THEORETICAL FRAMEWORKS OF REMOTE SENSING SYSTEMS BASED ON COMPRESSIVE SENSING

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ABSTRACT:

As an application of Compressive Sensing (CS) in remote sensing area, the theoretical frameworks of SAR and optical imaging system based on CS are investigated. The processes of data acquisition are mathematically described. After that the sparse representation of images corresponding to the two systems are also presented. Finally, the spare recovery is employed to retrieve images. Numerical simulations validated the feasibility of such imaging systems.

1. INTRODUCTION

Compressive Sensing (CS) provides us with a new theory for signal/image acquisition. Employing this theory, we can reconstruct signals with equivalent or better qualities (e.g.: resolution, SNR, etc.) by using less sensors, slower sampling rate, smaller data size or lower power consumption than that required in traditional system. Instead of uniform and periodical samples, CS measurements are formed by the inner products of signals with certain sensing matrix. The sparsity of signal is exploited to accurate recovery, and the measurements utilized are no longer depending on the signal's bandwidth but on the signal's sparsity. Generally speaking, the dimension of measurement vector is logarithmically with the dimension of signal (Candes, 2006; Candes, 2006b; Candes, 2006c; Donoho, 2006).

In this paper, we focus our research on the application of CS in remote sensing systems, and especially for optical and synthetic aperture radar (SAR) imaging system. In the area of optical compressive imaging, Baraniuk's group realized a Rice Single-Pixel CS Camera (Duarte, 2008), in which the Digital Micro-mirror Device (DMD) is served as sensing matrix. This camera suffers from an inherent inefficiency: sequential measurements are needed. But in many scenarios, there is a high-speed movement between the imaging sensor and target (such as spaceborne remote sensing), and the sequential multiple measurements is not permitted. A. Stern and B. Javidi proposed a random projection imaging system (Stern, 2007), in which the measurements are obtained within a single exposure by using a random phase mask. Enjoying sparse recovery, the more object pixels may be reconstructed and visualized than the number of pixels of the image sensor. But the design of sensing matrix and sparse recovery algorithm desire much improvement. Meanwhile, in the area of radar imaging, Baraniuk introduced a compressive radar imaging system (Baraniuk, 2007), but the simulation is too simple and far away from practical application. Besides, J. Romberg proposed a sampling strategy (Romberg, 2008) based on "random convolution", and discussed its application in radar and Fourier optics conceptually.

We investigated the theoretical frameworks of compressive radar and optical imaging systems, which involves: 1) mathematically reformatted the processes of data acquisition of SAR and optical imaging in the form of linear system, and then the imaging process becomes the inverse problem. Furthermore, a process called random phase modulation is specially designed for CS optical imaging system. 2) The sensing and sparse representation matrices are chosen according to the characteristics of data acquisition and images from SAR or optical imaging systems respectively. Due to the large data scale of two-dimensional imaging problem, we also give attentions to the computational efficiency of sparse recovery. Finally, in each system, numerical simulations are conducted to validate the feasibility. Especially, the data for CS SAR imaging is generated from professional electromagnetic scattering computing software which is similar to real SAR raw data.

2. COMPRESSIVE SAMPLING AND SPARSE RECOVERY

Different from the traditional uniform and periodical samples, the measurements in CS are the projections of the signal onto the sensing matrix, i.e.

$$\mathbf{y} = \mathbf{\Phi x}_0 + \boldsymbol{\varepsilon} \tag{1}$$

where $\mathbf{x}_0$ is the N-dimensional signal, $\mathbf{y}$ is the M-dimensional measurement, $\mathbf{\Phi}$ is the sensing matrix, $\boldsymbol{\varepsilon}$ stands for the noise in the data acquisition process, and its variance is σ^2. The dimension of $\mathbf{y}$ is far smaller than $\mathbf{x}_0$, i.e.: $M \ll N$. In order to reconstruct original signal, the sparsity of $\mathbf{x}_0$ is required, that is, with a representation matrix $\mathbf{\Psi}$, we can decompose $\mathbf{x}_0$ as $\mathbf{x}_0 = \mathbf{\Psi}\boldsymbol{\alpha}_0$, where the coefficient $\boldsymbol{\alpha}_0$ has at most K non-zero (or almost non-zero) components. Substitutes the signal's sparse representation into Eq. (1), then

$$\mathbf{y} = \mathbf{\Phi\Psi}\boldsymbol{\alpha}_0 + \boldsymbol{\varepsilon} \triangleq \mathbf{\Theta}\boldsymbol{\alpha}_0 + \boldsymbol{\varepsilon} , \tag{2}$$

where $\mathbf{\Theta}$ is a matrix compound by the representation and sensing matrix.

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The coefficient $\boldsymbol{\alpha}_0$, so the signal $\mathbf{x}_0$, can be recovered by solving a convex program

$$\min \|\boldsymbol{\alpha}\|_1 \quad \text{s.t.} \quad \|\mathbf{y}-\boldsymbol{\Theta\alpha}\|_2^2 \le \sigma^2 , \tag{3}$$

given the matrix $\boldsymbol{\Theta}$ obyes a Restricted Isometry Property (RIP) (Candes, 2006c).

Roughly speaking, the RIP has restrained the theoretical lower bound of number of measurements. For $M \times N$ -dimensional random sensing matrices whose entries are independently generated according to the Gaussian or Bernoulli distribution, when the sparsity of signal K and the dimension of signal N are given, the number of measurements M must obey $M \ge K \log N$.

Many algorithms have been developed to handle the optimization in Eq. (3). Basis Pursuit (BP) (Chen, 1999) is one of the first proposed methods. This method enjoys rigorous proofs of exact reconstruction, but suffers from heavy computationally burdens and can not be used in two-dimensional imaging which involves data with large scale. Algorithms proposed recently have improved computationally complexity without loss of precision. These algorithms include: Gradient Projections for Sparse Reconstruction (Figueiredo, 2007), Sparse Reconstruction by separable approximation (Wright, 2009), Spectral Projected Gradient (Van Den Berg, 2008), Fixed Point Continuation method (Hale, 2008) and its modification (Wen, 2008), Bregman iteration (Cai, 2008; Osher, 2008) etc.

The sparse recovery algorithm employed in this paper is Fixed Point Continuation (FPC) algorithm, which can solve large scale problem. We briefly describe it in Algorithm 1.

Algorithm 1 (FPC algorithm)

Input : $\mathbf{y}, \boldsymbol{\Theta}$; Parameter : $\bar{\mu}, \tau, \eta, xtol$

Initializtion: $\boldsymbol{\alpha}_k = 0, \boldsymbol{\alpha} = \tau \boldsymbol{\Theta}^T \mathbf{y}, \mu = \dfrac{1}{\|\boldsymbol{\Theta}^T \mathbf{y}\|_\infty}$

while $\mu \le \bar{\mu}$

  while $\dfrac{\|\boldsymbol{\alpha}-\boldsymbol{\alpha}_k\|_2}{\|\boldsymbol{\alpha}_k\|_2} > xtol$

    $\boldsymbol{\alpha}_k = \boldsymbol{\alpha}$

    $\mathbf{g} = \boldsymbol{\Theta}^T(\boldsymbol{\Theta}\mathbf{x}-\mathbf{y})$

    $\mathbf{b} = \boldsymbol{\alpha} - \tau \mathbf{g}$

    $\boldsymbol{\alpha} = \operatorname{sgn}(\mathbf{b}) \cdot \max\left\{|\mathbf{b}| - \dfrac{\tau}{\mu}, 0\right\}$

  end while

  $\mu = \min\{\eta\mu, \bar{\mu}\}$

end while

Output : $\boldsymbol{\alpha}, \mathbf{x}_0 = \boldsymbol{\Psi\alpha}$

The value of relevant parameter recommend are: $\bar{\mu} = 1\times10^{-6}$, $\tau = 1.999$, $\eta = 4$ and $xtol = 1\times10^{-5}$.

3. RADAR IMAGING BASED ON CS

Radar image is a reflection of the electromagnetic scattering of the illuminated target. The process of "Radar Imaging" is to obtain the scattering coefficients from radar echo (i.e. the inverse process of radar echo generation). Supposing $\sigma(p_x, p_y)$ a two-dimensional function which describes the scattering coefficients, and then the radar echo can be modeled as follows:

$$E(f,\varphi) = \iint \sigma(p_x, p_y) \exp\left\{ j\frac{4\pi f}{c}(p_x \cos\varphi + p_y \sin\varphi) \right\} dp_x dp_y \tag{4}$$

where $E(f,\varphi)$ is the radar echo, f is frequency of the electromagnetic wave, φ is the angle of radar observation, and c is the speed of light.

The basic principle of classical SAR imaging is employing Fast Fourier Transform (FFT) to reconstruct $\sigma(p_x, p_y)$. It is worth to mention that the actual radar echoes contain phase errors caused by non-ideal motion of target. These phase errors should be compensated before SAR imaging (Bao, 2006).

In this paper, we utilize CS for radar sampling and imaging. Firstly, under the hypothesis of "point scattering model" (Huang, 2006), the radar images are sparse in their original (space) domain, so we can take identity matrix as the sparse representation matrix.

Secondly, we will re-format the generation of radar echo in the framework of CS. Supposing the scattering coefficients $\sigma(p_x, p_y)$ can be represented by a $\sqrt{N}\times\sqrt{N}$ -dimensional matrix, and can be reshaped into a $N\times1$ -dimensional vector $\boldsymbol{\sigma}$. Matirx $\mathbf{F}$ stands for a $N\times N$ -dimensional Kronecker product matrix, which is produced by two identical $\sqrt{N}\times\sqrt{N}$ -dimensional Fourier transform matrices. Based on these, a discrete version of Eq. (4) can be described as

$$\mathbf{E} = \mathbf{F}\cdot\boldsymbol{\sigma} . \tag{5}$$

Where $\mathbf{E}$ is a matrix represents radar echoes. Furthermore, assuming an random index set $\boldsymbol{\Gamma} \subset \{1,\ldots,N\}$ obeys $|\mathbf{T}| = M \ll N$, then, a sub-matrix $\mathbf{F}_{\boldsymbol{\Gamma}}$ can be formed by selecting M rows from $\mathbf{F}$ according to $\boldsymbol{\Gamma}$. So the compressive radar sampling process can be described as

$$\mathbf{E}_{\boldsymbol{\Gamma}} = \mathbf{F}_{\boldsymbol{\Gamma}}\cdot\boldsymbol{\sigma} . \tag{6}$$

Note that the dimension of $\mathbf{E}_{\boldsymbol{\Gamma}}$ is far lower than $\mathbf{E}$.

Finally, the scattering coefficients vector $\boldsymbol{\sigma}$ is retrieved by sparse recovery as Eq. (3).

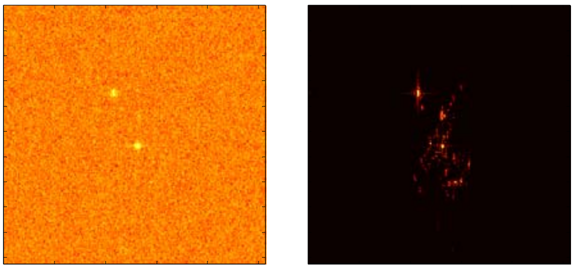
In our simulation, electromagnetic scattering computing software is employed to generate radar echoes of airplane A10. The overview of the 3D model of airplane A10 is shown in Fig. 1, and some radar parameters are list in Table 2.

The sensing matrix $\mathbf{F}_{\Gamma}$ is constructed by randomly selecting 30% rows of the 65536 × 65536 dimensional Kronecker product matrix $\mathbf{F}$, and compressive measurement is then generated. The imaging results from conventional FFT-based and sparse recovery methods are listed in Fig. 3. We can see from this figure that there are only two intense scattering points in the first image and other scattering points are missed. The sparse recovered image (Fig. 3(b)) clearly describes the outline of airplane. This comparison can validate the feasibility of the new principle for radar sampling and imaging based on CS.
Besides, the resolution of SAR image also can be enhanced via sparse recovery. The resolution of conventional correlation-based imaging is limited by the ambiguity function of the transmitted signal; while the resolution of the sparse recovery is determined by the accuracy of optimization and the discretization of scattering coefficients $\boldsymbol{\sigma}$.

Figure 1. An overview of the 3D model of A10

Table 2. Radar observation parameters

Carrier frequency	9G Hz (X band)
Bandwidth	200M Hz
Azimuth angles	44°to 47°
Polar angles	60°
Amount of samples	256 × 256



(a) (b)

Figure 3. Result comparing of the two methods based on 30% measured data. (a) conventional FFT-based method, (b) sparse recovery.

4. OPTICAL IMAGING BASED ON CS

The basic composition of the proposed random phase modulation and sparse sampling based imaging system is sketched in Fig. 4. The incident lights are firstly transformed to their frequency domain by a Fourier Transform (FT) lens. Then, the transformed lights will be leaded to a Spatial Light Modulator (SLM), and the SLM will add random phases to the lights which pass through its pixels. That is called random phase modulation in this paper. Subsequently, the modulated lights will be transformed back to space domain by an inverse Fourier Transform (IFT) lens. After that, the lights will be randomly and sparsely sampled by a two-dimensional detector array. A typical SLM consist of liquid crystal pixels, each independently addressed, acting as separate variable amplitude and phase modulator. In the proposed imaging system, the SLM is placed at the image plane of the FT lens.

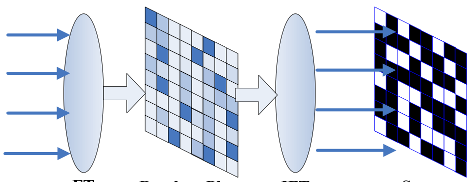


Figure 4. Schematics of the random phase modulation and sparse sampling system.

Suppose $\mathbf{x}_0$ is the reshaped $N \times 1$ -dimensional image (whose original dimension is $\sqrt{N} \times \sqrt{N}$), and its frequency spectrum is $\mathbf{F}\mathbf{x}_0$, where matrix is the same as defined in Eq. (5). The SLM can be mathematically described as a $N \times N$ diagonal matrix $\mathbf{\Sigma}$, whose non-zero entries are $\exp(-j\pi \cdot \varphi_n)$, $1 \le n \le N$, where $\varphi_n \sim \mathrm{Uniform}([-1,1])$. The sparse sampling is realized by a multiplication of a $M \times N$ matrix $\mathbf{S}$, whose rows are randomly selected from a $N \times N$ diagonal matrix whose entries in diagonal line obey (0, 1) binominal distribution. Then, the whole process of Fig. 4 can be mathematically described as

$$\mathbf{y} = \mathbf{S}\mathbf{F}^{-1}\mathbf{\Sigma}\mathbf{F}\mathbf{x}_0 \tag{7}$$

where $\mathbf{F}^{-1}$ stands for two-dimensional IFT matrix.

The function of the phase modulation (i.e. multiplication of matrix $\mathbf{\Sigma}$) is to translate phases of the spectrum to “noise like” modalities. So, when the modulated spectrum is transformed back to space domain, its energy will evenly spread out of the entire image plane. This means that each sample from the detector will carry some mixed information about the original image. If sufficient measurements (which is still much less than N) are obtained, we can reconstruct the image according to Eq. (3). In other words, the random phase modulation extended the space-bandwidth product of original signal. It, together with the sparse recovery, enables the system to acquire more detailed (high-resolution) images with fewer measurements.

The sparse representation in optical imaging is more complicated than radar imaging, where identity matrix is chosen. Borrowing the idea form JPEG and JPEG2000 stands, we employ Discrete Cosine Transform (DCT) and Discrete Wavelet Transform (DWT) to construct our sparse representation matrix.

The feasibility of the proposed random demodulation and spare sampling based optical imaging system is validated by a numerical experiment in this section. The original image in the simulation is show in Fig. 5(a). The measurements are generated according to Eq. (6), and $M/N \approx 0.5$. Fig. 5(b) shows an image from direct reconstruction, which is an inverse process of Eq. (6). The sparse recovery gives much better results. Two different images are shown in Fig. (c) and (d), which corresponding to DCT and DWT representation matrix respectively.

One of the superiorities of CS is reducing the measurement for image reconstruction. Fig. 6 shows the curve of RMSE of sparse recovery with DCT and DWT due to the variation of M/N .

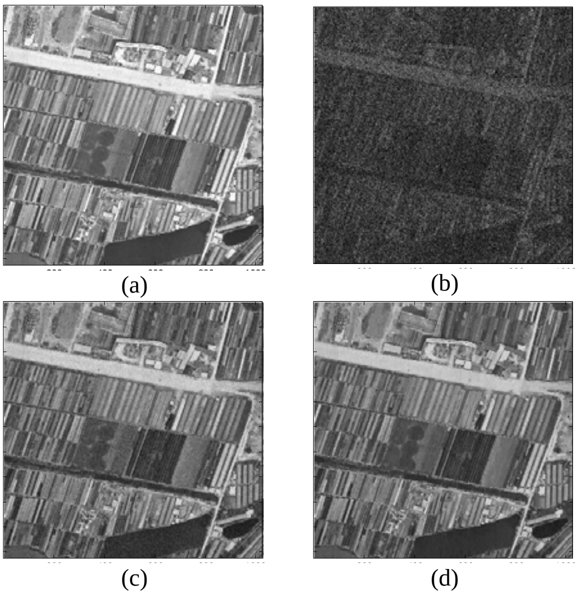


Figure 5. Result of the CS based optical imaging simulation. (a) original image, (b) direct reconstruction (RMSE: 0.30), (c) sparse recovery with DCT matrix (RMSE: 0.06), (d) sparse recovery with DWT matrix (RMSE: 0.05).

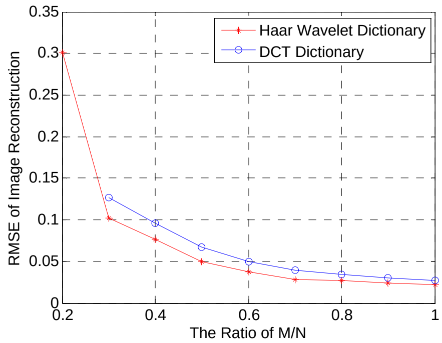


Figure 6. Curve of RMSE of sparse recovery with DCT and DWT due to the variation of M/N.

5. CONCLUSIONS

Benefit from its potential for alleviating the data sampling and storage system, CS theory has received more and more attentions. As an application in remote sensing area, the theoretical framework of SAR and optical imaging based on compressive sampling and sparse recovery is investigated in this paper. Numerical simulation validated the feasibility of the systems.

CS theory also can be employed in any signal acquisition system which can be re-formulated as an inversion of linear equations. However, the primary restriction is the computational efficiency in sparse recovery. Although we have emphasized this problem in our algorithm, the consumption of memory and time still substantially exceeds that of the conventional imaging method, particularly when the images are large.

Future research will focus on the further alleviation of the computational burdens. The other important point is the optimization of sensing matrix, which will permit exactly reconstruction with fewer measurements. The difficulty of physical realization will also be taken into account during the optimization.

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