

# A COMPARATIVE ANALYSIS OF TWO APPROACHES FOR MULTIPLE-SURFACE REGISTRATION OF IRREGULAR POINT CLOUDS

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## ABSTRACT:

In close range photogrammetry or laser scanning, it is often not possible to image or scan certain objects of interest or enclosed spaces from a single sensor station. In some cases, it is necessary to produce multiple irregular point clouds or surface models from different sensor locations, i.e. one unique point cloud from each sensor location. These separate point clouds or surface models belong to different coordinate systems, and in order to fuse all points in a single dataset, the surface models have to be registered to a common reference frame. This paper describes a methodology for performing registration of such multiple surface models. First, conjugate point-patch pairs are detected in the overlapping surface areas, and the transformation parameters between all neighbouring surfaces are estimated in a pairwise manner. Then, using the conjugate point-patch pairs, and applying the transformation parameters from the pairwise registration as initial approximations, the final surface transformation parameters are solved for simultaneously. This is done in a least-squares adjustment, where each surface is iteratively transformed to a common reference frame until the sum of the squared normal distances between the conjugate surface elements is minimized. This paper will show two ways of performing this least-squares adjustment. One is referred to as the coplanarity constraint method, and the other one as the modified weight matrix method. The paper will compare results for the multiple-surface registration of an artificial scoliotic torso mannequin using both approaches.

## 1. INTRODUCTION

In close range photogrammetry or laser scanning, it is often not possible to image or scan certain objects of interest or enclosed spaces from a single sensor station. For example, if a human torso or an inanimate object is being reconstructed, it is necessary to integrate multiple partial scans of the subject or object of interest, in order to generate a full 360° 3D model. These multiple partial scans must be overlapping, and they can be derived from stereo imaging or laser scanning. Another example would be that if a laser scanner is used to perform an as-built survey for an industrial plant, numerous instrument locations must be chosen in order to fill in the gaps from obstruction occlusions and thus get full coverage for the volume of interest. In such cases, separate irregular point clouds or surface models are produced for each of the sensor locations. These point clouds or surface models belong to different coordinate systems, and in order to fuse all points in a single dataset, the surface models have to be registered to a common reference frame. Thus, the integration of multiple point clouds is a registration procedure. In general, the elements of the registration paradigm could be classified in four categories: (1) geometric primitives, (2) transformation function, (3) similarity measure, and (4) matching strategy (Fonseca and Manjunath, 1996). The primitives are the domain in which information is extracted from the input data, e.g. points, lines, planes, triangular patches, etc. The transformation function is what mathematically describes the mapping process between the data, e.g. a 3D similarity transformation. The similarity measure is the necessary constraint for ensuring the correspondence of conjugate primitives. Finally, the matching strategy is the

controlling framework that uses the primitives, the transformation function, and the similarity measure to solve the registration problem (Brown, 1992).

A well-known algorithm for registering point clouds is the iterative closest point, also known as ICP (Besl and McKay, 1992). In the light of registration paradigm definitions, the ICP method uses points as the geometric primitives, it applies a 3D similarity transformation as the transformation function, and in terms of the similarity measure, it minimizes Euclidean distances between conjugate points in two overlapping surface models. The matching strategy is implemented in an iterative manner. More specifically, starting from an approximate estimate of the parameters of the transformation function relating two point clouds, hypothesized conjugate points are made by identifying the closest point in one of the datasets to a transformed point from the second one. Hypothesized matches are generated for all the points in the overlap area. These matches are then used to estimate a refined estimate of the transformation function parameters, which are then used to derive a new set of hypothesized matches. The process of hypothesized-match generation and parameters estimation are repeated until convergence. However, when dealing with irregular point clouds exact point-to-point correspondences between surface models cannot be guaranteed. Some variations of the ICP algorithm, which employ more appropriate geometric primitives, exist. For example, Chen and Medioni (1992) minimized the normal distance between points in one surface model and planes in another surface model. Such point-to-plane correspondence may be assumed to exist, however, the algorithm requires local plane fitting. Another example for a surface registration procedure is the one developed by Habib et

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al. (2006), where a modified iterated Hough transform is used as the matching strategy.

This paper will show two methods for performing surface registration of irregular point clouds. Both methods use point-to-patch correspondence, where points in one surface model and triangular irregular network (TIN) patches in another surface model serve as the geometric primitives. This choice of geometric primitives does not assume existing point-to-point correspondence, and at the same time, it does not require any local fitting. The only preprocessing that is necessary is the generation of the TIN patches, which is a common function in most geographical information system (GIS) or laser scanning software. The two methods, referred to as the coplanarity constraint and the modified weight matrix, also share the same transformation function. The only difference is the similarity measure used in the implementation of the least-squares algorithm when solving for the registration transformation parameters. The rest of the paper will describe the two methods in detail. In addition, the paper will also explain how the two methods were extended to handle not only pairwise registration between two overlapping surface models, but also multiple-surface registration in a network mode. Such multiple-surface registration is necessary in order to minimize any error propagating from the pairwise-surface registration.

## 2. METHODOLOGY FOR PAIRWISE-SURFACE REGISTRATION

Due to the irregular nature of point clouds describing surface models generated from close range photogrammetry or laser scanning, exact point-to-point correspondence cannot be assumed. In this paper, the geometric primitives chosen for the registration of point clouds are points and triangular patches. Thus, for any two overlapping surface models, one of the point clouds is kept as is, and the other one is converted to a TIN, i.e. one of the surface models is represented by the original points, and the other surface model is represented by the triangular patches from the TIN. It is important to note that the TIN patches are an acceptable primitive only in the cases when the TIN model represents the true physical surface of the reconstructed object or the scene of interest (Habib et al., 2010). This means that the surface models have to have point density high enough that no triangles are built across what would be a breakline. If this is true, then it can be assumed that point-to-patch correspondence between the overlapping surface models does exist (see Figure 1). In order to deal with cases where the TIN does not represent the physical surface (e.g. sparse areas in the point clouds), a threshold is implemented in the matching strategy (explained later on in this section). So, if it is safe to assume that point P in surface one ( $S_1$ ) corresponds to the triangular patch with vertices  $V_1$ ,  $V_2$ , and  $V_3$  in surface two ( $S_2$ ), then this point should coincide with the patch after applying the transformation parameters in equation (1).

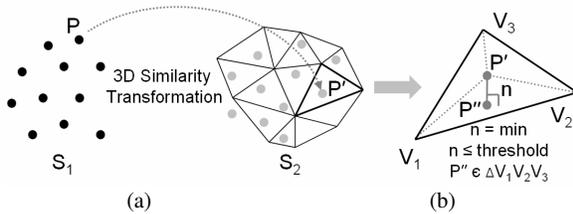


Figure 1. Surface model representation (a), and criteria for accepting correspondence between conjugate primitives (b)

$$\begin{bmatrix} X_{P'} \\ Y_{P'} \\ Z_{P'} \end{bmatrix} = \begin{bmatrix} X_T \\ Y_T \\ Z_T \end{bmatrix} + s \cdot \mathbf{R}_{3 \times 3}(\omega, \phi, \kappa) \cdot \begin{bmatrix} X_P \\ Y_P \\ Z_P \end{bmatrix} \quad (1)$$

where  $X_P, Y_P, Z_P$  = coordinates of point P  
 $X_T, Y_T, Z_T$  = translation parameters  
 $\mathbf{R}$  = rotation matrix defined by angles  $\omega, \phi, \kappa$   
 $s$  = scale factor  
 $X_{P'}, Y_{P'}, Z_{P'}$  = coordinates of the transformed point P'

The correct correspondence between points in surface  $S_1$  and triangular patches in surface  $S_2$  is established through an iterative procedure. It proceeds as follows: first, initial point-patch pairs are determined after applying approximate transformation parameters, which bring  $S_1$  in the reference frame of  $S_2$  (Figure 1a); then, these initial point-patch pairs are used to calculate a better estimate of the transformation parameters between  $S_1$  and  $S_2$ ; and the updated transformation parameters are used to determine a new set of point-patch pairs. The procedure is repeated until the transformation parameters converge, and there is no change in the point-patch correspondence. A point-patch pair is considered a valid conjugate match under three conditions (see Figure 1b). First, the particular triangular patch,  $\Delta V_1 V_2 V_3$ , must be the closest to the transformed point of interest, P'. Second, the normal distance,  $n$ , from the transformed point, P', to the patch must be within a certain threshold. This is the threshold mentioned earlier, which is incorporated in order to be able to deal with cases where the TIN does not represent the physical surface. Lastly, the projection of the transformed point onto the patch, P'', must be inside the patch. At the end of this registration procedure the goodness of fit between the two surfaces is evaluated by calculating the average normal distance for all the matched point-patch pairs (Habib et al., 2009). So far, the choice of geometric primitives, the transformation function, and the matching strategy for the pairwise-surface registration has been described. The next two subsections will explain the similarity measures used in the calculation of the registration transformation parameters for the coplanarity constraint and the modified weight matrix methods.

### 2.1 Coplanarity Constraint Method

In the coplanarity constraint method, P',  $V_1$ ,  $V_2$ , and  $V_3$  are assumed to be coplanar. This means that the volume of the pyramid, whose vertices are P',  $V_1$ ,  $V_2$ , and  $V_3$  in Figure 1b, should be zero. This can be mathematically expressed as the determinant in equation (2).

$$\det \begin{bmatrix} X_{P'} & Y_{P'} & Z_{P'} & 1 \\ X_{V_1} & Y_{V_1} & Z_{V_1} & 1 \\ X_{V_2} & Y_{V_2} & Z_{V_2} & 1 \\ X_{V_3} & Y_{V_3} & Z_{V_3} & 1 \end{bmatrix} = 0 \quad (2)$$

By using numerous point-patch pairs, which satisfy this coplanarity constraint, the transformation parameters relating the two surface models can be estimated. In order to obtain reliable estimates of these transformation parameters, variations in the topography of the surfaces are needed so that there are constraints in as many directions as possible.

## 2.2 Modified Weight Matrix Method

In the modified weight matrix method, the transformation function in equation (1) is used as the basis for the mathematical model for the similarity measure.

$$\begin{bmatrix} X_{V_1} \\ Y_{V_1} \\ Z_{V_1} \end{bmatrix} = \begin{bmatrix} X_P \\ Y_P \\ Z_P \end{bmatrix} + \mathbf{e}_{3 \times 1} \quad (3)$$

where  $\mathbf{e}$  = mis-closure vector between hypothesized conjugate points after applying the transformation function to point P in  $S_1$

As seen from equation (3), the similarity measure for this surface registration method is a point based approach, where it is assumed that one of the triangular patch vertices is conjugate to the transformed point P'. However, such correspondence is not necessarily true. To compensate for the fact that we are using a point-based procedure while using non-conjugate points in a point-patch pair, the weights associated with the similarity measure in equation (3) are modified. More specifically, the weight matrix is modified to ensure the minimization of the mis-closure vector in direction normal to the TIN patch in question. In other words, due to the lack of point-to-point correspondence, there would be a spatial offset between point P' in  $S_1$  and vertex  $V_1$  in  $S_2$  in all three directions. However, the weights in the least-squares adjustment are modified in such a way that the transformation parameters are estimated to minimize the spatial offset normal to the triangular patch. This is accomplished by the following sequence of operations (Aldelgawy et al., 2008):

1. Compute the rotation matrix, R, which transforms the coordinates of the point from the original coordinate system (X,Y,Z) to the local coordinate system of the triangular patch (U,V,W), where the U and V axes are within the patch plane and the W axis is normal to the patch plane
2. Compute the weight matrix in the (U,V,W) coordinate system according to the law of error propagation:

$$P_{UVW} = R \cdot P_{XYZ} \cdot R^T \quad (4)$$

where  $P_{XYZ}$  = weight matrix in the (X,Y,Z) coordinate system, and  
 $P_{UVW}$  = weight matrix in the (U,V,W) coordinate system

3. Modify the weight matrix in the (U,V,W) coordinate system by assigning zero for the weights along the triangular patch:

$$P'_{UVW} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & P_W \end{bmatrix} \quad (5)$$

4. Compute the modified weight matrix in the (X,Y,Z) coordinate system:

$$P'_{XYZ} = R^T \cdot P'_{UVW} \cdot R \quad (6)$$

5. Apply a point-based solution using least squares with the modified weight matrix,  $P'_{XYZ}$ .

Essentially, in the modified weight matrix method, the sum of the squared random errors along the triangular patch normal, are minimized. It is important to note that even though it seems like there are three observations equations for every point-patch pair, the net contribution of the constraint towards the redundancy estimation is one, because the rank of the modified weight matrix is one. Therefore, the redundancy for this method is equal to the redundancy of the coplanarity constraint one. Again, as with the coplanarity constraint, in order to obtain reliable estimates of the transformation parameters, variations in the topography of the surfaces are needed so that there are constraints in as many directions as possible.

## 3. METHODOLOGY FOR MULTIPLE-SURFACE REGISTRATION

The previous section explained two approaches for registering two overlapping surface models, a procedure referred to as pairwise registration. However, as explained in the introduction, in cases of a full 360° reconstruction of an object or a scene, there might be multiple surface models covering the volume of interest. Each surface model will be in a different reference frame, so it is necessary to register the multiple surfaces in a common reference frame. One way of achieving this is to choose the reference frame of one of the surface models as the common one, and then to sequentially register the rest of the surface models in the pairwise manner described before. However, the first and the last surface models might exhibit incompatibility due to errors propagated through the sequential registration process. This will be similar to a closed loop traverse in surveying, where the constraint that the first and last point coincide, has not been used. To avoid such an incompatibility, the multiple-surface registration has to be performed simultaneously, i.e. in a network mode. This procedure can be viewed as an extension to the pairwise registration. In particular, the pairwise registration procedure is used to accumulate a list of the corresponding point-patch pairs and estimate the transformation parameters between any two overlapping surface models. The multiple-surface registration, on the other hand, uses these corresponding point-patch pairs, and applies the transformation parameters from the pairwise-surface registration as initial approximates, to simultaneously solve for all the final surface transformation parameters. This is done in a least-squares adjustment, where each surface is iteratively transformed to a common reference frame until the sum of the squared normal distances between the conjugate point-patch pairs is minimized. This procedure is highly non-linear, so that is why the initial approximates from the pairwise-surface registration are necessary. It is also important to note that the transformation parameters for one of the surface models are kept fixed in order to define the datum for the final surface model. The transformation function for the multiple-surface registration is similar to the one in the pairwise-surface registration, except both point P, and triangular patch vertices  $V_1$ ,  $V_2$ , and  $V_3$  for a given conjugate point-patch pair are transformed to the reference frame of choice as shown in equations (7) and (8).

$$\begin{bmatrix} X_{P'} \\ Y_{P'} \\ Z_{P'} \end{bmatrix} = \begin{bmatrix} X_{T_i} \\ Y_{T_i} \\ Z_{T_i} \end{bmatrix} + s_i \cdot \mathbf{R}_{3 \times 3}(\omega_i, \phi_i, \kappa_i) \cdot \begin{bmatrix} X_P \\ Y_P \\ Z_P \end{bmatrix} \quad (7)$$

$$\begin{bmatrix} X_{V'_{1,2,3}} \\ Y_{V'_{1,2,3}} \\ Z_{V'_{1,2,3}} \end{bmatrix} = \begin{bmatrix} X_{T_j} \\ Y_{T_j} \\ Z_{T_j} \end{bmatrix} + s_j \cdot \mathbf{R}_{3 \times 3}(\omega_j, \phi_j, \kappa_j) \cdot \begin{bmatrix} X_{V_{1,2,3}} \\ Y_{V_{1,2,3}} \\ Z_{V_{1,2,3}} \end{bmatrix} \quad (8)$$

where 'i' denotes the transformation parameters between the surface represented by points and the common reference frame, and 'j' denotes the transformation parameters between the surface represented by triangular patches and common reference frame

The following subsections will explain how the coplanarity constraint and the modified weight matrix methods are applied to solve for the final surface transformation parameters in the multiple-surface registration.

### 3.1 Coplanarity Constraint Method

The mathematical model describing the coplanarity constraint for the multiple-surface registration is similar to equation (2), except that both point P and vertices  $V_1$ ,  $V_2$ , and  $V_3$  are transformed to the common reference frame. So, in this case, the volume of the pyramid with vertices  $P'$ ,  $V_1'$ ,  $V_2'$ , and  $V_3'$  should be zero. Mathematically, this is expressed as the determinant in equation (9).

$$\det \begin{bmatrix} X_{P'} & Y_{P'} & Z_{P'} & 1 \\ X_{V_1'} & Y_{V_1'} & Z_{V_1'} & 1 \\ X_{V_2'} & Y_{V_2'} & Z_{V_2'} & 1 \\ X_{V_3'} & Y_{V_3'} & Z_{V_3'} & 1 \end{bmatrix} = 0 \quad (9)$$

### 3.2 Modified Weight Matrix Method

In the modified weight matrix method for the multiple-surface registration, the transformation functions in equations (7) and (8) serve as the basis for the mathematical model for the similarity measure.

$$\begin{bmatrix} X_{V_1'} \\ Y_{V_1'} \\ Z_{V_1'} \end{bmatrix} = \begin{bmatrix} X_{P'} \\ Y_{P'} \\ Z_{P'} \end{bmatrix} + \mathbf{e}_{3 \times 1} \quad (10)$$

Again, due to the lack of point-to-point correspondence, there would be a spatial offset between point  $P'$  in  $S_1$  and vertex  $V_1'$  in  $S_2$  in all three directions. However, the weights in the least-squares adjustment are modified in such a way that the transformation parameters are estimated through minimizing the component of the spatial offset between non-conjugate points within a point-patch pair along the normal to the triangular patch. This is accomplished by implementing the defined procedure in section 2.2.

## 4. EXPERIMENTAL RESULTS AND DISCUSSION

The two methods for performing multiple-surface registration were tested using two datasets of an artificial scoliotic torso mannequin. The two datasets were derived from a photogrammetric system which had multiple cameras and projectors (Chang et al., 2009). The approximate dimensions of the torso mannequin were height of 60cm, width of 40cm, and depth of 25cm. Each dataset had four overlapping point clouds representing four partial models of the torso surface, i.e. the back, the front, and the two sides (see Figure 2 for a visual example). The point clouds for the back and the front had between 20,000 and 22,000 points, while the point clouds for the sides had between 16,000 and 18,000 points.

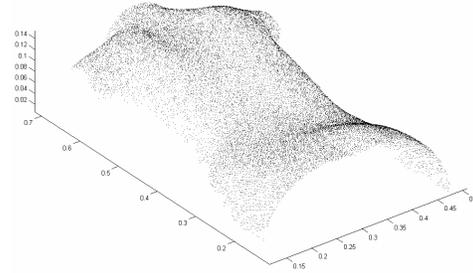


Figure 2. Example of a point cloud representing a partial model of the torso surface

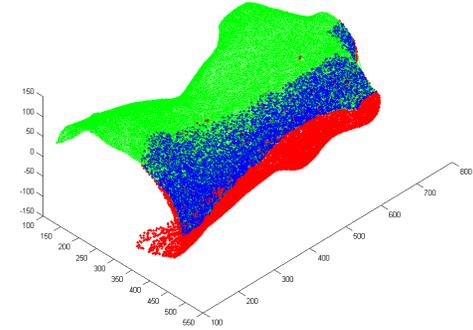


Figure 3. Example of a pairwise registration of two overlapping surfaces (green: non-matched points from the back model; blue: matched area; red: non-matched points from the side model)

As described in the previous sections, TIN models were first created for all the point clouds. In this way, pairwise registration could be performed between all the overlapping surfaces (e.g.  $S_2 \rightarrow S_1$ ,  $S_3 \rightarrow S_2$ ,  $S_4 \rightarrow S_3$ , and  $S_1 \rightarrow S_4$ ) for both datasets. The output from the pairwise registration was the set of transformation parameters between the neighbouring surfaces and the conjugate point-patch pairs in the overlapping regions. On average, there were about 7,000 conjugate point-patch pairs for each of the four overlapping regions in both datasets (see Figure 3 for a visual example for the pairwise registration of two overlapping surfaces).

	S <sub>2</sub>	S <sub>3</sub>	S <sub>4</sub>
X <sub>T</sub> [mm]	291.49 ±0.04	556.89 ±0.02	267.31 ±0.05
Y <sub>T</sub> [mm]	-26.56 ±0.02	-11.38 ±0.02	14.90 ±0.02
Z <sub>T</sub> [mm]	265.07 ±0.02	-27.01 ±0.03	-290.76 ±0.02
ω [°]	93.1381 ±0.1412	8.3861 ±0.0018	-89.2218 ±0.0886
φ [°]	92.8342 ±0.0029	-179.8875 ±0.0056	-84.4278 ±0.0029
κ [°]	-87.5881 ±0.1415	2.7046 ±0.0023	-92.1119 ±0.0893

Table 1. Transformation parameters and their standard deviations after multiple-surface registration using the coplanarity constraint for the first dataset

	S <sub>2</sub>	S <sub>3</sub>	S <sub>4</sub>
X <sub>T</sub> [mm]	291.51 ±0.04	556.91 ±0.03	267.34 ±0.05
Y <sub>T</sub> [mm]	-26.48 ±0.03	-11.31 ±0.03	15.06 ±0.02
Z <sub>T</sub> [mm]	265.07 ±0.02	-26.93 ±0.05	-290.72 ±0.02
ω [°]	92.9492 ±0.1508	8.3643 ±0.0039	-89.0556 ±0.0953
φ [°]	92.8210 ±0.0036	-179.8922 ±0.0081	-84.4546 ±0.0034
κ [°]	-87.4085 ±0.1509	2.7035 ±0.0044	-91.9283 ±0.0959

Table 2. Transformation parameters and their standard deviations after multiple-surface registration using the modified weight matrix constraint for the first dataset

The output from the pairwise-surface registration served as the input for the multiple-surface registration. That is, the final transformation parameters from the pairwise registration were used as the initial transformation parameters for the multiple-surface registration, and the detected conjugate point-patch pairs from the pairwise registration were used for both the coplanarity constraint and modified weight matrix methods for multiple-surface registration. In both datasets, the first surface was chosen as the common reference frame, so its transformation parameters were fixed as zeros for the translations and the rotations. Also, in all cases, the scale for the 3D similarity transformation was fixed as one, so technically a rigid body transformation was implemented. Table 1 shows the final transformation parameters and their standard deviations resulted from the multiple-surface registration for the first dataset using the coplanarity constraint method. Table 2 shows the same results for the modified weight matrix method. The average normal distances between the matched point-patch pairs for the transformed surfaces were in the order of 0.3mm for both methods.

	RMS <sub>X</sub> [mm]	RMS <sub>Y</sub> [mm]	RMS <sub>Z</sub> [mm]	RMSE <sub>XYZ</sub> [mm]
S <sub>2</sub>	0.07	0.02	0.09	0.12
S <sub>3</sub>	0.03	0.05	0.12	0.13
S <sub>4</sub>	0.09	0.19	0.05	0.22

Table 3. RMSE between point clouds registered by the coplanarity constraint and the modified weight matrix methods for the first dataset

It could be noticed that, even though quite close, the two sets of transformation parameters were not identical. In order to test their equivalency, the original points of surfaces S<sub>2</sub>, S<sub>3</sub>, and S<sub>4</sub> were transformed with both sets. Then, the root mean squared error (RMSE) between the resulted pairs of transformed points was calculated. As seen from Table 3, the total RMS values were less than the average normal distances of 0.3mm between the transformed surfaces. This means that the RMS values were less than the measurement noise, and the two sets of transformation parameters were deemed equivalent.

	S <sub>2</sub>	S <sub>3</sub>	S <sub>4</sub>
X <sub>T</sub> [mm]	270.19 ±0.04	529.99 ±0.02	259.44 ±0.05
Y <sub>T</sub> [mm]	-34.52 ±0.02	-11.16 ±0.02	23.37 ±0.02
Z <sub>T</sub> [mm]	257.08 ±0.02	-13.09 ±0.03	-270.17 ±0.02
ω [°]	-260.4508 ±0.0853	12.4952 ±0.0019	-85.3906 ±0.0709
φ [°]	94.8686 ±0.0030	179.6588 ±0.0060	-82.4158 ±0.0031
κ [°]	268.0451 ±0.0857	2.6986 ±0.0025	-90.2563 ±0.0717

Table 4. Transformation parameters and their standard deviations after multiple-surface registration using the coplanarity constraint for the second dataset

	S <sub>2</sub>	S <sub>3</sub>	S <sub>4</sub>
X <sub>T</sub> [mm]	270.24 ±0.04	529.92 ±0.03	259.32 ±0.05
Y <sub>T</sub> [mm]	-34.64 ±0.03	-11.25 ±0.03	23.45 ±0.02
Z <sub>T</sub> [mm]	256.94 ±0.03	-13.34 ±0.05	-270.33 ±0.02
ω [°]	-260.2185 ±0.0898	12.5197 ±0.0041	-85.4321 ±0.0747
φ [°]	94.8807 ±0.0037	179.6647 ±0.0085	-82.3933 ±0.0036
κ [°]	267.8207 ±0.0901	2.7146 ±0.0046	-90.3238 ±0.0755

Table 5. Transformation parameters and their standard deviations after multiple-surface registration using the modified weight matrix constraint for the second dataset

Similarly, Table 4 shows the final transformation parameters and their standard deviations for the multiple-surface registration using the coplanarity constraint for the second dataset. Table 5 shows the same results for the modified weight matrix method. Again, the average normal distances between

the transformed surfaces were in the order of 0.3mm, and the total RMS values showing the equivalency between the estimated transformation parameters by the coplanarity constraint and the modified weight matrix methods were at the 0.1mm level (see Table 6).

	RMS <sub>X</sub> [mm]	RMS <sub>Y</sub> [mm]	RMS <sub>Z</sub> [mm]	RMSE <sub>XYZ</sub> [mm]
S <sub>2</sub>	0.07	0.07	0.04	0.11
S <sub>3</sub>	0.06	0.03	0.08	0.10
S <sub>4</sub>	0.06	0.03	0.08	0.10

Table 6. RMSE between point clouds registered by the coplanarity constraint and the modified weight matrix methods for the second dataset

The only differences between the coplanarity constraint and the modified weight matrix approaches for multiple-surface registration was in terms of computing performance. More specifically, the coplanarity constraint method took about 30 iterations for the a-posteriori variance factor to converge to  $1 \times 10^{-10} [\text{mm}]^6$ , while the modified weight matrix method took about five iterations for the a-posteriori variance factor to converge to  $1 \times 10^{-15} [\text{mm}]^2$ . This difference was due to the fact that in the former case the errors minimized were volumetric, while in the latter case the errors were linear.

## 5. CONCLUSIONS AND RECOMMENDATIONS FOR FUTURE WORK

This paper presented two approaches for performing multiple-surface registration of overlapping point clouds. The two approaches shared the same geometric primitives, transformation function and matching strategy, but they differed in the similarity measure. In the coplanarity constraint method, the volume between conjugate points and triangular patches from the overlapping surfaces was minimized. Thus, this was a triangular patch based method. In the modified weight matrix method, for every conjugate point-patch pair, the distance between the point and one of the vertices of the triangular patch was minimized in the direction normal to the patch. Thus, in terms of the implementation, this was a point based method. The two approaches were tested with two point cloud datasets from a photogrammetric system, and they yielded equivalent results. However, since the former one minimized volumetric errors, while the latter minimized linear errors, the latter performed much quicker.

Future work will include optimizing the code to increase the speed of processing, and to be able to deal with datasets with less point density. Also, the two methods will be tested with more complex data derived from other 3D reconstruction modalities, e.g. a laser scanner. In addition, the multiple-surface registration procedures will be incorporated in other research projects dealing with applications such as change detection and infrastructure deformation monitoring.

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