STATISTICAL AND STRUCTURAL DESCRIPTIONS FOR IMAGE TO MAP REGISTRATION

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ABSTRACT:

The registration of imagery to maps is becoming increasingly important for a large number of applications. Today, this task still relies mostly on an operator, manually identifying corresponding points from the image and the GIS dataset for registration. The challenges for automating the process arise mainly from their dissimilar data structures (raster vs. vector), orientation variations, as well as occlusion-introduced extraction errors due to shadows, buildings or objects on the roads. In this paper, we present a robust automated approach that models road networks extracted from the two datasets as graphs, using statistical and structural descriptions of these road networks. The proposed approach starts by statistically analyzing local geometrical and topological properties of road networks such as orientation and number of connections. Such statistical similarity measures can be used to thin the number of potential matches when comparing a target road structure to a spatial database, resulting in computational efficiency. Subsequently, by considering the spatial distribution and structure similarity in a neighbourhood, we formulate a global compatibility function to measure the overall goodness of correspondence. We achieve an optimal matching by finding an optimal morphism that maximizes this compatibility function. The experimental results demonstrate the robustness of our approach.

1. INTRODUCTION

The registration of imagery to maps is becoming increasingly important for a large number of applications. Yet, this task still relies considerably on an operator, manually identifying corresponding points from the image and the GIS dataset for registration. One major challenge arises from their dissimilar data structures (raster vs. vector). In this context, objects from maps (vector data) contain no intensity information, which typically consists of the vital component in current promising algorithms in *image-to-image* registration. Such problem becomes further complicated when concerns include orientation variations, as well as occlusion-introduced extraction errors due to shadows, buildings or objects on the roads.

Feature-based approaches, where features represent information on higher level such as points and lines, are developed for such registration problem as the involved data have different data structure. Recent advances in collection capability of high-resolution imagery and sophisticated road extraction evoked a renewed research focus on using road networks for matching. For instance, Chen et al., (2004) developed a specialized point pattern matching algorithm using road intersections. The proper transformation between the two datasets is determined from a fraction of the detected road intersections using a brute-force point pattern algorithm. The direction and relative distance from the two datasets are assumed available and used as prior knowledge to prune the search space of possible mapping. Wu et al., (2007) proposed an automatic matching by breaking the global alignment problem into a set of localized domains (tiles). Within each tile, a least square optimization is applied to estimate the best translation locally. This group of work (Chen et al., 2004; Doytsher et al., 2001; Filin and Doytsher, 2000; Walter and Fritsch, 1999; Yu et al., 2004; Zhu et al., 2009) gives some promising results but they require the datasets are georeferenced and based on approximate transformation. Their misalignments are assumed only due to projection errors, inaccurate camera models, absence of precise terrain models, etc. These assumptions may not be satisfied in general scenarios. Importantly, selection of control points by operators for approximate transformation is usually required.

In this work, we developed an optimal and robust approach to automatically establish the matching between imagery and vector data invariant to their differences in scale, orientation, area of coverage, physical changes and extraction errors. Our automated approach models extracted road networks as attributed graphs and the matching is based on the relaxation labelling introduced by Hummel and Zucker (1983). There has been a great deal of effort in computer vision community devoted to graph matching. Matching techniques developed rely substantially on structure pattern, like the graph and sub-graph isomorphism approaches (Bruns and Egenhofer, 1996; Bunke, 1999; Jain and Wysotzki, 2002; Pelillo, 1999; Shaprio and Haralick, 1985). The major drawbacks in these graph-theoretical methods are their computational complexity and inability to handle inexact matching due to noise or corruptions in the graph. In our approach, the utilization of point networks and revised relaxation labelling provides the ability to utilize structures and geometric attributes derived from the network to improve the matching algorithm. A statistical analysis on local geometrical and topological properties of road networks is used to thin the number of potential matches when comparing a target road structure to a spatial database, thus achieving relatively efficient computation. These unique advantages serve both as the motivation for our work and constitute the main contributions of this paper.

The remainder of the paper is organized as follows: Section 2 describes the modelling of road networks with some definitions. The invariant attributes developed for relaxation matching are described and analyzed in Section 3. In Section 4, our revised relaxation labelling algorithm for matching is described in detail. Experimental results are presented in Section 5. Finally, Section 6 presents conclusion and outlines our future work.

2. NETWORK MODELLING

Spatial entities (road networks) from both data sets are first transformed into graph structure as input to our approach. In the graph, a *vertex* (also termed *nodes*) models each road intersection, *edges* of the graph represent the fact that there exist road segments joining two intersections, and an attributes set contains unary attribute attaching to each node, binary attribute attaching to each edge, ternary attribute attaching to every three related nodes, and if necessary, n-ary ($n \geq 3$) attribute associating to every related n nodes.

The extraction of road networks is not a topic addressed by this paper, as automatic road extraction from remote sensor data is a well-researched topic in computer vision, photogrammetry and remote sensing. In this work, we assumes that the data has been preprocessed using digital image processing and analysis techniques, e.g., Poullis and You (2010), and road networks have been detected in both datasets being registered. Figure 1 provides a graphic view of the graph over an image, where the road network is extracted. It is noted there is no edge joining the two green-colored vertices as the two road intersections represented by these vertices have no road segments joining them. The advantages of such modelling include that it models not only the topological structure of the road network (e.g. connectivity) but also its non-structural properties with the use of attributes. In addition, the important information for the edge set is whether or not there is a road joining two intersections. We do not need to know how many road segments connect them, which is significantly affected due to gap problems since one road segment between intersections may be extracted as several ones.

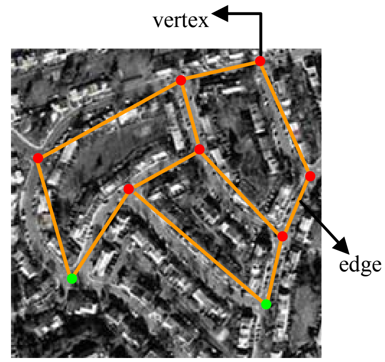


Figure 1. Graph representation of the extracted road network over the image where it is extracted

Let's denote the graph from the image as $G^{im}= (V^{im}, E^{im}, R^{im})$. In this notation, $V^{im} = \{v_1, v_2, \ldots, v_s\}$ is the set of s vertices in the graph representing road intersections and $E^{im}= \{e_1, e_2, \ldots, e_t\}$ is the set of t edges in the graph representing relationships between road intersections. The *degree* $d(v_i)$ of the vertex v_i is defined as the number of edges with v_i an endpoint. If there is an edge between vertex v_i and v_j, then v_i and v_j is said to be *adjacent* to each other and denoted by $v_i \sim v_j$. Vertices adjacent to v_i are termed *neighbors* of v_i. The *neighborhood* of v_i (denoted by $N(v_i)$) is the set of all vertices adjacent to v_i (Wallis, 2007). The *adjacency matrix* of the vertex set V^{im} is defined as follows:

$$a_{ij} = \begin{cases} 1 \ if \ \ v_i \sim v_j \\ 0 \ \ otherwise \end{cases} \tag{1}$$

The third element R^{im} is a set of attributes include unary attributes r_1 defined over V^{im}, binary attributes r_2 defined over E^{im}, ternary attributes $r_3(i,j,k)$ defined over any vertex with its two neighbors and n-ary attributes $r_n\ (i,j,k,\ldots,p)$ defined over any vertex with its n neighbors. The road network, hence, is defined in this manner through sets of vertices, edges and attributes among nodes. Similarly, the corresponding vector data can also be defined as $G^{db}=(V^{db}, E^{db}, R^{db})$ where members in each set are in uppercase notation. Using the above notations for these two networks, our aim in matching is to optimally correspond (label) vertices $V^{im}=\{v_1, v_2, \ldots, v_s\}$ in graph G^{im} to those from the set $V^{db}=\{V_1, V_2, \ldots, V_u\}$ in Graph G^{db} satisfying certain matching criteria.

3. INVARIANT ATTRIBUTES FOR LOCAL SIMILAIRTY

Invariant attributes are essential for matching as they can reduce ambiguities in local similarity and the corresponding search space. Developing invariant attributes, however, is a non-trivial issue. In one hand, as the involved imagery and GIS datasets may differ in terms of resolution, scale, coverage, and orientation in general, the conjugate features may also differ to a certain extent. On the other hand, as road networks usually involve high volume of data, it is important to develop attributes that require less computational efforts. In this section, we introduce attributes derived from the geometry and topology of road networks, which are invariant to translations, rotations and scale changes.

As the topological properties of a graph describe its structural characteristics and are not altered by 2D transformations (such as scaling or rotation), it is straightforward that the *degree*, a topological invariant of graphs, is an ideal unary attribute associated with each node. In our defined graph for the road networks, every node represents a road intersection where at least two roads join. The degree of any node must be equal to or greater than two.

Typically the mathematical Euclidean metric is an important measurement of the geometry. It is invariant to translations and rotations, but not to scale changes. The Euclidean distance between two nodes joined by an edge can not be used directly as a binary property. The *angle* formed by one node and its two neighbors, however, is invariant to 2D translations and appropriate to be applied as a ternary property. To overcome the variation of Euclidean distance to scales, a *relative distance* is proposed and can be used as one type of ternary property:

$$r_{ij,j\in N(i)} = \frac{D_{ij,j\in N(i)}}{\frac{1}{2}(D_{ij,j\in N(i)} + D_{it,t\in N(i)\cap t\neq j})} \qquad (2)$$

Where D_{ij} is the Euclidean distance between vertices i and j (j is a neighbour of i) and D_{it} is the Euclidean distance in i and t (t is a neighbour of vertex i and $t \neq j$). The denominator is the average of the Euclidean distances between vertexes i and its two neighbours. As vertices in the graphs denote road intersections, every vertex will have at least two adjacent vertices. In the case of more than two neighbours to current vertex i, j and t in the relative distance are selected randomly from its neighbourhood. Apparently, the property can be extended for higher-ordered property as:

$$r_{ij,j\in N(i)} = \frac{D_{ij,j\in N(i)}}{\frac{1}{m}\sum_{p_q\in N(i),q=1}^{m} D_{ip_q}} \qquad (3)$$

It easily understands that more and complex attributes, if desired, may be derived and used. Potentially additional attributes may contribute to improving performance of the next phase – matching extracted features. The derivation process and computing local matching, on the other hand, may be computationally expensive, particularly when the number and complexity of proposed attributes increase substantially. What's worst is that these additional attributes may be redundant to achieve a robust matching. At this point, proper attributes used for matching are critical and further analysis is necessary in terms of sufficiency. We will examine this issue in subsection 4.1.

4. MATCHING TWO ROAD NETWORKS

Accordingly, the road networks from the image and vector data are respectively defined through a graph embedded topological (e.g. neighbourhood) and geometric attributes (e.g. relative distance). Using the above notations for these two networks from two datasets, our aim in matching is to optimally correspond (label) nodes v_i in graph G^{im} to those in graph G^{db} satisfying certain matching criteria. Now the problem of matching an image to a vector data becomes a matching of attributed graphs.

Based on relaxation labelling, the matching process iteratively re-labels the data nodes with model nodes by changing their corresponding weights. The weights are optimized according to their local geometric and topological similarity. After each iteration, the global matching (i.e. global compatibility) is measured. The process reaches an optimal matching when the global compatibility measurement becomes unchanged or varies to a limited threshold.

4.1 Statistical Analysis on invariant attributes

As discussed, proposed attributes are preferred to be capable of describing sufficient patterns of the road networks geometrically and/or topologically, invariant to any 2D transformation, required for a robust matching, yet without comprising much on the complexity in deriving and exploring them. Unfortunately, this issue hasn't given rise to much attention in popular matching literature. Attributes are usually selected based on ad hoc decisions. In this subsection, we address the abovementioned issue using entropy concept. Originated from classical thermodynamics (Clausius, 1867), Entropy is a quantitative entity defined fundamentally via an equation. It has been extended to various new domains ever since as a measurement. In Shannon information theory (Shannon, 1948), the entropy is used to measure the uncertainty over the true content of a message (a string of binary bits). Mathematician Alfréd Rényi constructed the proper entropy for fractal geometries (Jizba and Arimitsu, 2001). In statistical mechanics, the entropy is defined as a function of statistical probability to measure the probability for a given macrostate. In this work, we introduce an entropy function as follows to measure how well a type of attribute describes a given data in terms of its pattern for matching:

$$Ep = \frac{1}{n-1}\sum_{i=1}^{n} | R_i - \overline{R} | \qquad (4)$$

where R_i is each measured attribute value, μ is the arithmetic mean of the population, n is the number of the R_i. A high entropy indicates salient pattern represented by such attribute, and a low entropy indicates weak pattern represented by the attribute. The defined entropy Ep is similar to unbiased estimator of sample variance σ^2 where the two parameters μ and σ^2 are estimated from the data itself. Attributes that has most distributed values is useful for salient patterns and should be selected for matching.

4.2 Local Similarity

Once we have constructed the attributed graphs from two networks and select proper attributes from statistical analysis, we proceed with their local similarity. Our aim at this stage is to measure the similarity in structure and geometry associated with each mapping nodes. The local similarity is also termed *the goodness of the local fit*, *local compatibility* or *the goodness of local mapping*. They are used interchangeably in this thesis. Given V_j from G^{db} as the current counterpart (label) of v_i in G^{im}, let $\{v_s, \dots v_q\}$ be neighbor vertices of v_i and $\{V_p, \dots, V_t\}$ be any neighbor vertices of V_j. We introduce an exponential function, a modified version from the work of Li (1992), to measure the goodness of such mapping ($v_i \to V_j$). Using the relative distance attribute as an example, the goodness of the local fit can be measured with $H(v_i, V_j)$:

$$H(v_i, V_j) = \exp\left(-\sum \frac{\min|r_{i,\{s,\dots,q\}} - R_{j,\{p,\dots,t\}}|}{\sigma}\right) \tag{5}$$

where σ is some parameter, $r_{i,\{s,\dots,q\}}$ is the relative distance from v_i to its neighbors in G_{im}, $R_{j,\{p,\dots,q\}}$ is the relative distance from V_j to its neighbors in G_{db}.

Use Figure 2 as an example. Let's assume we are considering labeling v_2 for V_1 in the process. According to Eq. 5, the summation component in $H(v_2, v)$ includes two ingredients since v_2 has two adjacent vertices v_1 and v_4. Specifically, relative distance r_{21} is compared with all relative distance $R_{1t,\ t\in\{2,3,4\}}$ and the most proximate one (say $t=2$) is used to calculate the absolute difference as one ingredient. The relative distance r_{24} is then compared with the rest of relative distance $R_{1t,\ t\in\{3,4\}}$ and the most proximate one (say t=4) is used to calculate the absolute difference as the second ingredient. With relative distance attribute and adjacency constraints, the vertex (t=3) in neighborhood of V_1 matches no neighbors of v1. We assign it to match *null*, a special label in V^{db}.

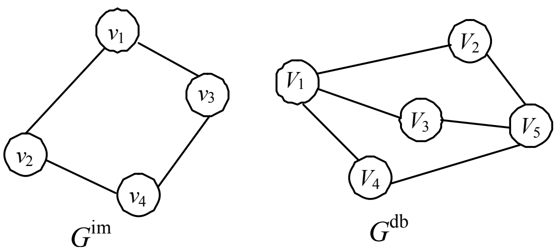


Figure 2. Two graphs used for exemplifying local similarity measure

The novel feature of this local consistency measure H is its compound exponential structure, which distinguishes it from many alternatives in the literature. The underlying advantages are that the constructed H function will not be affected by the presence of noise (i.e. the additional link V_3 in Figure 2) and the ambiguity will be reduced as low as possible. Similarly, the presence of noise (i.e. additional links) in V^{db} would not affect our measurement.

4.3 Global Compatibility

As discussed before, matching two attributed graphs that model road networks requires an optimal solution, which maximizes a sort of global compatibility. With the constructed local similarity H, we use the continuous relaxation labeling method introduced in (Hummel and Zucker, 1983; Rosenfeld et al., 1976) which require *no* threshold to determine whether a mapping is acceptable. As the continuous relaxation-labeling framework, probability values other than logical assertions (1 or 0) are attached to all possible assignments for each vertex in G^{db} and *null* vertex. The probability with which label V^{im} is assigned to vertex V^{db} (including *null*) is denoted by $p_\mu(\lambda)$ and satisfies:

$$0 \le p_u(\lambda) \le 1, \quad \lambda \in V^{im}, \mu \in \{V^{db}, null\} \tag{6}$$

and

$$\sum_{\mu} p_\mu(\lambda) = 1, \quad \lambda \in V^{im},\ \mu \in \{V^{db}, null\} \tag{7}$$

Let Θ be all available assignments with V^{im} to V^{db} and *null*. The global compatibility function (using relative distance attribute as an example) can be formed as:

$$\Lambda(\Theta \mid d,m) = \sum_{i,j,k} \sum_{a,b,c} H(v_i \to V_a, v_j \to V_b, v_k \to V_c) p_a(i) p_b(j) p_c(k) \tag{8}$$

where $v_j, v_k \in N(v_i)$ and $V_b, V_c \in N(V_a)$. This function is close in nature to the global gain in (Li, 1992). Yet, with the data structure and attributes we proposed in this work, the neighborhood can easily been identified through edges of graphs. This is important to network matching as it allows our algorithm to determine the local similarity based on the topological structure intrinsic inside the data itself, not any ad hoc information. Thus, the optimal labeling of G^m with G^{db} will be the one that maximizes the above function:

$$\Lambda(\Theta^*) = \max(\Lambda) \tag{9}$$

The gradient projection algorithm by Hummel and Zucker (1983) is used. The advantage of the algorithm lies in its projection operator which is based on a theory of consistency and still widely used in solving constrained optimization problems. An exhaustive discussion and proof can be referred to (Mohammed et al., 1983). With the algorithm, we can iteratively compute the length and direction of the updating vector to update p such that the global compatibility function Λ will increase with each updating of p. The iteration terminates when the algorithm converges, generally producing an unambiguous labeling (or matching).

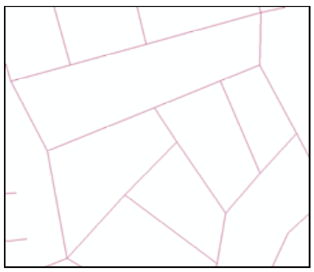
5. EXPERIMENTS

To evaluate the performance of our approach, experiments are performed in MatLab environment to find the correspondences in road networks from the high-resolution imagery and GIS vector data, where the two data sets present typical matching conditions for registration.

The used imagery and vector data are queried from the National Map Seamless Server[1]. The imagery has spatial resolution of 0.5m that covers some area of the county of Prince William, VA and has been orthorectified. The vector data covers larger area than the imagery and presents unknown orientation differences. The two datasets are shown in Figure 3a and b respectively.

[1] http://seamless.usgs.gov/

(a) The imagery



(b) The Vector data

Figure 3. The imagery and vector data with differences in orientation, scales and coverage

Figure 4 demonstrates aforementioned differences when the image is directly superimposed on the vector data.

From the datasets, two attributed graphs can be derived and they are marked as *G* and *H* below (Figure 5), where *G* represents the attributed graph built from the road network from the satellite image; *H* represents the attributed graph built from the road network in the vector data.

For each graph, we derive two types of attribute based on the datasets: degree on each node and relative distance between nodes. Before proceeding to the matching process, we analyze these properties using entropy analysis. The analysis is significant as it allows us to quantitatively measure the distribution of attribute values, an important factor for reducing local ambiguity. Based on the analysis, it comes to a decision about whether or not both types of attribute should be used or additional attributes are needed. Figure 6 shows their distribution graphically ordered by nodes and their corresponding entropy using Eq.4. Node 7 in Fig. 5a shows a salient degree-*4* (there are four road segments in the image incident to the intersection represented by node 7) from others. Similarly, node 4 and 9 in Fig. 5b. Because the majority of nodes in (a) and (b) have close degrees (same degree in this case), accordingly their entropy used to evaluate the distribution is very small, close to zero. This means the degree attribute is not appropriate to use for matching due to their proximate values. The relative attribute, however, has great entropy (96.92 and 67.76), representing distinguishable patterns locally and thus appropriate and sufficient for matching.

Figure 4. Superimpose the vector data on the image

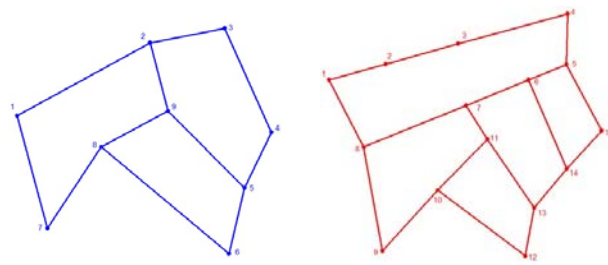


Figure 5. Attributed graphs G (left) from the imagery and H (right) from the vector data

Table 1. Initial weights for matching

	1	2	3	4	5	6	7	8	9
1	0.0769						0	0.0769	
2	0.0769						0	0.0769	
3	0.0769						0	0.0769	
4	0						0.5	0	
5	0.0769						0	0.0769	
6	0.0769						0	0.0769	
7	0.0769						0	0.0769	
8	0.0769						0	0.0769	
9	0	…	…	…	…	…	0.5	0	…
10	0.0769						0	0.0769	
11	0.0769						0	0.0769	
12	0.0769						0	0.0769	
13	0.0769						0	0.0769	
14	0.0769						0	0.0769	
15	0.0769						0	0.0769	
null	0						0	0	

It should be noted that, although the degree attributes score low entropy, they still include useful information for initiating the assignments to start the matching process. The underlying rationale lies in that a matched pair mostly likely has the same number of degree. Our initial assignment shows in Table1. The numbers in blue denotes nodes from graph *G* and the numbers in red are nodes from graph *H*. The special node in *H* is represented as *null*. Note that the sum of each column is equal to 1, satisfying the condition addressed by Eq. 7.

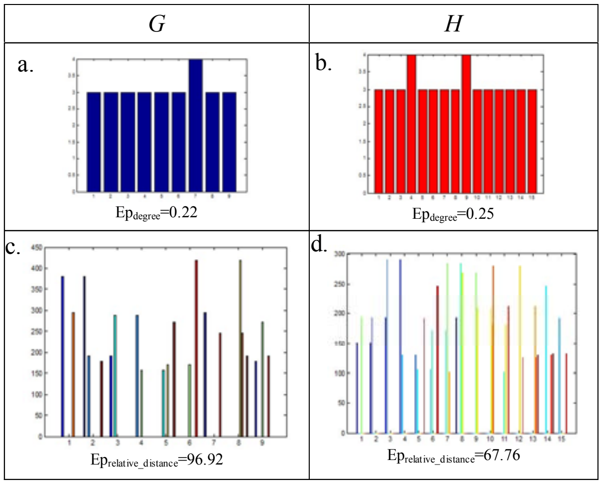


Figure 6. Graphical view of derived attributes over each node (x-axis shows the node # as shown in Figure 5)

We demonstrate the novel feature of the local consistency measures in Figure 8, where stars are global compatibility over iteration with relative distance attribute. One may note that the global compatibility has a steep increase with first iteration. This characterizes the local similarity measurement we proposed having the advantage to fast approach the optimal matching locally without much 'confusion'.

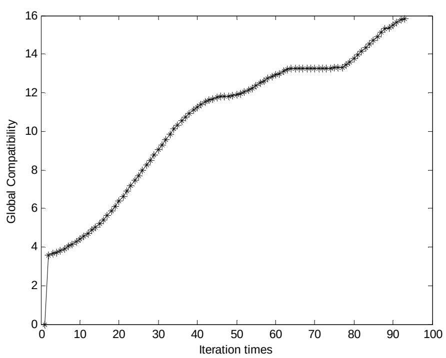


Figure 8. Variation of the global compatibility over iterations

To evaluate the performance of our approach when errors present in the data, we assume there is a new road appearing in the image between intersections denoted by node 1 and node 8 in graph *G*, but the vector data hasn't been updated yet. There is no edge connecting node 8 and 10 in graph *H*. Furthermore, there is a road in the vector data between intersections denoted by node 3 and 5 in graph *H*. This road on the image, however, is missed due to extraction errors. Figure 9 shows that, under presented errors, the global compatibility present a similar characteristic (steep increase) as in Figure 8, representing a distinct feature of our approach.

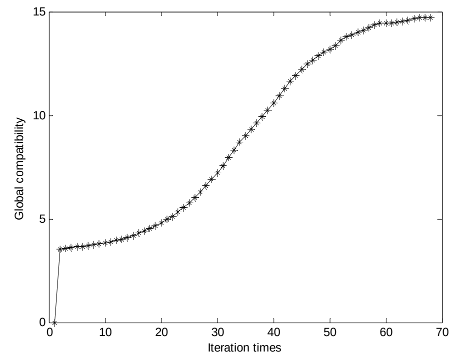


Figure 9. Comparison under inexact matching

Both matching results are summarized in Table 2, where the correspondences between nodes as a result of the network matching are shown column-wise. It can be easily seen that all nodes were matched correctly despite of coverage and orientation differences, as well as presented errors between these two networks.

Table 2. Matching result from both experiments

	Matching result								
G	1	2	3	4	5	6	7	8	9
H	8	7	6	14	13	12	9	10	11

6. CONCLUSIONS

This paper introduced a novel matching approach to the registration problem based on graph matching. It offers the ability to utilize information about the topology and geometry of a network to establish correspondence. The ability to utilize both allows us to reduce the ambiguity of local consistency, especially when inexact matching takes place. The statistical analysis of attributes provides a quantitative measure to support the attribute selection in matching. Furthermore, the approach does not require user input for the acceptance of local similarity, other than detecting road intersections through image processing. Thus our approach offers a robust and general solution to the image-to-x registration problem using networks.

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