

A Markov Random Field Model for Individual Tree Detection from Airborne Laser Scanning Data

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ABSTRACT:

Small-footprint Airborne Laser Scanning (ALS) holds great potential in forest inventory as it surpasses traditional remote sensing techniques in terms of rapid acquisition of 3D information of trees directly. With the increasing availability of high density ALS data, the derivation of more detailed individual tree information, such as tree position, tree height, crown size and tree species, becomes possible from ALS data exclusively. However, single tree detection is a critical procedure for tree-wise analysis in order to retrieve more accurate individual-tree-based parameters. The presented research highlights a novel Markov Random Field to model the configuration of single trees in ALS data in which a global optimum to isolate individual trees can be achieved and addressed difficulties of individual tree detection problem in terms of problem representation and objective function. Firstly, local maxima are overpopulated from the CHM recovered from ALS data using a circular type of window filter with variable size. Then trees are modelled as objects at the centre of the extracted local maxima and attributed with other features retrieved from CHM image. The neighbourhood system is set up by TIN and energy functions are carefully designed to incorporate constraints for penalizing false trees and favour true ones. Finally, the optimal tree models are obtained through an energy minimization process. The method is applied on ALS data acquired from a coniferous forest and experimental results show a good detection rate.

1. INTRODUCTION

Small-footprint Airborne Laser Scanning (ALS), as an active remote sensing technology, allows for rapid acquisition of accurate 3D information of Earth topography and features in large scale. ALS gains popularity in forest survey quickly due to its unique capability of penetrating the tree canopy and providing relatively direct measurement of 3D structural information of trees, as well as the elevation of terrestrial surface under the canopy in forested area. This advantage makes it an alternative of tradition technology or even preferred one in the acquiring some forest parameters.

Recent development of commercial small-footprint and full-waveform ALS system make the advantage stand even out. The practice of ALS in forestry study has extended from the formerly extraction of stand-based forest parameters, to the derivation of more detailed individual-tree-based information, such as tree position, tree height, crown size and tree species, with the increase of point density (Brandtberg, 2007; Hyypä, et al., 2008). The potential of ALS data in forest inventory is still being exploited by various researches.

However, in order to implement tree-wise analysis of forest, it is essential to detect individual trees from ALS data first. Extensive researches have been done to isolate single trees using ALS data and many of them used the methods extended from such procedures using aerial photos or satellite images (Chen, et al., 2006). The outer geometry of trees can be directly recovered from ALS point clouds and the peaks and valleys on the recovered model can be better estimations of treetop positions and crown edges, than that obtained from photos or images.

2. RELATED RESEARCH

In this case, most methods focused on the reconstruction of the canopy height model (CHM) and the methods developed for optical imagery were transferred to detect trees in CHM. Most of those approaches were segmentation-based methods and fall in the scope of low-level vision techniques. To the best knowledge of the author, those methods include but not limited to: seed-based region growing (Solberg, et al., 2006), valley following (Leckie, et al., 2003), watershed segmentation (Pyysalo and Hyypä, 2002) and its variance marker-controlled watershed (Chen, et al., 2006). One main drawback of those methods is that empirical values are often used to set key parameters in the solution, which makes them scene sensitive. One example is that to increase the detection success rate of local maxima, which is helpful in seed-based region growing and marker-controlled watershed, in terms of false positives and negatives using variable window size, preliminary knowledge of tree height and crown size of the study area is needed first, whether it is from field survey or experience. Also, in the post-processing of segmentation results, thresholds were also determined subjectively to remove or merge segments with certain size or according to other heuristic rules (Chen, et al., 2006).

Recently, some applications utilizing the information contained in the 3D ALS point clouds to detect single trees were reported. Reitberger et al. (2009) tried to detect tree stems beneath the tree canopy using full-waveform ALS data, and combine them with local maxima detected on CHM as seeds to segment trees in 3D using a graph-cutting algorithm. One possible limit of this method is that stem detection may be influenced by ALS resolution, canopy closure, undergrowth vegetation and tree species, which determine the straightness of the stem. Raham and Gorte (2009) attempted to delineate tree crown based on densities of high points from high resolution ALS and got

comparable result with CHM-based approach. However, the main defect of the method is that the point distribution in the tree crown is not only related to tree species, but can be also affected by many factors such as scan rate, scan direction, scan angle, scan pattern and shadow effects from neighbourhood trees.

Under such a situation, there is a reasonable demand in the perspective of computer vision that a more generalized way for single trees detection from remote sensing data to be developed. Perrin et al. (2005) employed marked point process to extract tree crown from aerial photo of a plantation. This method considered the image as a realization of a marked point process and searched for the best configuration in a completely stochastic way. The method produced good result from the image. However, as the nature of the method ignored any priori information can be extracted from the image, the process of searching for global optimum takes a long time to converge and its computing cost is expensive. Descombes and Pechersky (2006) presented a three state Markov Random Field model to detect tree crown from aerial image and define the problem as a segmentation problem. Although the approach defines a template of tree crown and works on a local mask, it actually calculates on the pixel level. As well, this model does make use of any knowledge can be extracted from the image.

3. ORGANIZATION OF THE PAPER

The presented work places the detection of individual trees in a stochastic framework using a novel Markov Random Field model, but bring it to high level by addressing the difficulties of individual tree detection problem in terms of problem representation and objective function. In our approach, trees are modeled as objects in the image by extracting priori information from ALS image. Then a Markov Random Field model is defined on the objects. The configuration is updated towards the global optimum at which point we get the detected trees. The method is particularly interesting for following reasons:

- i. it extracts priori features from ALS data and models trees as objects, so the sites in the Markov Random Field model are objects, not pixels;
- ii. it allows to introduce *a priori* knowledge of objects, as well as consider likelihood between the represented object and image, which is the property and advantage offered by a Markov Random Field model;
- iii. it greatly reduces the searching space by modelling objects and their relationships at high level and makes the computation much less heavy.

To present our method, we first introduce how we represent trees as objects in the ALS data and how the neighbourhood system is defined. Then we propose the energy formulation based on a data term which measures how features extracted from the data support the object as an individual tree, and a contextual term which take into consideration some interactions between neighbouring objects. The model updating and optimization are followed at the third place. Finally, experimental results of the method on ALS data acquired from a coniferous forest are presented.

4. THE PROPOSED MODEL

We introduce in this section about how the proposed Markov Field Model is built in three parts: problem representation, objective function design and model optimization.

4.1 Markov Random Field

Markov Random Field (MRF) was first introduced into computer vision community by Geman and Geman (1984). The appeal of MRF theory is that it provides a systemic framework to encode contextual constrains into the priori probability and MRF based approaches has been successfully applied to modeling both low level and high level vision problems (Li, 1994).

In a probabilistic frame, a random field X is said to be a Markov Random Field, if the value of random variable $X_s \in X$ only depends on its local environment through a neighborhood system \mathbb{V} defined as (Li, 1994):

$$\begin{cases} s \notin \mathbb{V}(s) \\ \forall r \in S \setminus \{s\}, s \in \mathbb{V}(r) \Leftrightarrow r \in \mathbb{V}(s) \end{cases} \quad (1)$$

In the measurable space $(\Omega, \mathcal{F}, \mathbf{P})$, MRF model can be described by the probability law $\mathbf{P}(X = x)$ the event x to be a realization of X as:

$$\forall x \in \Omega, \forall s \in S, \mathbf{P}(X_s = x_s | X_r = x_r, r \in S \setminus \{s\}) = \mathbf{P}(X_s = x_s | X_r = x_r, r \in \mathbb{V}_s) \quad (2)$$

The Bayesian model is then used to solve the inverse problem of how to retrieve the best configuration \hat{x} given the observations D . By Bayesian law, which relates the *a priori* and conditional probability, the *a posteriori* probability can be written as:

$$P(X|D) = \frac{P(D|X)P(X)}{P(D)} \propto P(D|X)P(X) \quad (3)$$

The problem is then converted into maximizing the *a posteriori* probability (MAP) problem:

$$\hat{x}_{MAP} = \arg \max_{x \in \Omega} P(X|D) \quad (4)$$

Which is equivalent to

$$\hat{x}_{MAP} = \arg \min_{x \in \Omega} \left(-\log(P(D|X)) - \log(P(X)) \right) \quad (5)$$

According to the Hammersley-Clifford theorem (Besag, 1974), a MRF field and a Gibbs field are equivalent. The *a priori* probability of X can there written as:

$$P(X = x) = Z_c^{-1} \times e^{-U_c(x)} \quad (6)$$

Where $Z_c = \sum_{x \in \Omega} e^{-U_c(x)}$ is normalization constant and U_c the priori energy, or contextual energy as referred to in 4.2.

Let the conditional probability be expressed also in the exponential form:

$$P(D|X = x) = Z_d^{-1} \times e^{-U_d(x)} \quad (7)$$

Where $Z_d = \sum_{x \in \Omega} e^{-U_d(x)}$ is normalization constant and U_d the likelihood energy, or data energy as referred to in 4.2.

Under the Markovian hypothesis, solve equation (5) with equation (5) and (7). The problem searching for the MAP configuration is equivalent to finding minimum global energy U as sum of data term U_d and contextual term U_c :

$$U = \alpha U_d + (1 - \alpha) U_c \quad (8)$$

In our problem domain, we aim to model trees as objects in the ALS data and fit the detection of individual trees in a high-level MRF labeling problem. The problem representation of the model is specified below.

4.1.1 Representation of trees: In our study, we represent a tree using a treetop point and a crown radius. Later, some other features are also extracted from the ALS image as properties of trees. The extraction process can be divided into two parts: local maxima extraction, and crown boundary points and radius calculation. The second part will be described in next section 4.1.2, which is related with the building of neighborhood system. As treetops are good representation of trees, after CHM is reconstructed from ALS data, local maxima are overpopulated using a variable circular window with relatively small size, to ensure in the set of local maxima contains all the treetops in the image as shown in Figure 1. So the goal of the MAP-MRF labeling is to label all local maxima which are true treetops as “true”, and all the otherwise as “false”. Initially, all the local maxima are assumed to be true treetops thus labeled as “true”.

4.1.2 Neighborhood system: The sites in a configuration is related with each other via neighborhood system, which is another important task in the designing of a MRF model. A Delaunay TIN is used in our research to build up relationships between neighboring treetops, as shown in Figure 1. In this way, Delaunay TINs are built and updated during the optimization process using the local maxima label as “true”. And “false” local maxima will be pruned from the graph during the updating process. The neighbor system could then help us examine the interaction between two “true” treetops easily.

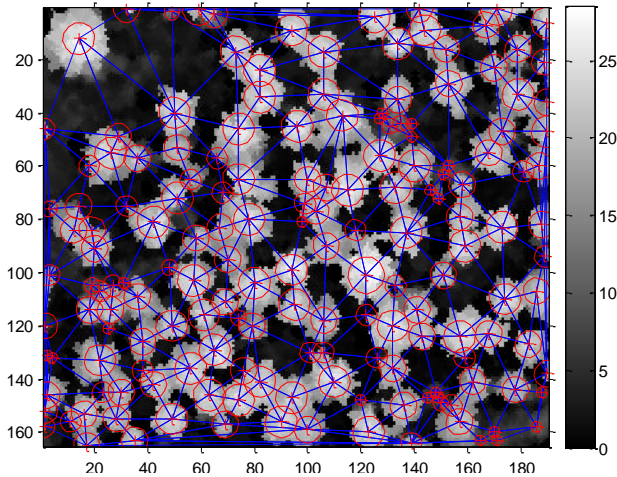


Figure 1: The local maxima extracted using variable circular window size. Delaunay TIN is used to build the neighborhood system.

The neighborhood system designed in such a manner is also advantageous for extracting some other properties for the treetops, namely crown boundary points and radius as mentioned in 4.1.1. A image profile between two adjacent “true”

treetops is extracted from the CHM image. It is then reasonable to say the deepest valley point on the profile is the safest separation of the two trees, if the two treetops are “true”. From the treetop points to the separation points, we can find all the possible boundary points. Those boundary points are then used to determine the directional average boundary radius, average boundary radius and most possible boundary points on each profile. Those features extracted from the image are then used with the local maxima to represent a tree object in the MRF model.

4.2 Energy Formulation

Design of energy functions, or objective functions, is another critical issue in MRF. For energy functions will map a solution to a real number measuring the quality of the solution in terms of goodness or cost, the formulation has to carefully determine how various constrains to be encoded into the function, in order to get the optimal solution. According to equation (8), the global energy U of the model is comprised of data energy U_d and contextual energy U_c .

4.2.1 Data Energy: The data energy indicates the likelihood of the objects of trees in relation with the features extracted from the ALS data, or how well those features support the treetop point as “true”. In the calculation of this term, we incorporate four kinds of constrains, which are specified below.

Symmetric (U_d^s)

The “true” treetops are assumed to locate in the central part of the crown, whereas the “false” ones at the edge part of the crown. Therefore, the determined crown shapes of “true” treetops are expected to be more symmetric. This function (see Figure 2) is then used to penalize treetops with asymmetric crown given by equation (9).

$$U_d^s(s) = \begin{cases} \sin\left(\frac{\pi}{2\varepsilon_s}(\Delta R(s) - \varepsilon_s)\right) & \text{if } 0 \leq \Delta R(s) \leq 2\varepsilon_s \\ 1 & \text{if } \Delta R(s) > 2\varepsilon_s \end{cases} \quad (9)$$

Where s is a treetop, $\Delta R(s)$ is the radius difference ratio of s , given by equation (10).

$$\Delta R(s) = \sqrt{\frac{1}{N} \sum_{i=1}^N (\overline{R}_s^i - \overline{R}_s)^2} / \overline{R}_s \quad (10)$$

Where \overline{R}_s^i is the directional average boundary radius of treetop s , \overline{R}_s is the average radius of s and N the number of profiles of s .

Boundary Radius Constrain (U_d^r)

This scoring function was set to penalize the local maxima located closely to the edge part of a crown according to the number of radius of most possible boundary points under certain threshold given by equation (11).

$$U_d^r(s) = \begin{cases} 1 & \text{if } n_b(s) \geq 2 \\ 0 & \text{if } n_b(s) = 1 \\ -1 & \text{if } n_b(s) = 0 \end{cases} \quad (11)$$

Where s is a treetop and $n_b(s)$ is the number of radius of most possible boundary points under threshold, with is set as $\min(\mathbb{B}, 0.1\overline{R})$.

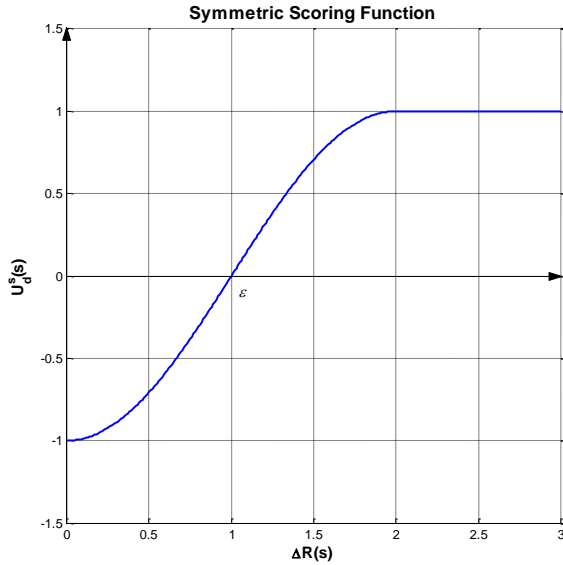


Figure 2: Symmetric scoring function.

Boundary Point Depth (U_d^d)

The valley depth of the most possible boundary point in the profile between two local maxima indicates the possibility of the local maxima to be “true” treetops or not. So boundary point depth scoring function (see Figure 3) is given by equation (12).

$$U_d^d(s) = \begin{cases} \sin\left(\frac{\pi}{2\varepsilon_d}(d(s) + \varepsilon_d)\right) & \text{if } 0 \leq d(s) \leq 2\varepsilon_d \\ -1 & \text{if } d(s) \geq 2\varepsilon_d \end{cases} \quad (12)$$

Where s is a treetop, $d(s)$ is the boundary point depth ratio given by equation (13).

$$d(s) = (h_s - h_{b(s)}) / (h_s - h_o) \quad (13)$$

Where h_s is the height of treetop s , $h_{b(s)}$ is the height of the most possible boundary point and h_o is the threshold set for to stretch the value of high difference ratio, which is set as 5m.

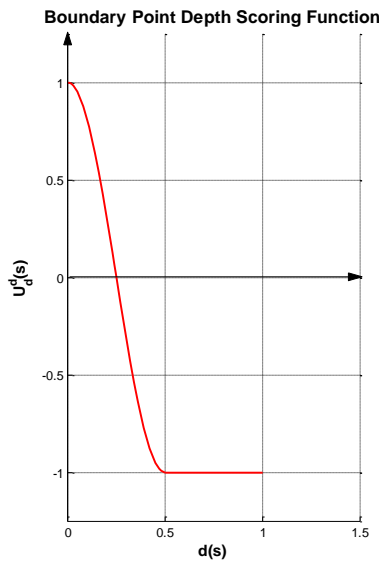


Figure 3: Boundary point depth scoring function

Area Constrain (U_d^a)

So far data energy can be calculated as equation (14):

$$U_d = U_d^{\{s,r,d\}} = w_s U_d^s + w_r U_d^r + w_d U_d^d \quad (14)$$

where $w_s + w_r + w_d = 1$.

However, to more heavily penalize the trees which were apparently not “true” from empirical knowledge and accelerate the convergence rate of optimization process, an area constrain is added given by equation (15).

$$U_d^a(s) = \begin{cases} 1 & \text{if } \bar{R} \leq 3 \\ -1 & \text{otherwise} \end{cases} \quad (15)$$

where s is treetop s and \bar{R} is the average radius of s .

Finally, the data energy is computed as equation (16).

$$U_d = \max_{\{s,r,d\}}(U_d^{\{s,r,d\}}, U_d^a) \quad (16)$$

4.2.2 Contextual Energy: The contextual energy introduces *a priori* knowledge concerning the objects layout. It is natural for us to incorporate a constraint that penalizes severe overlapping of tree crowns. However, the design of constrain to penalize over-pruning situations might result in too much gaps on the tree crowns. To address this problem, the two scoring functions are proposed in detail below.

Profile Connectivity (U_c^c)

As it is mentioned, this scoring function is used to penalize the over-pruning situation during the optimization process which leads to gaps between tree crowns. The disconnected ratio of i -th profile of treetop s is defined by equation (17).

$$c(p_s^i) = \left| R_{p_s^i}^{outer} - R_{p_s^i}^{mp} \right| / \bar{R}_s \quad (17)$$

where p_s^i is the i -th profile of treetop s , \bar{R}_s is the average radius of s , and $R_{p_s^i}^{outer}$, $R_{p_s^i}^{mp}$ are the radius of the outermost and most possible boundary points of s on the i -th profile.

The disconnect ratio of treetop s is then calculated as equation (18).

$$c(s) = \max(c(p_s^i)) \quad (i = 1 \sim N) \quad (18)$$

where N is the number profiles of treetop s .

Then the disconnected ratio $c(s)$ is used as input to calculate the profile connectivity score given by equation (19).

$$U_c^c(s) = \begin{cases} 1 & \text{if } c(s) \geq c_2 \\ \sin\left(\frac{\pi}{c_2 - c_1}\left(c(s) - \frac{c_1 + c_2}{2}\right)\right) & \text{if } c_1 \leq c(s) \leq c_2 \\ -1 & \text{if } c(s) \leq c_1 \end{cases} \quad (19)$$

Profile Overlap (U_c^o)

Similarly, a profile overlap scoring function is designed to penalize severely overlapped tree crowns. The overlapping ratio of i -th profile of treetop s is defined by equation (20).

$$o(p_s^i) = (\overline{R}_s + \overline{R}_{s'} - l_p) / l_p \quad (20)$$

where p_s^i is the i -th profile of treetop s , \overline{R}_s is average radius of s , $\overline{R}_{s'}$ is the average radius of s' which is connected with s by profile p_s^i , and l_p is the length of p_s^i .

The profile overlap ratio of treetop s is then calculated as equation (21).

$$o(s) = \max(o(p_s^i)) \quad (i = 1 \sim N) \quad (21)$$

where N is the number profiles of treetop s .

Then the profile overlap score of treetop s is given by equation (22).

$$U_c^o(s) = \begin{cases} 1 & \text{if } o(s) \geq o_2 \\ \sin\left(\frac{\pi}{o_2 - o_1} \left(o(s) - \frac{o_1 + o_2}{2}\right)\right) & \text{if } o_1 \leq o(s) \leq o_2 \\ -1 & \text{if } o(s) \leq o_1 \end{cases} \quad (22)$$

At last, the contextual energy is computed as equation (23).

$$U_c = w_c U_c^e + w_o U_c^o \quad (23)$$

where $w_c + w_o = 1$.

4.2.3 Parameters Settings: There are three categorizes of parameter setting in the model: physical parameters, and weights and thresholds.

The physical parameters have a physical meaning in the application and are fixed according to the scene. There are two physical parameters in the model, h_o set as the height of low vegetation and the threshold used to penalize local maxima which locate near the edge of tree crowns, both in the scoring functions of data energy.

Weights are assigned to data energy and contextual energy in the calculation of global energy, respectively α and $(1 - \alpha)$. And more are used in the computation of data energy and contextual energy respectively, as more than one constrains are incorporated in the model. The settings of weights are basically intuitive and tuned through trial and errors.

Thresholds are also necessary in the design of the scoring functions, as we want to set tolerances to different constraints. For example, we can set a smaller tolerance of ε_s in the symmetric scoring function, if we want to penalize more effectively about treetops with asymmetric crown. It is the same case with ε_d , c_1 and c_2 , o_1 and o_2 . In our application, we set $\alpha = 0.6$, $\varepsilon_s = 1$, $\varepsilon_d = 0.25$, $c_1 = 0.5$, $c_2 = 1.5$, $o_1 = 0.4$ and $o_2 = 0.8$.

4.3 Model Optimization

In the preliminary tests of our method, we used relatively simple model evolving scheme to find the configuration of objects with “minimum” global energy. This scheme simulates a discrete Markov Chain $(X_t)_{t \in \mathbb{N}}$ on the configuration space in which only death process is considered. For each iteration, the

site with the highest global energy in the configuration is regarded as the weakest site and removed from the configuration. The iteration continues until there are only 3 sites left, which is the minimum number of points required to construct a TIN.

As the initial site number is finite and relatively small, the iterations can be completed in a short period of time and the configuration with the minimum global energy during the Markov Chain evolving process can be recovered quickly.

5. EXPERIMENTAL RESULTS

The ALS data used for this study was acquired by Riegl LMS-Q560 in a coniferous forest area about 60km east to Sault Ste. Marie, Canada. The point density is about 30 pt/m². The ALS data was first processed into CHM image with a resolution of 0.5m. We just used the highest point in each cell to reconstruct the CHM and no smoothing operation was done to it.

After CHM was prepared, the *a priori* information was extracted from the data and trees are then modeled as object in the data using local maxima and crown radius as shown in figure 4. As can be seen from the figure, trees are over-populated with a total number of 169. Crown radiuses are reasonably extracted from the data.

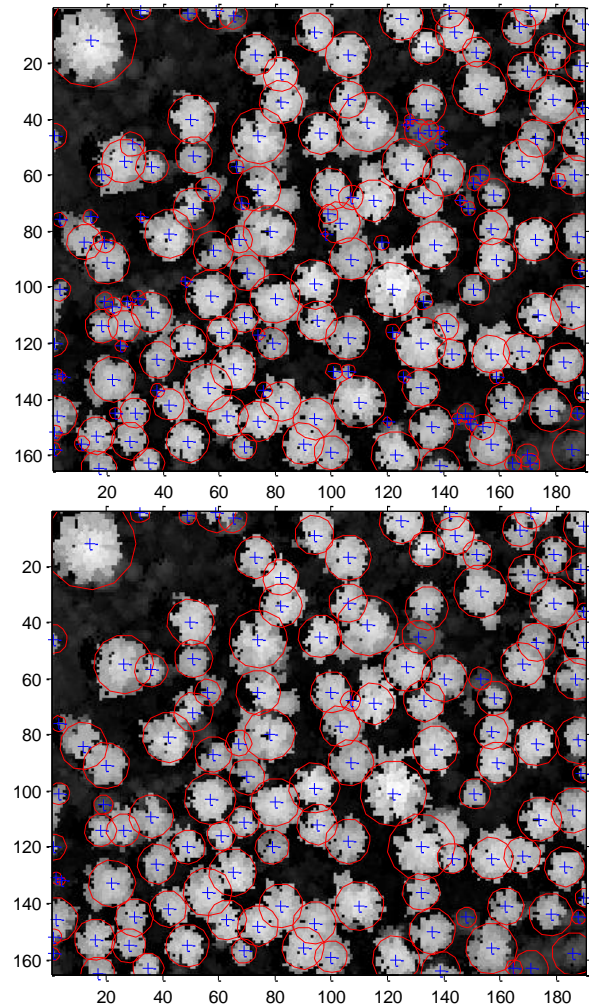


Figure 4: Result of tree detection: (Up) initial configuration with 169 tree objects shown in red circle; (Below) optimal configuration when minima global energy reached, with 127 trees labeled as true.

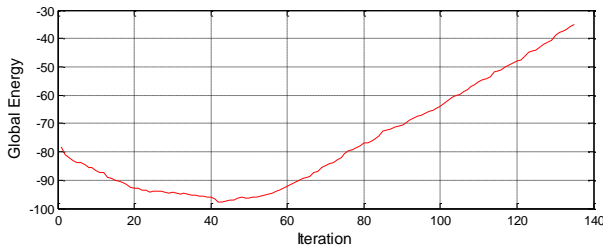


Figure 5: Global Energy Curve

Figure 5 shows the corresponding global energy curve during the searching for optimal configuration. As can be seen from those figures, at the beginning, the global energy decreases quickly with the removal of false trees, and reaches the minima of global energy get at 42 iterations. Later, the global energy start to increase when trees are over-pruned, which indicates the effectiveness of the designed energy functions in our model. Finally, 127 trees are detected from the CHM data. What we can interpret as well is that in the optimal configuration, the “false” treetops located on the crown edge and trees with overlapping crowns are removed at a high accuracy, when compared with the initial configuration. At the meantime, trees with big crown are kept even with some extend of overlaps.

6. CONCLUSION AND FUTURE WORKS

We have presented in this paper detecting individual trees from ALS data. The innovation of the method is formulating the tree detection in the data as a high-level MRF labeling problem, and highlights the problem representation and energy function design in the Markov Random Field model. In this approach, trees are modeled as objects with treetops, crown radius and some other features extracted from the data and the data is regarded as configurations of those objects. Then, neighborhood system is proposed to introduce relationships between the objects and energy functions are carefully designed to incorporate the constraints in model. Finally, the optimal configuration is found through an energy minimization process. The experimental result shows a good detection rate of single trees in the data.

The advantage of the method lies in that low level vision method is first used to extract priori information from the data, and trees are abstracted from that information in a high level. Then the problem is formulated using a Markov Random Field model. In such a way, the size of configuration space is greatly reduced and much less computation will be needed in the searching of optimal solution. Furthermore, under such a mathematic framework, other features or constraints extracted from data or even other sources, which help in the detection of trees, e.g. stems detected underneath the canopy cover, can be easily added and integrated into the current model without having to alter the structure of algorithm. However, there are still some issues to be studied in the future in order to improve the method. The first one will be model optimization. A RJMCMC embedded simulated annealing is suggested to be introduced for searching the configuration space more thoroughly to get the optimal configuration. Secondly, it will also be interesting to explore some algorithm can be employed to help find best weighting coefficients automatically. Finally, more investigation will be implemented to find out which scoring functions designed play more significant roles in the penalization of false trees and detection of true trees. Also, this method will be applied to more datasets to test its feasibility to forests of different types or structures.

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