently confirmed by the surveyor from the mining company. This example demonstrates the feasibility of monitoring mine-induced subsidence by a DInSAR technique with a high temporal resolution (i.e. one day).
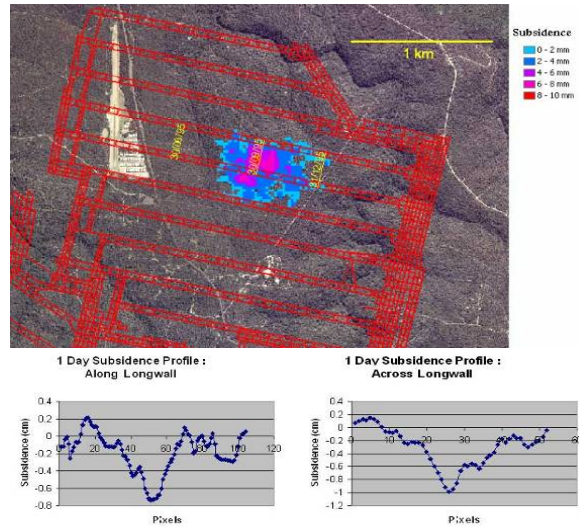

Figure 2. ERS tandem DInSAR interferogram overlaid with a mine plan and analysis of the result: the two subsidence profiles drawn along and across the longwall (Ge et al., 2007).

2.2.2   Reclaimed land monitoring. This example is concerned with reclaimed coastal land monitoring. The test area was located on the south coast of the Korean Peninsula, as reported in Kim, et al. (2005). The accuracy of DInSAR results was evaluated with the estimated subsidence rate from field measurements obtained by a magnetic probe extensometer at 42 ground stations. The correlation coefficient R between the 2D radar measurements and the field survey was 0.87 with an rms error of 1.4cm. The 2D subsidence map estimated by differential interferograms from 7 pairs of JERS-1 radar images is shown in figure 3.
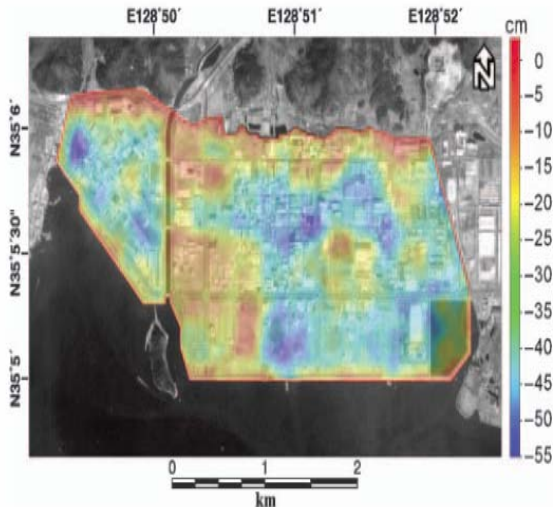
Figure 3. The 2D subsidence map estimated by accumulation of DInSAR measurements. The maximum subsidence was about 60cm over 352 days (from Kim, et al., 2007).

The reclaimed coastal land is quiet similar to the reclaimed land in area subsiding due to underground coal mining activities. This example demonstrates the potential of DInSAR for monitoring post-mining areas and reclaimed land in subsidence area.

3. ADVANCED DINSAR TECHNIQUES

Conventional DInSAR applications are limited due to temporal and geometrical decorrelation and atmospheric inhomogeneities. Temporal decorrelation makes InSAR measurements difficult to use over vegetated areas and where the electromagnetic profiles and/or the positions of the ground radar scatterers change with time within the resolution element. Geometrical decorrelation limits the number of image pairs suitable for interferometric applications and prevents full exploitation of the data set available. Atmospheric inhomogeneities create an atmospheric phase screen superimposed on each SAR image that can seriously compromise accurate deformation monitoring. Some advanced DInSAR techniques have been developed to overcome such limitations.

3.1 Permanent Scatterers (PS) InSAR

To overcome the limitations caused by decorrelation, the PSInSAR technique, developed by the SAR processing group in Milan, Italy, has been employed in the monitoring of slow but continuous ground subsidence (Ferretti et al., 2000; 2001). The PSInSAR approach is based on the exploitation of long time-series of interferometric SAR data (typically at least 15-20 images). PSs are only slightly affected by decorrelation and can be used to estimate and remove the atmospheric phase screen. The sparse PS grid is analoguous to a high spatial density geodetic network, allowing ground deformation measurements (along the ling-of-sight (LOS) direction) with high accuracy (0.1-1mm/yr for the average LOS deformation rate and 1-3.5mm for single measurements). More details on a formal precision assessment of PS results can be found in Colesanti et al. (2003). Here one application of PSInSAR for detecting

surface displacement caused by underground coal mining is mentioned.

3.1.1 Post-mining subsidence. This example deals with subsidence in an abandoned coal mining area located in Gaeun, Korea, reported by Jung et al. (2007). From November 1992 to October 1998, 25 JERS-1 SAR interferometric pairs were used, and the crack level obtained by a 1997 field survey was used to validate the subsidence estimated by PSInSAR. The maximum subsidence was measured by PSInSAR as 11.2cm over 6 years. The mean subsidence rate of one class was found to be 0.5cm/year, where cracks were most severely developed. The radar-estimated subsidence agrees well with the estimate based on crack level data. Result is shown in figure 4.
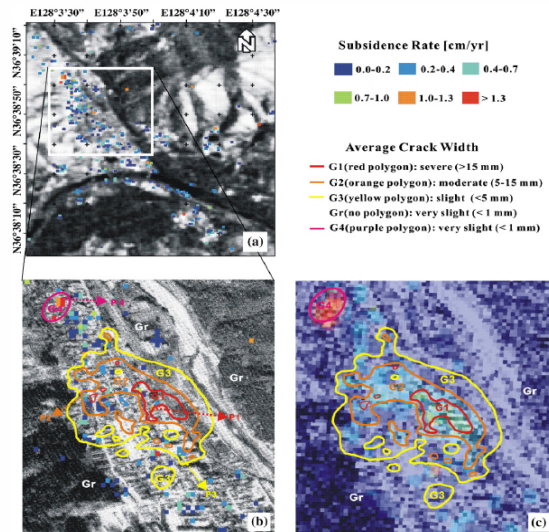

Figure 4. Comparison between measured crack levels and estimated subsidence rate. (a) Location of the final persistent scatterers and their subsidence rates; (b) aerial photograph overlaid with crack level and estimated subsidence rate at each persistent scatterer; and (c) 2D subsidence field (from Jung et al., 2007).

Although the PSInSAR measured mining subsidence in some cases, it is limited to the observation of continuous and temporally linear subsidence. To precisely measure non-linear subsidence it is necessary to develop a more sophisticated non-linear subsidence retrieval model suitable for monitoring realistic subsidence patterns across underground mining areas.

3.2 Small Baseline Subset (SBAS)

The Small Baseline Subset (SBAS) approach was proposed by Berardino et al. (2002), and permits the detection of surface deformation and its analysis for space-time characteristics. The key point of the SBAS technique, in addition to the use of multi-look interferograms, is that the data pairs involved in the generation of the interferograms are carefully selected in order to minimise the spatial and temporal separation (or baseline) between the acquisition orbits, thus mitigating the decorrelation phenomena.

The SBAS procedure permits the production of mean deformation velocity maps and, at the same time, deformation time series for each coherent pixel across the monitored area. SBAS's capability to detect deformation phenomena has been

already been demonstrated in several different applications, mostly based on exploiting European Remote Sensing (ERS) SAR data, acquired in C-band. One typical example of the use of the SBAS technique is to investigate surface displacement caused by underground coal mining.

3.2.1   Mine subsidence-SBAS. The example here deals with areas of rapid deformation caused by mining and agricultural activities in the Crescent Valley, Nevada, USA, as reported by Gourmelen et al. (2007). For each SAR acquisition, 5 interferograms with the smallest perpendicular baseline (as long as it is less than 400m) and temporal separation less than 4 years were selected for processing. The results are shown in figure 5.
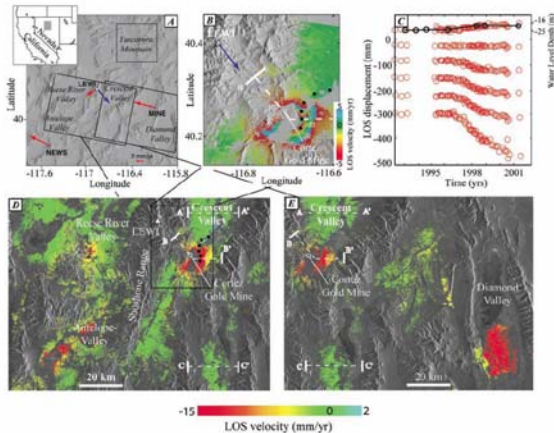

Figure 5. InSAR deformation velocity maps. (a) Shaded relief of the study area and permanent GPS stations (black boxes are locations of InSAR derived rate maps); (b) rate velocity map over Crescent Valley (subset of figure 5d); (c) time series of deformation from 1992 to 2002, from (top) away to (bottom) within the Crescent Valley subsidence area (black circle are water level at a well collocated with the top time series showing linear relationship between surface deformation and water level); (d, e) rate velocity maps of the subsidence at Crescent Valley, Diamond Valley, Reese River Valley and Antelope Valley (from Gourmelen et al., 2007).

The phase unwrapping errors and the unfiltered atmospheric phase artifacts are the main error sources in the deformation retrieval process. The development of advanced techniques for improving the performance of such algorithms is still an active research topic.

3.3   Interferometric Point Target Analysis (IPTA)

The GAMMA remote sensing research group in Switzerland developed a method they called interferometric point target analysis (IPTA) (Werner et al., 2003). The example mentioned here combines IPTA and conventional DInSAR to detect mine subsidence, as reported by Wegmuller et al. (2005).

3.3.1   Mine subsidence-IPTA. In this hybrid approach atmospheric phase corrections were determined for all pairs between the 10 available JERS scenes with IPTA using a linear deformation model that excluded the fast moving areas. The IPTA-derived atmospheric phase corrections were spatially interpolated and used to correct the corresponding conventional differential interferograms. These differential interferograms were spatially unwrapped and converted to deformation maps. The main advantage of this approach is that it is possible to combine atmospheric corrections based on the entire data series with the good spatial coverage of the conventional differential interferograms. The results are shown in figure 6.
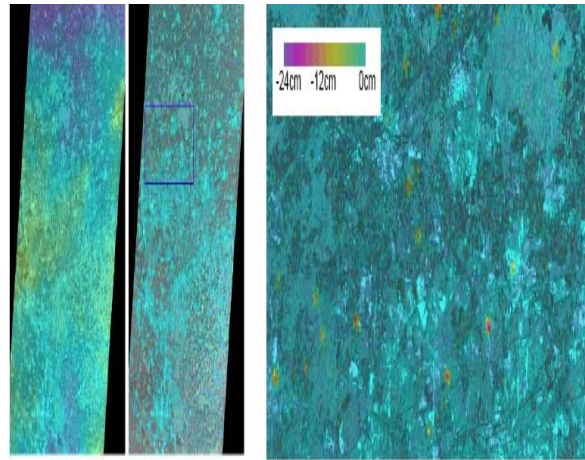

Figure 6. Subsidence map for Ruhr mining area from JERS differential interferometry (19980713_19980826, 44days, B⊥ 352m). The left image shows the low-frequency phase correction applied, the centre image results from the corrected differential interferogram, both for a 92km x 127km area, and the right image is the deformation map for a 32km x 16km subsection (from Wegmuller et al., 2005).

The limitations of IPTA are similar to those of PSInSAR mentioned earlier. For accurate results, one should investigate point scatterers behaving as coherent targets only on over a limited number of images, not over the whole time series of observation.

4.   DISCUSSION

4.1   Limitations

1) For displacement mapping, SAR imagery should ideally be acquired from exact repeat orbits to eliminate the effects of topography on the measured signal. Although topographic effects are corrected for using a DEM during the processing, DEMs of adequate quality are not always available and residual effects may remain. Pairs of SAR scenes suitable for interferometric change detection can be selected to minimise topographic effects. As the existing sensors most commonly used (ERS, RARDARSAT) were not designed to optimise interferometric processing, only a subset of the available SAR acquisitions is typically suitable for interferometric change detection. Future acquisitions frequently will not pair with any other SAR acquisitions of interest as the orbit tracks are not repeated with sufficient accuracy.

2) SAR interferometry only measures LOS range displacements, however horizontal displacement is as important as vertical subsidence information as it can be used to better understand the mechanism of mining subsidence. Using SAR pairs from ascending orbits and SAR pairs from descending orbits for approximately the same period, one can assess whether the opposed look angles reveal differences attributable to horizontal displacements (Hoffmann and Zebker, 2003). However, in general, there are few available ascending acquisitions and hence it is difficult to resolve the horizontal displacement components without additional assumptions. Nevertheless, additional information will generally be necessary to obtain a 3D displacement field from interferometry using space-based sensors in polar orbits (Wright et al., 2004).

3) The global availability of SAR acquisitions is somewhat limited. Many areas have few or no acquisitions unless the area of interest previously was imaged. For example, there are subsiding areas in Mexico and in the People's Republic of China that have significant aquifer-system compaction problems but with limited ERS SAR coverage (Galloway and Hoffman, 2007).

4) Commercially available processing software is expensive and can involve a significant investment to develop the skills necessary to generate high quality interferograms. There are now some free/open source packages available, but the users have to be familiar with DInSAR principles, and typically advanced DInSAR techniques are not included in these packages.

4.2 Future Developments

Satellite-borne SAR sensors most commonly used in the past for making interferometric measurements were optimised for SAR imagery applications and not for interferometry. An important consequence is that the satellite orbital configurations and control were suboptimal, resulting often in degraded interferogram quality. Future sensors can be expected to improve upon this, making InSAR more generally applicable.

The integration of multiple satellite and ground-based technologies for monitoring regional land displacements improves accuracy, reliability, and the value of subsidence information. This combination can be used to overcome the limitations associated with each technique, and to provide improved mapping, monitoring, and even controlling of mining-induced subsidence. Some approaches for integrating GPS and InSAR techniques have been reported in, e.g., Ge et al. (2001a; 2001b; 2004a; and 2004b).

The future of InSAR for mining-induced subsidence will likely include new operational and research applications that will improve the evaluation of the sustainability and management of mining area through integrated monitoring and prediction of mining-induced subsidence, which will in turn lead to an enhanced understanding of the mechanisms for mining-induced subsidence.

It is anticipated that future applications of conventional DInSAR as well as advanced DInSAR techniques will continue to improve upon the definition of mining-induced subsidence models. As more SAR data suitable for interferometry become available, better models, predictions and mine management can be expected.

5. CONCLUDING REAMRKS

Overall, future scenarios involving multi-frequency and multi-LOS SAR observation, coupled with reduced revisit time options, are extremely promising for the development and application of conventional and advanced DInSAR techniques for monitoring, modelling, or even predicting, underground coal mining-induced ground subsidence. This may finally lead to the use of DInSAR technologies as part of routine surveillance operations.

ACKNOWLEDGEMENTS

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