THE IMPLEMENTATION AND ANALYSIS OF KALMAN FILTER PHASE UNWRAPPING ALGORITHM OF INSAR

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KEY WORDS: InSAR, Phase unwrapping, Kalman filter, Noise reduction, Algorithm implementation, Result analysis

ABSTRACT:

Phase unwrapping is the difficulty and hotspot in research because it is the key step in InSAR. The general phase unwrapping methods need usually use some filtering algorithms to eliminate noise firstly, and then to phase unwrapping, so as to ensure the quality of unwrapping. The Kalman filter overcomes this drawback, and transforms the phase unwrapping problem into state estimate to deal with phase unwrapping and noise reduction at the same time.Choosing simulated data to processing, it is shown that Kalman filter phase unwrapping method is very feasible and effective in the respect of unwrapping and noise restraining through the analysis and evaluation of the results.

1. INTRODUCTION

InSAR provides critical information about disaster and emergency responses related to volcanic eruptions, earthquakes, landslides, flooding, and high-resolution DEM generation, etc. In order to infer the cause that is responsible for the observed deformation or generate a DEM, InSAR images have to be unwrapped. The general phase unwrapping algorithms are broadly divided into two categories. The first is based on the path following algorithms[1-3],such as Goldstein's branch cut algorithm. The second is based on the minimum norm algorithms[4-7],such as minimum Lp-norm algorithm.The former is a local algorithm, its advantages are that it can isolate the phase branch points, prevent the local phase error in the spread of the whole integral region, and gain the accurate unwrapping phase in the region of better coherence. But it is difficult to get the best integral path, and easily lead to errors transmission or the isolated region under the strong noise conditions.While the latter is a global algorithm, the excellence is that operation is stable, and it do not need to identify the residues. But they are "through" the residues instead of "bypass" them, so it is likely to result in errors transmission.

In addition to the above two categories algorithms, people have taken more and more attention in using optimal estimation algorithms to phase unwrapping, such as Network flow model [8-10], Bayesian model [11], Kalman filter model [12-16] and Grid-based filter model [17]. Although phase unwrapping methods have developed rapidly in the past 20 years and achieved fruitful results, but also be widely used. However, due to the complexity of phase unwrapping and its affected by many factors, phase unwrapping has still been the difficult and research hotspot in InSAR. A large number of residues are generated in phase unwrapping process because the interferograms are impacted by noise. Thus it seriously affects the quality of phase unwrapping, and even causes its failure. Therefore, the general phase unwrapping methods must be carried out to eliminate noise before unwrapping. Kalman filter transforms the phase unwrapping issue into a state estimation

problem, the real phase is solved through the establishment of the dynamic equation and observation equation of phase. But the biggest feature of this method is to achieve phase unwrapping and simultaneous noise reduction, overcoming the drawback that the general methods must be implemented prior to the elimination of phase noise before phase unwrapping.

The basic model and algorithm of Kalman filter are briefly introducted in this paper, then it is to evaluate the performance of the algorithm in two aspects of unwrapping and filtering using simulated data for experiment.

2. KALMAN FILTER MODEL OF PHASE UNWRAPPING

2.1 Observation Model

Consisting of in-phase and quadrature components of the complex interferogram as two noisy observations of the true interferometric phase[13]. In order to writing conveniently, substituting again the n, m dependence by a 1-D k dependence, we let

$$y(k) = \begin{bmatrix} \mathrm{Re}\left\{\dfrac{z(k)}{a(k)}\right\} \\ \mathrm{Im}\left\{\dfrac{z(k)}{a(k)}\right\} \end{bmatrix} \overset{\Delta}{=} \begin{bmatrix} \cos(\varphi(k)) \\ \sin(\varphi(k)) \end{bmatrix} + \begin{bmatrix} v_1(k) \\ v_2(k) \end{bmatrix} \qquad (1)$$

$$= h(x(k)) + v(k)$$

$\varphi(k)$ is the true unambiguous phase, since there is no difference between true and wrapped phase in the complex Domain.Thus, any filter directly operating on the complex data, rather than on the phases calculated from the complex values, will implicitly not suffer from the wraparound effect.

2.2 State-Space Model

A simple but very effective state-space model for the wanted unambiguous phase is given in [16]. Starting with the

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interferometric phase in a discrete-time representation, we assume that

$$x_1(k+1) = \overset{\wedge}{\varphi}(k+1) = x_1(k) + u(k)$$
$$u(k) = \hat{u}(k) + w(k) \tag{2}$$

3. KALMAN FILTER AND PHASE UNWRAPPING

Established for the above mentioned state-space model, if the statistical properties of noise is known, we can accord to the following extended Kalman filter for state estimation [18]:

$$\hat{x}_{k,k-1} = \Phi_{k,k-1}\hat{x}_{k-1,k-1} + q \tag{3}$$
$$P_{k,k-1} = \Phi_{k,k-1}P_{k-1}\Phi_{k,k-1}^T + Q \tag{4}$$
$$J_k = P_{k,k-1}A_k^T(A_kP_{k,k-1}A_k^T + R)^{-1} \tag{5}$$
$$\hat{x}_k = \hat{x}_{k,k} = \hat{x}_{k,k-1} + J_k(L_k - A_k\hat{x}_{k,k-1} - r) \tag{6}$$
$$P_k = P_{k,k} = (I - J_kA_k)P_{k,k-1} \tag{7}$$

where J_k =gain matrix

$\hat{x}_{k,k-1}, P_{k,k-1}$ =predictive value and its covariance matrix, respectively

$\hat{x}_{k,k}$, $P_{k,k}$ =filter value and its covariance matrix, respectively

A_k = linearized observation matrix

A_k is given by

$$A_k = \frac{d}{dx(k)}h(x)|_{\hat{x}_{k,k-1}}$$
$$= [-\sin(\hat{x}_{k,k-1}), \cos(\hat{x}_{k,k-1})]^T \tag{8}$$

Phase slope estimation q ,the measurement noise covariance R , and the driving noise covariance Q can be calculated in [14]. The specific implementation of two-dimensional Kalman filter phase unwrapping can be seen in [12] and [13].

4. EXPERIMENTAL RESULTS AND ANALYSIS

The simulated data is used in two different experiments under the different conditions of coherence. From the view of visualization and quantitation, Kalman filter phase unwrapping algorithm is analyzed and evaluated in two aspects of the filter effect and unwrapping quality.
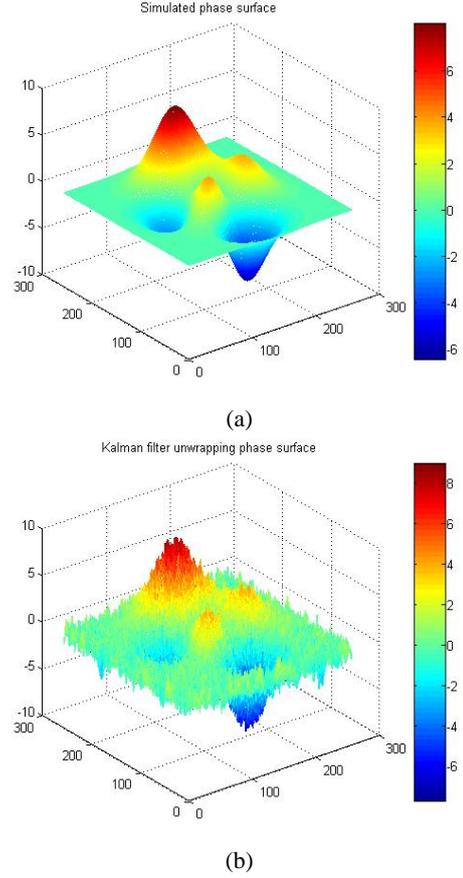
4.1 Experiment I

(a)

(b)

Figure 1. (a) True phase surface(without noise)
(b) Kalman filter unwrapping phase surface

As shown in Figure 1 (a), we simulate a phase surface. In experiment, we deal with it to wrap phase (Figure 2 (a)),
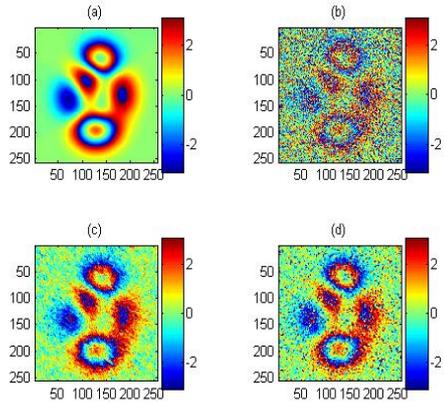

Figure 2. (a) Wrapped phase image(without noise). (b) Noisy wrapped phase image(coherence=0.4). (c) Rewrapped phase image of unwrapping result of Kalman filter. (d) Wrapped phase image after ML filter.

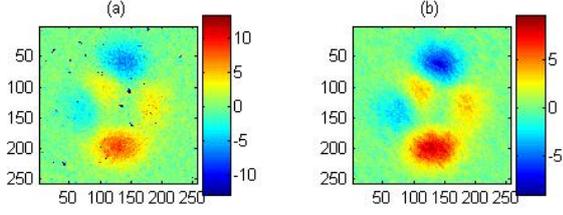adding phase noise which the coherent is 0.4, the noisy wrapped phase can be seen in Figure 2 (b).

Figure 3. (a) Unwrapping result of Goldstein's branch cut algorithm after ML filter. (b) Unwrapping result of Kalman filter.

Using Goldstein's branch cut method after prefiltering and Kalman filter phase unwrapping algorithm to deal with respectively, and the results are in Figure 3 (a) and (b). Here we use Maximum Likelihood(ML) filter as a prefiltering approach. The Kalman filter unwrapping phase surface is shown in Figure 1 (b). The rewrapped phase of unwrapping result of Kalman filter is in Figure 2 (c); meanwhile, the result after doing a ML filter to wrapped phase directly is in Figure 2 (d).
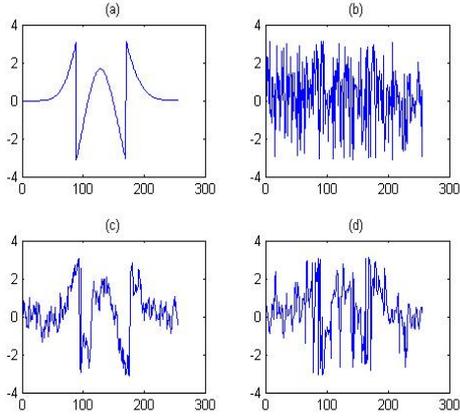

Figure 4. The profile of the first 200 line corresponding to Figure 2 (a) (b) (c) (d), respectively.

Visual evaluation of filter effect: By comparing Figure 2 (c) with Figure 2 (b) and Figure 2 (d) separately, we can see the fringes in Figure 2 (c) become more clear and bright. Figure 4 is shown the profile of the first 200 line corresponding to Figure 2 (a) (b) (c) (d), respectively. The curves in Figure 4 (c) become more smooth after contrasting Figure 4 (c) with Figure 4 (d). This shows that compared with the ML filter, Kalman filter deals with phase unwrapping and noise effective elimination simultanously.

Quantitative evaluation of filter effect: We subtract noisy wrapped phase(coherence=0.4) from the filter result of Kalman filter and ML filter separately, and then to compare their differences. Note: The filter result of Kalman filter is the rewrapped phase of unwrapping result of Kalman filter. From Table 1, we can see that the mean abs.phase error and mean square error of Kalman filter are both less than the ML filter. It is indicated that when the coherence is 0.4, the Kalman filter is a little better than the ML filter.

Filter method	Mean abs.phase error(rad)	Mean square error (rad^2)
ML filter	1.3169	0.0115
Kalman filter	1.0840	0.0080

Table 1. Difference comparison between the results of two filter methods and noisy wrapped phase (coherence=0.4)

Visual evaluation of unwrapping quality: Contrasting Figure 1 (a) with (b), the Kalman filter does well in restoring a real surface phase. Comparison of Figure 3 (a) and (b), it can be seen unwrapping result of Goldstein's branch cut algorithm (Figure 3 (a)) is poorer than that of Kalman filter method (Figure 3 (b)).Despite of prefiltering, the containing noise causes branch cut method can not determine the correct branch-cut, forming some isolated regions (dark blue spot regions in Figure 3 (a)).

Phase unwrapping method	Max abs.phase error (rad)	Min abs.phase error (rad)	Mean abs.phase error (rad)	Mean square error (rad^2)
Goldstein's branch cut algorithm after ML filter	9.1480	0	0.6471	0.0025
Kalman filter unwrapping Algorithm	3.3330	2.8312×10^{-6}	0.5017	0.0021

Table 2. Difference comparison between the results of two unwrapping methods and true phase (without noise)

Quantitative evaluation of unwrapping quality: We subtract true phase from unwrapping result of Kalman filter and branch cut method separately, and then to compare their differences. From Table 2, we can see although the min. abs. phase error of Kalman filter method is very close to that of branch-cut method after prefiltering, the max. abs. phase error, mean abs.phase error and mean square error are less than prefiltering branch-cut method. This shows that in the condition that the coherence is 0.4, even if it does prefiltering to noisy wrapped phase, the result of branch-cut method is still unsatisfactory, but Kalman filtering approach is relatively good.

On the whole, the result of Kalman filter phase unwrapping is satisfactory in poorer coherence.

4.2 Experiment II

Adding phase noise which the coherence is 0.75 to the true phase surface(Figure 1(a)), we get the noisy wrapped phase (Figure 5 (b)). Using Goldstein's branch cut method without prefiltering, after prefiltering and Kalman filter phase unwrapping algorithm to deal with respectively, the results are
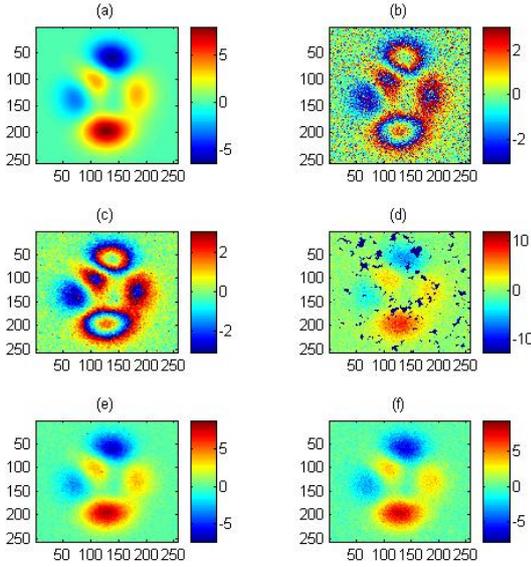
Figure 5. Some results. (a)True phase image(without noise). (b)Noisy wrapped phase image(coherence=0.75). (c)Wrapped phase image after ML filter. (d)Unwrapping result of Goldstein's branch cut algorithm without prefiltering. (e)Unwrapping result of Goldstein's branch cut algorithm after ML filter. (f) Unwrapping phase image of Kalman filter.

in Figure 5 (d) , (e) and (f). Here we use ML filter as a prefiltering approach.

Visual evaluation of unwrapping quality: Contrasting Figure 5 (d) with (e), it can be seen the unwrapping result of Goldstein's branch cut algorithm after prefiltering (Figure 5 (e))is better than that without prefiltering (Figure 5 (d)). This is because without prefiltering, forming many isolated regions (dark blue regions in Figure 5 (a)).

Quantitative evaluation of unwrapping quality: We subtract true phase from the unwrapping result of branch cut method without prefiltering, that of branch cut method after prefiltering, that of Kalman filter separately, and then to compare their differences. From Table 3, we can see branch cut method after prefiltering is better than that without prefiltering in max. abs. phase error, min. abs. phase error, mean abs.phase error and mean square error. It is to say that in the conditions that the coherence is 0.75, it is in favor to phase unwrapping by using prefiltering to noisy wrapped phase, the result is better. Although Kalman filtering approach is slightly

Figure 6. (a)Wrapped phase image(without noise). (b)Noisy wrapped phase image(coherence=0.75). (c)Rewrapped phase image of unwrapping result of Kalman filter. (d)Wrapped phase image after ML filter.

Figure 7. The profiles of the first 200 line corresponding to Figure 6 (a) (b) (c) (d), respectively.

larger than branch cut method after prefiltering in min. abs. phase error and mean abs.phase error, it is smaller than that in max. abs. phase error and mean square error.

Phase unwrapping method	Max abs.phase error (rad)	Min abs.phase error (rad)	Mean abs.phase error (rad)	Mean square error (rad^2)
Goldstein's branch cut algorithm without prefiltering	12.8180	0	0.7757	3.0301×10^{-3}
Goldstein's branch cut algorithm after ML filter	3.6132	0	0.2479	9.6835×10^{-4}
Kalman filter unwrapping algorithm	2.5645	1.1563×10^{-5}	0.4198	9.1433×10^{-5}

Table 3. Difference comparison between the results of three unwrapping methods and true phase (without noise)

In general, the result of Kalman filter phase unwrapping and branch cut method after prefiltering are close in better coherence.
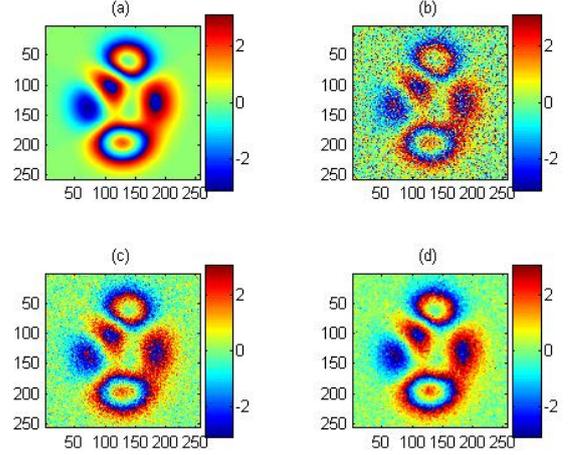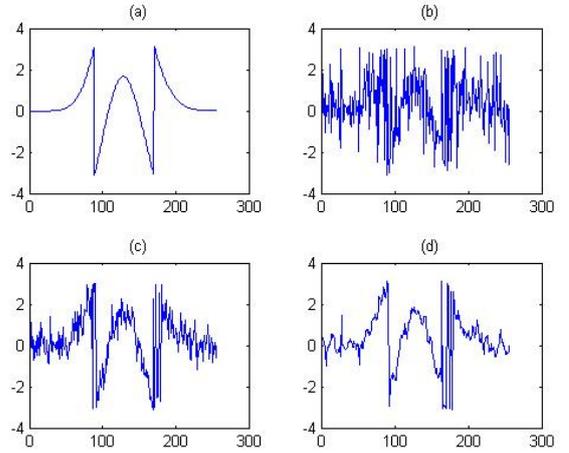
Visual evaluation of filter effect: By comparing Figure 6 (d) with 6 (b) and 6 (c) separately, we can see the fringes in Figure 6 (d) become more clear and bright. Figure 7 is the profile of

the first 200 line corresponding to Figure 6 (a) (b) (c) (d), respectively. The curves in Figure 7 (d) become more smooth after contrasting Fig 6 (c) with Figure 6 (d). This shows that Kalman filter is poorer than ML filter in the condition that the coherence is 0.75.

5. CONCLUSION

Using simulated data to experiment, the results are showed that we can choose Goldstein's branch cut method after prefiltering in the situation of better coherence; meanwhile, in the less coherence case, we use Kalman filter algorithm. Because it does unwrapping and filter at the same time, eliminating the noise effectively, indicating its advantages in unwrapping and filter. However, when the terrain is steep and slope is larger, the unwrapping results will be bad and cause errors transmission. Therefore, how the Kalman filter algorithm taking into account the impact of the terrain will be a future research direction.

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