THE EFFECT OF SOUND WAVE BENDING ON UNDERWATER ACOUSTIC POSITIONING ACCURACY

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ABSTRACT：

Underwater acoustic positioning system solves the problem of how to position and navigate object underwater. Therefore, it can help people to locate the area of disaster to search and succor. But, because there are some differences between water and atmosphere, the factors that influence position will be different. Hence, in this paper, first, the feasibility of apply space resection model underwater is analyzed based on the propagation law of acoustic ray using real surveying data of water property. And then, the relationship between the underwater acoustic positioning accuracy of one positioning model and sound wave bending was deduced.

1. INTRODUCTION

Underwater acoustic positioning system(Wang Quan, Cheng Pengfei,2006, Wang Zemin,2005) is a system which used sound wave to position the object under water. The reason of this system choose sound wave as the positioning signal but not electromagnetic wave is that the electromagnetic wave attenuate quickly along with the transmitted distance becomes long in the water. And this is the big difference from this system to GNSS. However, most underwater acoustic positioning system should use GNSS to position the fixture on the surface of water.

Traditionally, the underwater acoustic positioning system includes three classes: Long Base Line System, Short Base Line System and Ultra-Short Base Line System. In 1991, James Youngberg advanced a concept named underwater GPS. Now, it is can be called underwater GNSS(UGNSS) which is based on a surface buoys network that measures the time-of-fight of acoustic signals emitted by an acoustic transmitter mounted on a n object.

2. POSITIONING MODEL

The primary positioning method of underwater acoustic positioning system is angle resection and space resection. Supposed the emanatory time of signal from acoustic transmitter is t_0, the arrival time to the acoustic receiver is t_i, and the average sound speed on this travel rode is $\upsilon_{s,i}$, then, the positioning model of space resection is：

$$\rho = \upsilon_{s,i}(t_i - t_0) \tag{1}$$

The coordinate of acoustic transmitter is (x, y, z), while the

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coordinate of acoustic receiver is (x_i, y_i, z_i). To be noted that the coordinates of acoustic receivers are surveyed by GNSS. Then, the geometrical distance between this two points is

$$\rho' = \sqrt{(x_i - x)^2 + (y_i - y)^2 + (z_i - z)^2} \tag{2}$$

i.e.

$$\upsilon_{s,i}(t_i - t_0) \approx \sqrt{(x_i - x)^2 + (y_i - y)^2 + (z_i - z)^2} \tag{3}$$

Actually, there are two methods to resolve this model. One is resolve formula(3) directly, the other is make difference between two distances of sound waves with formula

$$\upsilon_{s,i}(t_i - t_0) - \upsilon_{s,j}(t_j - t_0) = \\ \sqrt{(x_i - x)^2 + (y_i - y)^2 + (z_i - z)^2} - \\ \sqrt{(x_j - x)^2 + (y_j - y)^2 + (z_j - z)^2} \tag{4}$$

Whatever used in work, both of them have a same problem that how long the sound wave bending is allowed. This problem determines the positioning accuracy finally.

3. SOUND WAVE BENDING

The difference between actual distance and geometrical distance from pinger to receiver is

$$d\rho = \rho - \rho' \qquad (5)$$

While ρ is actual distance(curvilinear distance) between pinger and receiver, and ρ' is the geometrical distance.

The bigger the $d\rho$, the lower the accuracy will be。When $d\rho$ is too big, the space resection model would not work .

3.1 The travel rule of acoustic ray—Snell law

Snell law(R.J.Ulak . Hong Shen(translate,1972)) is a transmittable law of sound wave, which describes the refractive characteristic of acoustic ray. Divided the seawater into thousands of thinnest and symmetrical layers, and suppose that there are refraction effects except reflective effects between two layers, sign the layers with number $1, 2, \cdots, n, n+1, \cdots$, there are relations that

$$\frac{\cos\chi_1}{c_1} = \frac{\cos\chi_2}{c_2} = \cdots$$
$$= \frac{\cos\chi_n}{c_n} = \frac{\cos\chi_{n+1}}{c_{n+1}} = \cdots \qquad (6)$$

Where, χ_i is the grazing angle of acoustic ray in every layer , c_i is the sound speed of each layer, $i = 1, 2, \cdots, n, n+1, \cdots$。 And then the Snell law can be denote as

$$\cos\chi / c = \text{constant} \qquad (7)$$

Where χ is the grazing angle, which is ransacked by the ray and the plane of sound speed.

Snell law could be expressed to be the relationship of an angle of refraction and the sound speed as:

$$\frac{\sin\theta_{n+1}}{c_{n+1}} = \frac{\sin\theta_n}{c_n} \qquad (8)$$

3.2 The trajectory of acoustic ray

In the initial study of underwater acoustic, the water was divided into a series of uniform layers. The sound speeds are different between two contiguous layers. And the acoustic rays in these thinnest layers are line. But the travel direction of the acoustic ray changes when the ray travels from one layer to another. The schematic diagram of the trajectory of acoustic ray is as fig 1.
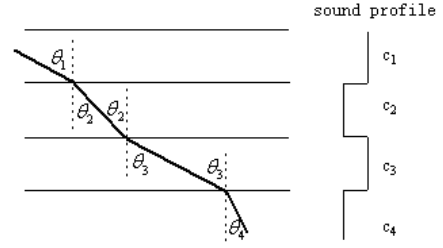

Fig 1.   the refraction of acoustic ray in layered medium

where c_1, c_2, c_3, c_4 are sound speed of corresponding layer, θ_i is the emission angle of corresponding layer

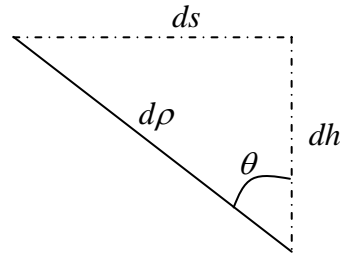### 3.3 The propagation distance of acoustic ray

Fig 2.   the unit of acoustic ray

Suppose that the numerical sound speeds are constant in every layer, and the travel roads of ray in these layers are line as is shown in fig 1. There is a refraction effect which changes the transmitting direction of acoustic ray between two layers. The relations between angles and side length (depth, horizontal distance and slant range on the propagation path) of one unit in figure 2 are as follows:

$$d\rho = \frac{dh}{\cos\theta} , \qquad (9)$$

$$ds = \tan\theta dh .$$

Suppose that B is the point which acoustic arrives, and its depth is z. P is the sound source, and its depth is z_0. Therefore, the real length of the acoustic ray is

$$\rho_{BP} = \rho = \int_{z_0}^{z} \frac{1}{\cos\theta} dh \qquad (10)$$

The horizontal distance of the acoustic ray is

$$D_{BP} = S = \int_{z0}^{z} \tan\theta dh \qquad (11)$$

When the depths change, the θ also change. But the angle θ in each layer could be calculated by Snell law with sound speed in each layer and emission angle in the sound source.

Water will be divided into n layers of the same speed of sound, the layers within the constant speed of sound transmission, the formula above can be write as :

$$\rho = \sum_{i=1}^{n} \frac{dh}{\cos \theta_i} \qquad (12)$$

$$S = \sum_{i=1}^{n} \tan \theta_i dh \qquad (13)$$

3.4 Valuation of sound wave bending

One of the big factor that made sound wave bending is water character. The change of water character made a change in the sound speed of water. The sound speed can be calculated by experiential formulas which have relationship with water character. There are surveying data that can be used to research the sound wave bending. This research can be divided to two steps. First, the sound speed of water is calculated using date of temperature, salinity and pressure of water which was surveyed by Argo buoy(Sun Chaohui, Xu jianping, 2008) from the water depth of 2000 m and 8000 m to the sea surface by Mackenzie experiential formula(Alkan R.M. et al. 2006, Yang Shi'e,1994). Then, Given an emission angle of sound ray from 0o to 80o, using formulas before, the sound wave bending is calculated. And the results is shown in figure3, figure 4, table 1 and table 2.
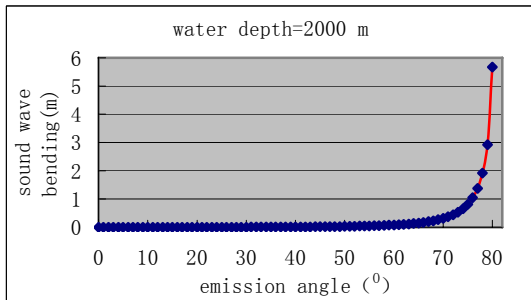

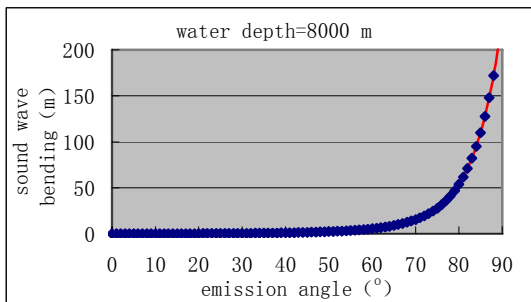Figure 3.   sound wave bending (the sound wave emitted from 2000m depth)

Figure 4.   sound wave bending (the sound wave emitted from 8000m depth)

Emission angle（°）	0	20	40	60	75	80
Sound wave bending（m）	0	0.001	0.012	0.082	0.816	5.674

Table 1.   the value of sound wave bending at some emission angles(the sound wave emitted from 2000m depth)

Emission angle（°）	0	40	60	80	89
Sound wave bending （m）	0	0.999	5.503	53.788	200.329

Table 2.   the value of sound wave bending at some emission angles(the sound wave emitted from 8000m depth)

Two of the figures and tables show the sound wave bending at cases study area of 47.956oN and -128.971oW from 0o to 80o and 0o to 89o of sound emission angles. According to figures and tables above, it can be clearly seen that the sound wave bending grows as the sound emission angles increasing. And there are a sharp increase in the value of sound wave bending after 75o where the water depth is 2000m while the rapid increase point is at 70o where the water depth is 8000m.

Also from the two tables, it can be noticed that the sound wave bending are smaller in 2000m depth than in 8000m depth when the emission angle is same. And, the sound wave bending is too big while the water depth is 8000m and the emission angle is more than 80o.

In conclusion, sound wave transmit from pingers at a depth underwater to receivers at surface of water. The deeper the location of pingers under water, the bigger the sound wave bending will be. The bigger the emission angle, the bigger the sound wave bending will be. In fact, these factors would finally decrease the positioning accuracy.

3.5 The effect of sound wave bending

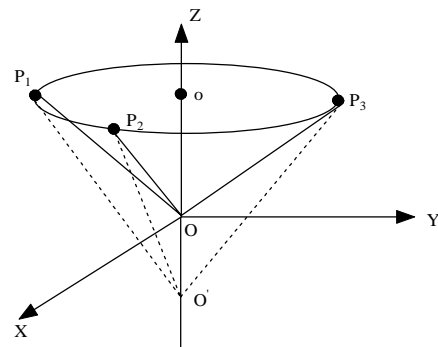Taken the location of pinger as the centre, a coordinate system of right hand was establish(shown in figure 5.).

Figure 5.   Coordinate system of surveying station

Suppose that the signal receivers are situated at P_1, P_2, P_3 which are at a circle . The pinger is just under the centre of this circle. So the distances from this pinger to the three receivers are same. And the coordinates of these points are $P_1(0,-Y,Z)$, $P_2(X,0,Z)$, $P_3(0,Y,Z)$, $O(0,0,0)$.

The position of pinger could be calculated by three of the receivers without errors if there have no random mistakes, systemic errors. But this situation is impossible in fact. There are always some factors that effect the positioning accuracy, such as sound wave bending. In case of there are sound wave bending when sound wave transmitted from pinger to arrivers, the surveying distances (the left value of formula (3)) will be longer than the real distances(the right value of formula (3)).

As a result, the location of pinger will be set at $O^{'}(X_{O^{'}}, Y_{O^{'}}, Z_{O^{'}})$. Supposed that the sound wave bending is same along with three sound ray and there are no other influencing factors, the coordinates of $Q^{'}$ can be described as $(0, 0, Z_{O^{'}})$, and the relationship between distances and coordinates are

$$\begin{aligned} \rho + \Delta\rho &= \sqrt{(-Y)^2 + (Z - \Delta Z)^2} \\ &= \sqrt{(X)^2 + (Z - \Delta Z)^2} \\ &= \sqrt{(Y)^2 + (Z - \Delta Z)^2} \end{aligned} \tag{14}$$

Because $\Delta Z < 0$, $X = Y$,

$$\Delta Z = Z - \sqrt{(\rho + \Delta\rho)^2 - X^2} \tag{15}$$

where, ρ is the real distance from pinger to receivers, and $\Delta\rho$ is the distance error which caused by sound wave bending, and ΔZ is the positioning error of pinger.

From formula (15) we can see that the relation between ΔZ and $\Delta\rho$ is nonlinear. A section plane of figure 5 is described at figure 6.
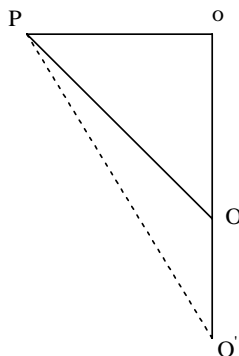

Figure 6.   A section plane of coordinate system

From the relationship between three of the triangular boundary that

$$\overline{OO^{'}} > \overline{PO^{'}} - \overline{PO} \tag{16}$$

i.e.

$$\overline{OO^{'}} > \Delta\rho \tag{17}$$

So, the positioning error of pinger is bigger than sound wave bending.

4.   CONCLUSION

Marine accounted for 71 percent of Earth's surface area, are hidden wealth of resources and energy. Human beings in the world today is facing a "Growing global population, Land and energy resources more grim" crisis. The development of marine resources is the future of economic development trends. In this process, underwater positioning and navigation technology would play an important role. It can help to make sure the accurate location of development area, positioning the workers and so on. And it can also help to search the area of disaster and give help to succour.

Underwater positioning system choose sound wave as the positioning signal. Sound wave bending is one of the great factor that influence the positioning accuracy. This paper analyse the theory of sound wave bending and calculate it with actual data. The data shows that the water depth and emission angle of sound wave are the great influence factors at underwater positioning system.

REFERENCES

Alkan R.M.，Kalkan Y.，Aykut O.，TüRKYE, 2006．Sound Velocity Determination with Empirical Formulas & Bar Check[J]．Munich，Germany. http://www/fig.net/pub/pig2006/papers/ps04.

R.J.Ulak．Hong Shen(translate,1972)．Water Acoustic Principle in engineering．*National Defense Industry Publishing House*, pp. 113-123.

Sun Chaohui, Xu jianping, 2008. Application of Argo Data in the Analysis of Water Masses in the Northwest Pacific Ocezn. Marine Science Bulletin.

Wang Quan, Cheng Pengfei, 2006. Underwater positioning system based on DGPS. *Science of Surveying and Mapping*, China. pp. 18-21.

Wang Zemin, 2005. Underwater and High Precision 3D Navigation and Positioning System. *Acoustics and Eleclrical Engineering*. China. pp. 1-3.

Yang Shi'e, 1994. The Transmitting Principle of Water Acoustic. *Harbin Engineering Publishing House*, China.