

ON THE SELF-CALIBRATION OF LONG FOCAL LENGTH LENSES

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ABSTRACT:

One of the practical impediments to the adoption of long focal length cameras in close-range photogrammetry is the difficulty in network exterior orientation and self-calibration that can be encountered with the collinearity equation model when the camera field of view is smaller than around 10° . This paper reports on an investigation that examined two avenues for improving the self-calibration of long focal length cameras. The first makes use of orthogonal decomposition, and specifically the Singular Value Decomposition as a prospective means to better accommodate the ill-conditioned observation and normal equations of the bundle adjustment that arise for very narrow field of view images. The second is a re-examination of the linearization of the additional parameter model for self-calibration, and especially determination of the coefficients in the design matrix corresponding to the interior orientation parameters. Through the use of both simulated and real test networks, it is shown that while the adoption of numerical analysis tools that are better suited to ill-conditioned equation systems offer little benefit in practical terms, a simple extension of the generally adopted partial derivatives for principal point coordinates has a significant, positive impact on the stability and accuracy of the self-calibration of cameras with long focal length lenses. It is demonstrated that the proposed 'correction' to the functional model allows the stable recovery of calibration parameters for cameras with fields of view down to 3.5° .

1. INTRODUCTION

The growing use of digital SLR cameras in close-range photogrammetry has been accompanied by an increasing desire to perform measurements over long distances, for applications in construction engineering, deformation monitoring and traffic accident reconstruction, for example. Lenses of long focal length are often warranted in such cases in order to keep the spatial resolution high and to optimize the angular measurement precision. However, there are practical impediments to the adoption of very long focal length lenses on small format cameras, these centering principally upon potential difficulties in analytical orientation and consequently self-calibration. As focal length increases, so the field of view becomes narrower. This can impact adversely on the performance of the conventional central perspective, collinearity equation model, since the bundle of rays can approach, in effect, a parallel projection.

Fully automatic camera self-calibration generally employs object point arrays in which some or all of the targeted points are coded. A push-one-button operation is expected and so long as well-known principles such as convergent imaging, the use of orthogonal camera roll angles and, desirably, use of a 3D target distribution are adopted, a successful outcome can be anticipated for medium and wide angle lenses. In applying network orientation with self-calibration to images with very narrow fields of view images, however, problems can arise through overparameterization, ill-conditioning and subsequent numerical instability in the normal equations of the bundle adjustment. The appearance of numerical problems might be anticipated when the field of view drops below 10° , which is equivalent to a 200mm lens on a 35mm format digital SLR camera. Recovery of satisfactory camera calibration parameters is often precluded in such 'weak geometry' cases due to linear dependencies that arise between the interior and exterior orientation parameters.

The topic to be examined in this paper concerns how to enhance the stability of the self-calibrating bundle adjustment in the presence of very long focal length lenses, the aim being two-fold: to improve the robustness and precision of recovery of camera and exterior orientation parameters, and to possibly extend the applicability of self-calibration to sensors with even narrower fields of view. In seeking to overcome problems encountered in the self-calibration of cameras with narrow fields of view, two prospective approaches are available, both of which will be considered here. The first is to investigate means to better accommodate numerical ill-conditioning, and the second is to look to the formulation of the functional model. This could range from a re-examination of the existing collinearity equations with additional camera calibration parameters, through to formulation of alternative mathematical models, for example affine projection.

2. MATRIX DECOMPOSITION

The estimation of camera parameters in close-range photogrammetry is generally accomplished by bundle adjustment in which the highly overdetermined linear observation equation system

$$Ax = b \quad (1)$$

is solved via the least-squares normal equations

$$Nx = u \quad (2)$$

Here, $N = A^T A$ and $u = A^T b$, A being the design or configuration matrix and x the vector of unknown parameters. The vector x is generally obtained through the inversion of matrix N , but there are various alternatives to the solution of the normal equations that do not involve this explicit matrix inversion. These include triangular decompositions, such as LU and Cholesky, which produce an upper and/or lower triangular matrix where the solution can be readily computed via backward or forward substitution.

Another useful and practical way to solve normal equation systems is via widely used orthogonalization techniques, such as Householder, Gram-Schmidt, Given matrices, QR, eigenvalue and the Singular Value Decomposition (SVD). Orthogonal decomposition of the configuration matrix A leads to unitary and diagonal matrices, allowing for simple solution of the linear systems. Additionally, the need to create the normal equations is circumvented, which is desirable in the sense that the Condition Number of N is the square of that of A .

As a component of the reported investigation, an analysis of selected numerical techniques applicable to poorly conditioned or near-singular linear models has been performed. A range of triangular and orthogonal factorisations were tested and their numerical properties and stability evaluated. It was found that the characteristics and simplicity of the SVD make it a logical choice for application to the self-calibrating bundle adjustment of networks of images with narrow fields of view.

2.1 Singular Value Decomposition

The SVD is a particularly revealing, complete orthogonal decomposition. It provides a convenient expression for directly obtaining the least squares solution and the norm of the residuals $p = \|Ax - b\|$, while having the potential to provide better insight into the nature of the ill-conditioning problem. The SVD of an $m \times n$ matrix $A \in R^{m \times n}$ of rank r is $A = U\Sigma V^T$, where U is an $m \times m$ orthogonal matrix, Σ is an $n \times n$ diagonal matrix and V is an $n \times n$ orthogonal matrix. For an SVD solution, Eq. 1 is recast as

$$U\Sigma V^T x = b \quad (3)$$

and the solution is calculated by

$$x = \sum_{i=1}^n \frac{u_i^T b}{\sigma_i} v_i \quad (4)$$

Typically, one hopes to find an index κ such that all coefficients $p_i = \frac{u_i^T b}{\sigma_i}$ are acceptably small for $i \leq \kappa$, all singular values σ_i for $i \leq \kappa$ are acceptably large, and the residual norm

$$p^2 = \|Ax - b\|_2^2 = \sum_{i=r+1}^m (u_i^T b)^2$$

is reasonably small. This technique can be used in order to successfully solve ill-conditioned systems via an approximation to a corresponding well-conditioned system. By ignoring the small singular values, or treating them as zero, a more satisfactory solution can be achieved. Additionally, linear column dependencies, that in photogrammetric orientation arise through projective coupling of parameters, can be identified. The columns of the matrix V associated with small singular values may be interpreted as indicating a high degree of linear dependence between columns of A . More detailed analysis of the SVD is presented in Golub & Reinsch (1970), Reinsch & Wilkinson (1971), Chan (1982) and Golub & Van Loan (1996).

2.2 Bundle Adjustment and the SVD

In order to fully evaluate the SVD in the handling of orientation and calibration of long focal length cameras, the algorithm proposed by Golub et al. (1970) and Reinsch et al. (1971) was implemented in the *Australis* software platform. The self-calibrating bundle adjustment was initially set up as a direct solution of Eq. 1, since this is the most straightforward implementation and it is more numerically stable than using the normal equations. The SVD was also implemented for bundle adjustment utilizing normal equations and incorporating both the standard and reverse fold-in techniques in order to compare

solution stability against the direct solution of Eq. 1. The pseudo-inverse produced by the SVD was also used to overcome the rank defect in the network. This was accomplished by zeroing out the seven smallest singular values.

2.3 Testing Regime

In order to ensure the robustness of the SVD algorithm, various tests with existing multi-image close-range photogrammetric data sets were performed. In the case of wide angle lenses the calibration results should be the same as those obtained via 'normal' bundle adjustment, and this was found to be so for all the cases examined. Differences in the estimated precision of the object points arose in some cases; however Fraser (1982) has shown that use of the pseudo-inverse does not lead to a minimal a posteriori variance of object point coordinates and thus the different standard error values were to be expected.

It is noteworthy that, in the case of the normal equations, incorrect results were obtained whenever the SVD was used to decompose a matrix that contained the interior orientation parameters of the camera jointly with other parameters. In such cases the decomposition was not accurate, probably due to numerical issues with the calculation of the normal equations. A minor modification to the standard and reverse fold-in algorithms was applied so that the matrix being decomposed holds either of the exterior orientation or XYZ coordinates, depending on the fold-in approach used. This problem did not arise in the direct solution of Eq. 1 via the SVD.

In order to gain further insight into the calibration of long focal length lenses, a number of simulated and real image networks, with camera fields of view ranging from 13° down to 4° , were initially solved via the normal bundle adjustment approach. The principal aim was to identify the properties and the application limits of the current model. Even though a firm figure for the minimum allowable value cannot be given, special attention was typically required when the field of view fell below 10° . Unsuccessful and unsatisfactory self-calibrations were then re-evaluated via application of the SVD in the expectation of improved numerical behaviour. A numerical analysis of the linear system was performed for these cases, which showed no abnormalities or any linear or near-linear dependencies between the parameters compared to the wide angle datasets.

While improved results were anticipated through use of the pseudo-inverse by means of the SVD, the results obtained were instead the same for all practical purposes to those achieved using a regular bundle adjustment employing Cholesky decomposition of the normal equations. Thus, further insight was indeed gained into the self-calibration of narrow-field-of-view cameras, namely that the application of numerical tools such as orthogonal decomposition does not enhance the recovery of calibration parameters, and so practical limits to focal length still apply. Attention needed instead to be turned to the second option mentioned above, namely the formulation of the functional model.

3. PARTIAL DERIVATIVES OF INTERIOR ORIENTATION PARAMETERS

3.1 Calibration Model

The well-known 8-parameter 'physical' camera calibration model developed by Brown (1971) has been found to be near universally applicable in close-range photogrammetry. For long focal length lenses, however, a careful selection of parameters

has to be performed, especially as the correlation between camera parameters, and interior and exterior orientation parameters increases with increasing focal length. The properties of long focal length lenses are well recognised and have previously been noted by Wiley & Wong (1995), Noma et al. (2002), Labe & Förstner (2004), and Fraser & Al-Ajlouni (2006).

The calculation of principal distance and principal point coordinates is of equal importance for long as for short focal length lenses, in spite of the opportunities for projective compensation in the photogrammetric orientation of narrow field of view imagery. Also, the radial distortion is metrically very significant and needs to be taken into account for any photogrammetric application. Radial distortion is universally modelled via the well-known odd-ordered polynomial expression comprising terms to seventh order. However, for zoom lenses the third-order coefficient K_1 is usually sufficient to describe the radial distortion profile; the maximum radial distortion occurs at the minimum zoom focal length and it decreases as the zoom focal length increases (Fraser & Ajlouni, 2006).

The image coordinate correction model adopted for self-calibration of long focal length lens can then comprise a 4-parameter subset of Brown's model:

$$\begin{aligned} x_{corr} &= x - x_p + (x - x_p)K_1r^2 - \frac{x}{c} dc \\ y_{corr} &= y - y_p + (y - y_p)K_1r^2 - \frac{y}{c} dc \end{aligned} \quad (5)$$

where c is the principal distance and r the radial distance, ie $r = \sqrt{(x - x_p)^2 + (y - y_p)^2}$.

3.2 Partial Derivatives

Upon linearization of the collinearity equations to form the configuration matrix A , partial derivatives with respect to the unknown calibration parameters forming Eq. 5 are determined. Traditionally, this has led to coefficients of -1 for the parameters x_p and y_p , resulting in a model that has served the photogrammetric community well for 40-odd years. However, given the impact of even the smallest inaccuracies in the ill-conditioned and consequently unstable equation system for the self-calibrating bundle adjustment when cameras with very long focal lengths are involved, it behoves us to look again at the determination of partial derivatives of the image correction model, especially terms for the principal point coordinates.

As is apparent from Eq. 5, the correct terms for x_p and y_p in the configuration matrix A of partial derivatives are not the commonly employed

$$\begin{matrix} & x_p & y_p \\ \frac{\partial x}{\partial y} & \begin{pmatrix} \dots & -1 & 0 & \dots \\ & 0 & -1 & \dots \end{pmatrix} \end{matrix}$$

but instead

$$\begin{matrix} & x_p & y_p \\ \frac{\partial x}{\partial y} & \begin{pmatrix} \dots & -1 - K_1r^2 - 2(x - x_p)^2K_1 & (x - x_p)r^2 \\ & (y - y_p)r^2 & -1 - K_1r^2 - 2(y - y_p)^2K_1 \dots \end{pmatrix} \end{matrix}$$

It will be shown that this small expansion or 'correction' to the configuration matrix for self-calibration can greatly enhance the recovery of interior orientation parameters in the self-calibration of cameras with long focal length lenses, even though the

magnitude of K_1 might only be of the order of 10^{-5} . In situations where the correlation between camera parameters is not high, the presence of small errors in the coefficients of the A matrix might not have a significant impact upon interior orientation parameter estimation, and consequently upon exterior orientation and object space coordinates. However, the opposite can be true where there is strong projective coupling, as with narrow field of view imagery.

4. EXPERIMENTAL EVALUATION

4.1 Preliminary Testing

Acquisition of images with long focal length lenses can pose practical difficulties for a number of reasons. Thus, in order to validate the proposed model for cameras with narrow fields of view, and also to assess its practicability, initial tests were carried out by using mostly simulated data where more control over the complete process was possible. This stage of experimental evaluation examined the impact of the expanded partial derivative terms within a 'standard' self-calibration; the SVD was not employed for the solution of the bundle adjustment.

For the first stage of testing, a new, comprehensive photogrammetric network simulator was developed. Coded and normal targets were used as test objects to facilitate automatic network orientation and self-calibration for focal lengths ranging from 100-400mm (field of view of 13.5° - 3.4°). The developed software provided fine-scale control over the camera interior and exterior orientation, and the object points, while supporting the creation of fictitious images. Thus, it was possible to test various network configurations and very encouraging results were obtained. Self-calibrations with the new partial derivative terms led to successful and correct outcomes across the full range of focal lengths tested, even down a field of view of 3.4°.

4.2 Self-Calibration Networks

The investigations with simulated data revealed promising results and consequently further testing with real imagery was performed. A Nikon D200 camera was employed, first with a zoom lens set at 300mm (field of view of approximately 4.5°), and then with a separate lens of 400mm focal length (field of view of approximately 3.4°). Tape was used in both cases to keep the focal length and focus fixed. Silver retroreflective coded targets of 6mm diameter were employed to facilitate automatic exterior orientation, whereas an array of single silver retro-targets formed the main target field. In both cases an external Nikon Speedlight SB 800 flash was used to illuminate the target array.

4.2.1 Case 1

Case 1, the geometry of which is shown in Figure 1, comprised a 7-station convergent network in which three images per station were recorded, at zero, 90° and -90° roll angles, over a camera-to-object distance of 70m. The approximate distance between adjacent camera stations was 12m. A total of 88 coded and 25 single targets were used. Three such 21-image networks were acquired, the lens being a Nikon ED AF NIKKOR 70-300mm 1:4-5.6D zoom lens, fixed at 300m.

With the expanded coefficients for principal point coordinates in the configuration matrix, the self-calibration of the three networks was successful and no numerical stability issues were

encountered. The calibration results showed high repeatability between the different networks, with the precision attained for the interior orientation parameters being standard errors of around 0.15mm for c and better than 0.01mm for x_p and y_p . The overall accuracy of object point coordinate determination from one of the three networks is provided in Table 1.

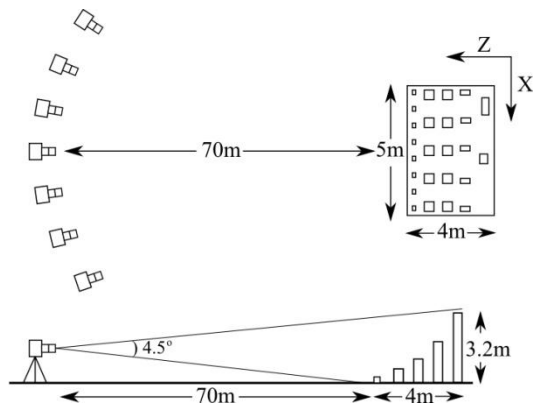


Figure 1. 7-station, 21-image network configuration for Case 1, 300mm focal length.

σ_x	0.09 mm	1:70,000
σ_y	0.06 mm	1:98,000
σ_z	0.21 mm	1:29,000
Mean Std. Error σ_{XYZ}	0.12 mm	1:51,000
RMS of xy residuals	0.83 μ m	

Table 1. Precision of object point coordinates for Case 1.

The three networks were also solved with the conventional self-calibration model (coefficients of -1 for x_p and y_p in the A matrix) in order to compare the results. The self-calibration solution, however, was unstable and yielded implausible results for the interior orientation, with estimates of the standard error for c , x_p and y_p being several mm. The poor determinability and repeatability of the camera interior orientation also adversely impacted upon the accuracy of the computed exterior orientation and object point coordinates.

4.2.2 Case 2

The network geometry for Case 2, illustrated in Figure 2, was similar to that of Case 1, with the distinction being that the camera-to-object distance was 100m, the overall convergence angle somewhat less (7m between adjacent stations), and the employed focal length was 400mm, the lens being a Nikon 80-400mm f/4.5-5.6D VR zoom lens. For this network, 94 coded and 25 single retroreflective targets were used. A second set of 18 images was acquired from six additional stations to independently assess the integrity of the initial self-calibration, and especially to assess the stability of recovery of camera interior orientation. The camera stations for this network were positioned between those of the 7-station configuration.

The two self-calibrating bundle adjustments utilising the expanded A -matrix were again free of numerical stability issues. The estimates of the interior orientation for the two networks displayed no significant differences, even though the geometry was weaker than in Case 1 and the field of view narrower, at 3.4° . The estimated standard errors were 0.8mm for c and better than 0.01mm for x_p and y_p . For this case the recovery of a stable

calibration via the traditional additional parameter model (coefficients of -1 for x_p and y_p in the A matrix) was not possible, which highlights the utility of the new approach.

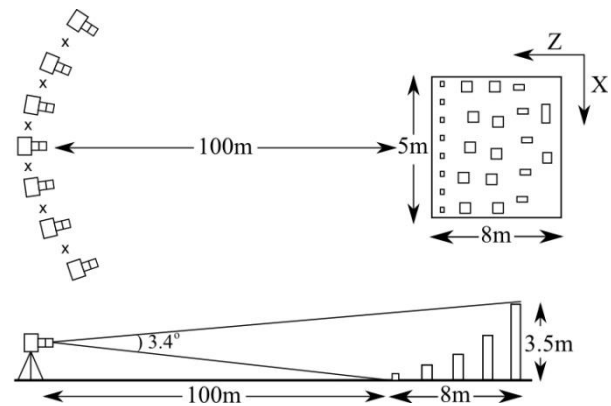


Figure 2. 7-station, 21-image network configuration for Case 2, 400mm focal length.

As a final processing step, the two networks were combined in a 39-image bundle adjustment in order to improve the precision of recovery of calibration parameters and object point coordinates. The results are summarized in Table 2 and the camera station configuration illustrated in Figure 3.

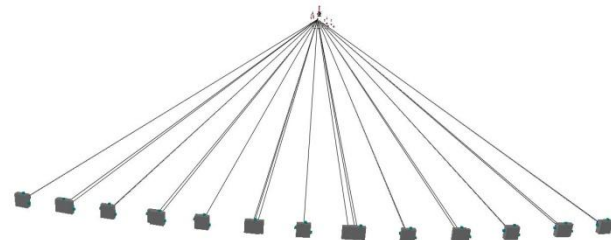


Figure 3. 13-station, 39-image network configuration for combined Case 2, 400mm focal length.

σ_x	0.12 mm	1:72,000
σ_y	0.30 mm	1:30,000
σ_z	0.54 mm	1:17,000
Mean Std. Error σ_{XYZ}	0.32 mm	1:28,000
RMS of xy residuals	1.3 μ m	

Table 2. Precision of object point coordinates for the 39-image combined network of Case 2.

It is noteworthy that the 80-400mm zoom lens of Case 2 had very pronounced chromatic aberration which degraded image quality and thus the accuracy of centroiding image points. The adverse impact of chromatic aberration can be seen in the higher RMS value of image coordinate residuals, and consequently also in poorer than anticipated precision of object point determination.

4.3 Macro Lens

A network of images recorded with a macro lens formed Case 3. Here, a Nikon D80 fitted with a Sigma AF 105mm f/2.8 EX macro DG lens was used to record images of a 50 cent coin from a setback distance of 40cm. Since there is no focal length ring on this lens, the principal distance value can only be

guessed prior to calibration from the magnification index, which was set to 1:3. The calibrated value turned out to be 134.8mm (field of view of approximately 10°). A convergent network of nine basic camera positions was established, with three orthogonally rolled images being taken at each, as indicated in Figure 4.

To facilitate automatic exterior orientation and self-calibration, 17 coded targets were placed around the object. The coded targets were specially designed for this project and they comprised black dots with an approximate diameter of 0.5mm, printed on normal paper.

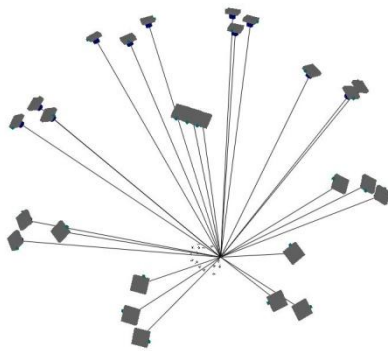


Figure 4. 27-image network configuration for Case 3, macro lens network, 135mm focal length.

Even though the field of view was almost double that in the previous cases, and the network displayed the strong geometry of a ring configuration, the conventional self-calibration model failed to provide either stable or plausible interior orientation results. The proposed approach with modified partial derivative terms, on the other hand, was able to provide a correct solution for the interior orientation. The resulting precision for the object space coordinates is summarised in Table 3.

σ_x	1.2 μm	1:49,000
σ_y	1.5 μm	1:40,000
σ_z	0.8 μm	1:68,000
Mean Std. Error σ_{xyz}	1.2 μm	1:50,000
RMS of xy residuals	1.1 μm	

Table 3. Precision of object point coordinates for the 27-image macro lens network.

5. CONCLUSION

In its adoption of two possible approaches to enhance the performance of self-calibration of close-range, narrow field of view cameras this investigation has yielded two noteworthy findings. The first is that the SVD can provide enhanced insight into both the numerical conditioning of a bundle adjustment solution and the quantifiable extent of projective coupling between parameters, but it appears not to offer an avenue for improved photogrammetric orientation of long focal length imagery. This finding is useful as it indicates that in order to handle the photogrammetric orientation of very narrow field of view imagery, the focus of attention needs to be upon the formulation of the functional model and not necessarily upon prospects for enhancing the solution of an ill-conditioned set linear equations.

The second finding, which is of practical significance, is that the recovery of camera calibration parameters in a network of very narrow field of view images is greatly enhanced through use of expanded partial derivative expressions for the principal point offsets x_p and y_p . The experimental tests presented have illustrated that whereas issues of numerical stability can be expected to appear with the traditional approaches when the field of view is around 10°, the expanded coefficients can facilitate stable and accurate self-calibration in cases where the camera field of view is only 3.5°. Moreover, what is appealing about the approach discussed is its simplicity, since the required changes to software systems to implement the expanded partial derivative terms are trivial.

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7. REFERENCES

- Brown, D.C., 1971. Close-Range Camera Calibration. *Photogrammetric Engineering*, 37: 855-866.
- Chan, T., 1982. An Improved Algorithm for Computing the Singular Value Decomposition. *ACM Trans. on Mathematical Software (TOMS)*, 8: 72-83.
- Fraser, C.S., 1982. Optimization of Precision in Close-Range Photogrammetry. *Photogrammetric Engineering and Remote Sensing*, 48: 561-570.
- Fraser, C.S., 1997. Digital Camera Self-Calibration. *ISPRS Journal of Photogrammetry and Remote Sensing*, 52: 149-159.
- Fraser, C.S. and Al-Ajlouni, S., 2006. Zoom-Dependent Camera Calibration in Digital Close-Range Photogrammetry. *Photogrammetric Engineering and Remote Sensing*, 72, 1017.
- Golub, G., and Van Loan, C., 1996. *Matrix Computations*. Johns Hopkins Univ Pr.
- Golub, G. H., and Reinsch, C., 1970. Singular Value Decomposition and Least Squares Solutions. *Numerische Mathematik*, 14: 403-420.
- Labe, T., and Förstner, W., 2004. Geometric Stability of Low-Cost Digital Consumer Camera. *International Archives of Photogrammetry and Remote Sensing*, 35: 528-535.
- Noma, T., Otani, H., Ito, T., Yamada, M. and Kochi, N., 2002. New System of Digital Camera Calibration. *International Archives of the Photogrammetry, Remote Sensing and Spatial Information Sciences*, 34, 54-59.
- Reinsch, C. and Wilkinson, J., 1971. *Handbook for Automatic Computation, Vol. 2, Linear Algebra*, Springer, London.
- Wiley, A. and Wong, K., 1995. Geometric Calibration of Zoom Lenses for Computer Vision Metrology. *Photogrammetric Engineering and Remote Sensing*, 61: 69-74.