

RIGOROUS MODELING OF GPS RESIDUAL ERRORS FOR PRECISE POINT POSITIONING

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ABSTRACT:

Generally, neglecting higher-order ionospheric delay can cause a range bias of several centimetres depending on satellite elevation, azimuth, and the ionospheric and geomagnetic conditions. Recent research has shown that such residual errors are correlated, which contributes to the slow convergence of precise point positioning (PPP) solution. In this paper, we attempt to rigorously model all GPS errors and biases, including the second-order ionospheric delay. Raw GPS measurements from a global cluster of international GNSS services (IGS) stations are first corrected for the effect of second-order ionospheric delay. The corrected data sets are then used as input to the Bernese GPS software to estimate the precise orbit and satellite clock corrections. Such precise products have been used in all of our PPP trials. NRCan's GPSPACE software is modified to accept the second-order ionospheric corrections as well as the new NOAA tropospheric correction product. To examine the effect of rigorous error modeling on the PPP solution, new data sets from several IGS stations were processed using the modified GPSPACE software. It is shown that the effect of second-order ionospheric delay can significantly affect the PPP solution convergence and its accuracy estimation.

1. INTRODUCTION

Ionospheric delay is considered one of the major GPS error sources, which is typically in the order of 5m to 15m. However it can reach over 150m under extreme solar activities, at midday and near the horizon (El-Rabbany, 2006). As the ionosphere is a dispersive medium, it causes frequency-dependent advances to carrier-phase measurements and delays to code measurements (Datta-Barua et al., 2006). Up to 99.9% of the ionospheric delay can be eliminated through linear combination of GPS observables on L1 and L2 frequencies (Bassiri and Hajj, 2003; Hofmann-Wellenhof, 2008). This combination is known as first-order ionosphere-free linear combination. However, this combination is still contaminated with higher-order ionospheric delay terms. Neglecting the higher-order ionospheric delay can lead to a range bias of up to several centimetres depending on satellite elevation, azimuth, and ionospheric and geomagnetic conditions (Hoque and Jakowski, 2007, 2008).

Typically, higher-order ionospheric delay corrections are not accounted for when estimating the IGS precise orbit and clock corrections. This leaves a residual error component, which is expected to be temporally correlated. This in turn affects the PPP solution convergence and the accuracy of the estimated parameters. To overcome this problem, higher order ionospheric delay corrections must be considered when estimating the precise orbit and clock corrections and when forming the PPP mathematical model. In this paper we restrict our discussion on the second-order ionospheric delay as it is much higher than the remaining higher order terms.

The second-order ionospheric delay results from the interaction of the ionosphere and the magnetic field of the Earth (Hoque and Jakowski, 2008). It depends on the slant total electron content (STEC), magnetic field parameters at the ionospheric pierce point, and the angle between the magnetic field and the direction of signal propagation. As indicated earlier, second-order ionospheric delay should be accounted for in the two

stages, i.e., while the estimation process of the GPS satellite orbit and clock corrections and in PPP observation equations. This paper demonstrates how the second-order ionospheric delay is estimated. The impact of the second-order ionospheric delay on the accuracy of the estimated GPS satellite orbit and clock corrections is also investigated. The effect of accounting for the second-order ionospheric delay on the PPP solution is also examined. It is shown that neglecting the second-order ionospheric delay introduces an error in the order of 2cm in GPS satellite orbit and clock corrections. In addition, accounting for the second-order ionospheric delay and applying the NOAA tropospheric corrections significantly affect the PPP convergence time and the accuracy of the estimated parameters.

2. GPS OBSERVATION EQUATIONS

The mathematical models of GPS observables, code and carrier-phase, could be written as (Hofmann-Wellenhof et. al., 2008; Leick, 2004):

$$P_i = \rho^{\lambda} + \frac{q}{f_i^2} + \frac{s}{f_i^3} + e_i \quad (1)$$

$$\Phi_i = \rho^{\lambda} - \frac{q}{f_i^2} - \frac{s}{2f_i^3} + \varepsilon_i \quad (2)$$

The ionosphere-free linear combination can be formed to eliminate only the first-order ionospheric delay as,

$$P_{IF} = \rho^{\lambda} + \frac{s}{f_1 f_2 (f_1 + f_2)} + e_{IF} \quad (3)$$

$$\Phi_{IF} = \rho^{\lambda} - \frac{s}{2f_1 f_2 (f_1 + f_2)} + \varepsilon_{IF} \quad (4)$$

$$q = 40.3 \int N dl = 40.3 * STEC \quad (5)$$

$$s = 7527 * c * B_0 * \cos(\theta) * STEC \quad (6)$$

where, P_i = pseudorange measurements on frequency f_i
 Φ_i = carrier-phase measurements on frequency f_i
scaled to distance
 P_{IF}, Φ_{IF} = first-order ionosphere-free code and carrier-phase, respectively
 ρ^\wedge = includes the geometric range, receiver clock error, satellite clock error, and tropospheric error
 e_i = hardware delays, multipath and other unmodeled errors on frequency f_i
 ε_i = hardware delays, multipath, initial phase bias, initial ambiguity parameter, and other unmodeled errors on frequency f_i
 e_{IF}, ε_{IF} = the first-order ionosphere-free combination of e_1, e_2 and $\varepsilon_1, \varepsilon_2$, respectively
 $STEC$ = the slant total electron content
 B_0 = the magnetic field at the ionospheric pierce point
 θ = the angle between the magnetic field and the propagation direction (Figure 1)
 N = the electron density
 c = the speed of light in vacuum

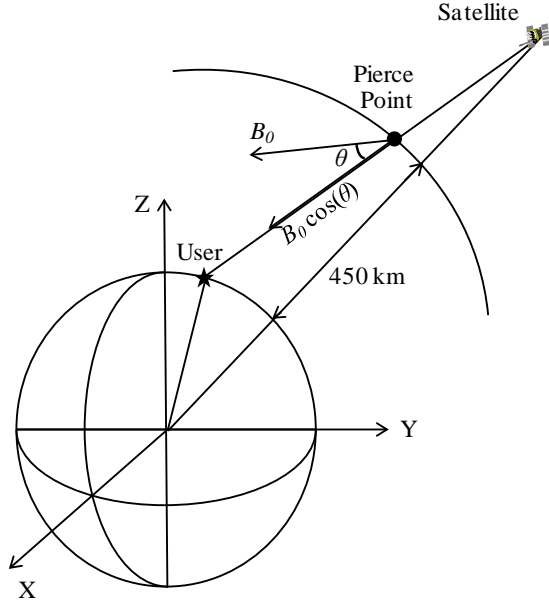


Figure 1. Magnetic Field and Propagation Direction

3. COMPUTATION OF STEC

One way to obtain the slant total electron content can be done by using external source of total electron content (TEC), e.g., local or global ionospheric models. The ionosphere exchange (IONEX) format was developed by the IGS for the exchange of global ionosphere maps (GIM) (Schaer et al., 1998a). GIMs are produced with a 2-hour temporal resolution and a 2.5° (latitude) by 5° (longitude) spatial resolution on a daily basis as rapid global maps. The rapid global maps are available with a delay

of about 12 hours, while the final maps are available with a delay of three days (Schaer et al., 1998b). An example of the regional ionospheric models is the ionospheric model developed by the NOAA space environment center. It is known as the United States total electron content (US-TEC) and covers regions across the continental US (CONUS), extending from latitude 10° to 60° North and from longitude 50° to 150° West. The US-TEC maps have a spatial resolution of 1°×1° and a temporal resolution of 15 minutes (Rowell, 2005).

Alternatively, STEC can be estimated by differencing the pseudorange observables, raw or smoothed values, on both frequencies. However, this method requires a priori information about satellite and receiver differential code biases. Values of satellite differential code bias are published by the international GNSS service (IGS) along with the corresponding values of GPS receivers occupying IGS stations. Non IGS receivers, however, must be calibrated to obtain the receiver hardware delay, which represents a drawback of this method.

$$STEC = [(P_2 - P_1) + c(DCB_{rP1-P2} + DCB_{s_{P1-P2}}^s)] \left(\frac{f_2^2}{f_1^2 - f_2^2} \right) \left(\frac{f_1^2}{40.3} \right) \quad (7)$$

where, DCB_r, DCB_s^s = receiver and satellite differential hardware delay, respectively

4. MAGNETIC FIELD MODEL

Geomagnetic field of the Earth can be approximated by a magnetic dipole placed at the Earth's center and tilted 11.5° with respect to the axis of rotation. The magnetic field inclination is downwards throughout most of the northern hemisphere and upwards throughout most of the southern hemisphere. A line that passes through the center of the Earth along the dipole axis intersects the surface of the Earth at two points, referred to as the geomagnetic poles. Unfortunately, dipole model roughly accounts for 90% of the Earth's magnetic field at the surface (Merrill and McElhinny, 1983). After the best fitting geocentric dipole is removed from the magnetic field at the Earth's surface, the remaining part of the field, about 10%, is referred to as non-dipole field. Both dipole and non-dipole parts of the Earth's magnetic field change with time (Merrill and McElhinny, 1983). The dipole approximation is more or less valid up to a few Earth radii; beyond this distance the Earth's magnetic field significantly deviates from the dipole field because of the interaction with the magnetized solar wind (Houghton et al., 1998).

A more realistic model for the Earth's geomagnetic field, which is used in this paper, is the international geomagnetic reference field (IGRF). The IGRF model is a standard spherical harmonic representation of the Earth's main field. The model is updated every 5 years. The international association of geomagnetism and astronomy (IAGA) has released the 11th generation of the IGRF in December 2009. IGRF11 coefficients are based on data collected from different sources including geomagnetic measurements from observatories, ships, aircrafts, and satellites (NOAA, 2010). The relative difference between the dipole and IGRF models ranges from -20% in the east of Asia up to 60% in the so-called south Atlantic anomaly (Hernández-Pajares et al., 2007).

5. IMPACT OF SECOND-ORDER IONOSPHERIC DELAY ON THE DETERMINATION OF SATELLITE ORBIT AND CLOCK CORRECTIONS

To investigate the effect of the second-order ionospheric delay on the determination of the GPS satellite orbit and clock corrections, a global cluster consisting of 84 IGS reference stations is used (Figure 2). GPS measurements for the 84 IGS stations are downloaded from the IGS website for DOY041 of the year 2010. The raw data are first corrected for the second-order ionospheric delay. The corrected data along with the broadcast ephemeris are used as input to the Bernese GPS software to estimate the satellite orbit and clock corrections.

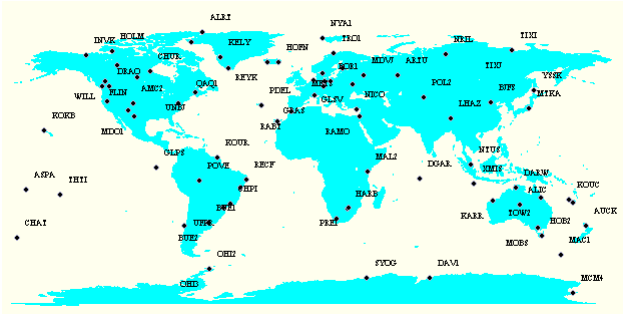


Figure 2. Global Cluster of 84 IGS Stations

Our results show that the effect of second-order ionospheric delay on GPS satellite orbit is up to 2cm at maximum for all GPS satellites. Figures 3 and 4 show the results for PRN04 and PRN24, respectively, as examples.

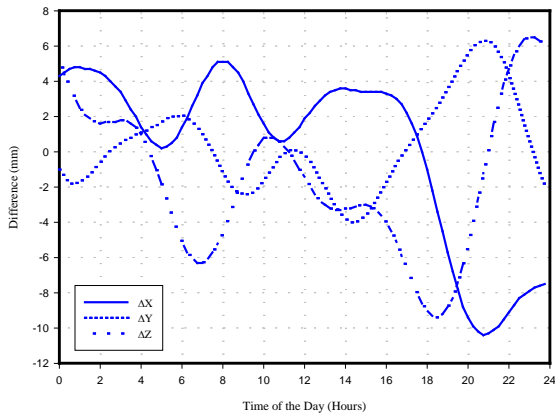


Figure 3. Effect of Second-Order Ionospheric Delay on PRN04's Coordinates (X, Y, Z)

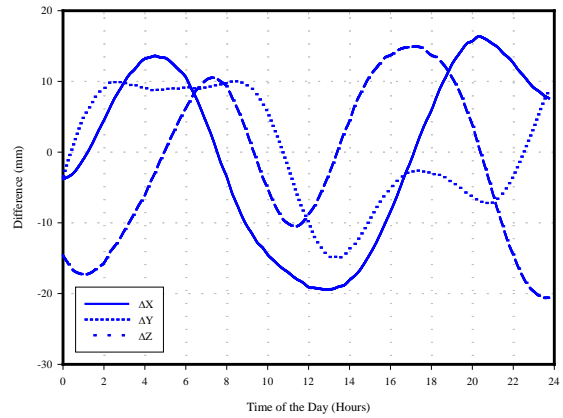
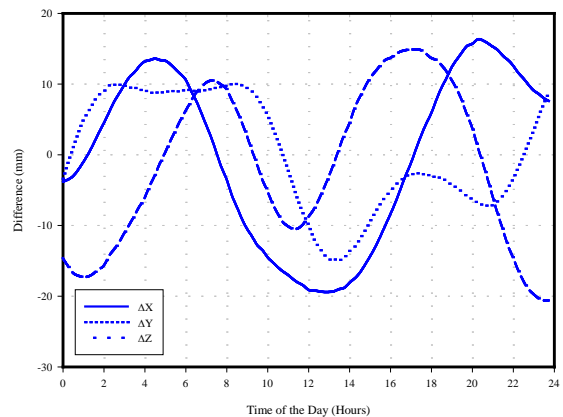


Figure 4. Effect of Second-Order Ionospheric Delay on PRN24's IGS05 Coordinates (X, Y, Z)



The effect of second-order ionospheric delay on the estimated satellite clock solution was found to be within 2cm and less than 1cm in most cases. Figure 5 shows the corresponding values for PRN30, as an example. These values are comparable to the corresponding values of the IGS analysis centers and the final IGS clock corrections. Figure 6 shows the difference in the clock solution for the same satellite between IGS final and CODE analysis center after removing clock offset and drift.

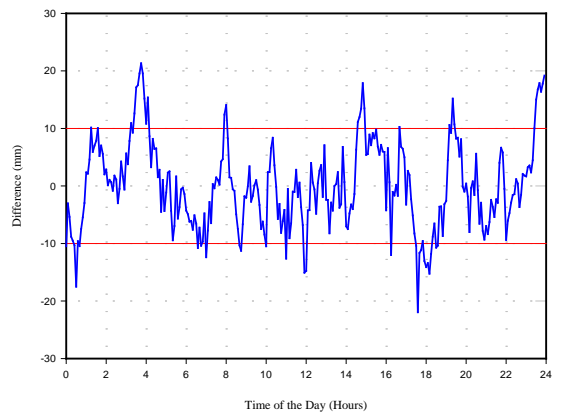


Figure 5. Effect of Second-Order Ionospheric Delay on PRN30 Clock Corrections



Figure 6. Clock Differences between IGS final and COD Products for PRN30

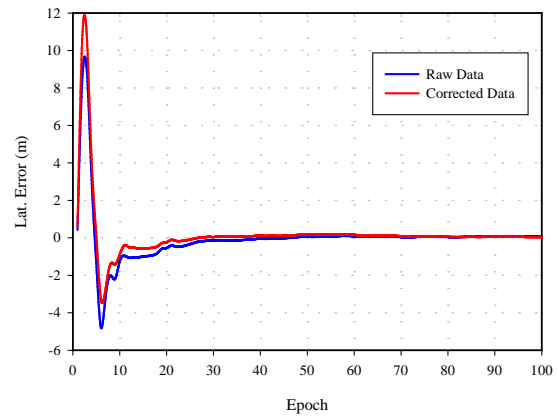


Figure 8. Latitude Improvement after Applying Second-Order Ionospheric and NOAA Tropospheric Corrections at WDC4 IGS Station, DOY41, 2010

6. EFFECT OF SECOND-ORDER IONOSPHERIC DELAY CORRECTION ON PPP SOLUTION

The GPSPACE PPP processing software, which was developed by Natural Resources Canada (NRCAN), was modified to accept the second-order ionospheric correction as well as the newly developed NOAA tropospheric correction product. GPS data from 14 IGS stations (Figure 7) were processed using the modified GPSPACE. The data used were the ionosphere-free (with both first- and second-order corrections included) linear combination of pseudorange and carrier-phase measurements, the estimated precise satellite orbit and clock corrections (from the previous step), NOAA tropospheric corrections, and Vienna mapping function (VMF) coefficients. The results show that improvements were attained in all three components of the station coordinates. Figures 8 through 13 show the 3D solutions obtained with and without the second-order ionospheric and NOAA tropospheric corrections included, for stations WDC4 and NIST. As can be seen, the amplitude variation of the estimated coordinates during the first 15 minutes is reduced when considering the second-order ionospheric delay and applying NOAA tropospheric corrections. In addition, the convergence time for the estimated parameters is lower by about 15%. The final PPP solution shows an improvement in the order of 3 mm in station coordinates. Similar results were obtained for the remaining stations.

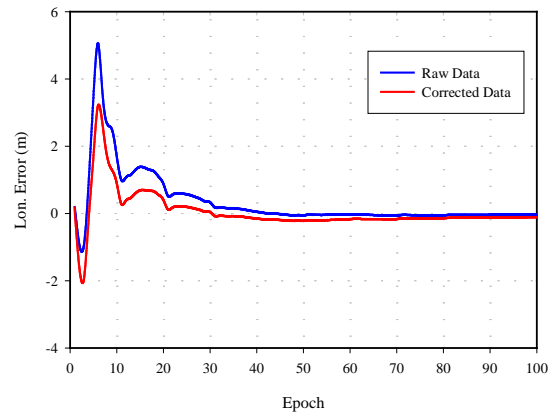


Figure 9. Longitude Improvement after Applying Second-Order Ionospheric and NOAA Tropospheric Corrections at WDC4 IGS Station, DOY41, 2010



Figure 7. IGS Stations Used to Test the Rigorous Models

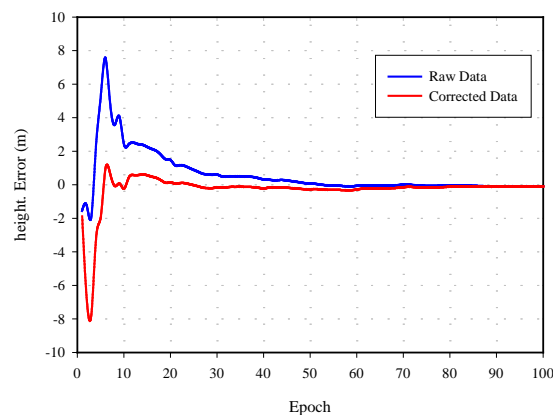


Figure 10. Ellipsoidal Height Improvement after Applying Second-Order Ionospheric and NOAA Tropospheric Corrections at WDC4 IGS Station, DOY41, 2010

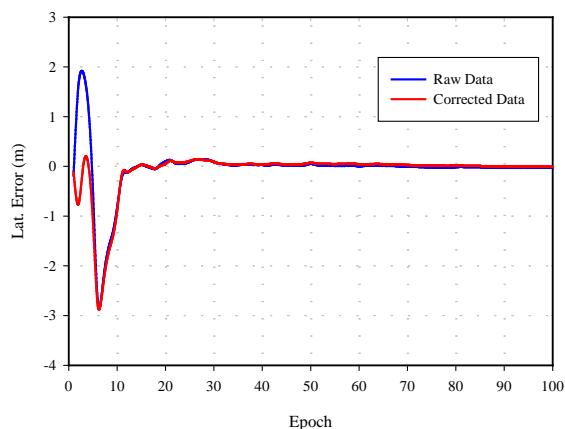


Figure 11. Latitude Improvement after Applying Second-Order Ionospheric and NOAA Tropospheric Corrections at NIST IGS Station, DOY41, 2010

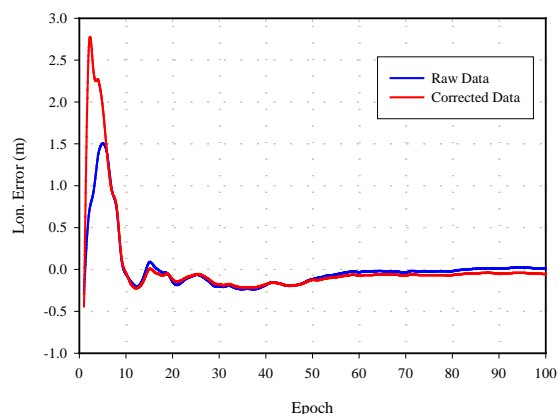


Figure 12. Longitude Improvement after Applying Second-Order Ionospheric and NOAA Tropospheric Corrections at NIST IGS Station, DOY41, 2010

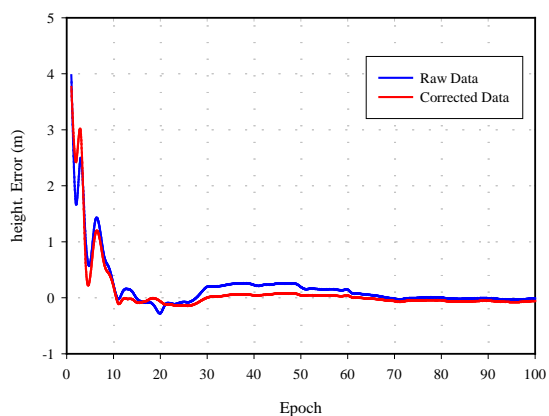


Figure 13. Ellipsoidal Height Improvement after Applying Second-Order Ionospheric and NOAA Tropospheric Corrections at NIST IGS Station, DOY41, 2010

7. CONCLUSIONS

It has been shown that rigorous modelling of GPS residuals error can improve the PPP convergence time and solution. STEC derived using the code measurements on L1 and L2 along with the IGRF geomagnetic field model (IGRF11) were used to estimate the correction for the second-order ionospheric delay. A global cluster consisting of 84 IGS stations is used to estimate the GPS satellite orbit and clock corrections after accounting for the second-order ionospheric delay. It has been shown that neglecting the second-order ionospheric delay can produce an orbital and satellite clock error up to 2cm. As well, accounting for the second-order ionospheric delay and applying the NOAA tropospheric corrections can improve the final PPP coordinate solution by about 3 mm and improve the convergence time of the estimated parameters by about 15%.

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