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A NEW RIGOROUS SENSOR MODEL FOR RADAR IMAGERY BASED ON EXTERIOR ORIENTATION ELEMENTS

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ABSTRACT:

Imaging equations are always considered as the most essential and basic part in photogrammetry and image mapping with remote sensing images. Rigourism and conciseness should be held as their basic characteristics.In this paper,using sensor exterior orientation including three lines and two attitude elements as the orientation parameters, new imaging equations for side-looking radar or SAR image positioning were derived. The model was based on the distance condition between sensor and object point and the azimuth condition of sensor scanning plane including antenna and radar beam center. Three forms of range-coplanarity equation were derived, the first form was in the tangent plane rectangular coordinate system, the second was in the geocentric rectangular coordinate system, and the third was the range-coplanarity equation with coordinates of image point as explicit function. The model could be easily used for side-looking radar and SAR image processing in photogrammetry field.

1. INTRODUCTION

1.1 The existing radar imaging equations

（1）Based on Range-Doppler condition model

R-D model is built with distance conditions and Doppler conditions, It is F.Leberl model when Doppler frequency value is set to zero.[1,2].

（2）Collinearity equation model:

The imaging equation based on collinearity equation is similar with that based on optical images. This equation uses exterior orientation elements as its orientation parameters. Generally, Radar images are processed as optical linear array sensor images. Some scholars have made a modification on the collonearity equation, e.g. G.Konecny et al.[3] put forward an improved collinearity equation in which the effect of terrain on image point location was taken into account.

（3）Polynomial model:

The application of general polynomial model in radar images is similar to that in optical images. The imaging mechanism was not considered in this model, while the polynomial model was used in it to convert all images by the same translation type.

1.2 The deficiency of R-D and collinearity equation model in Image Positioning

Doppler frequency is directly correlated with the speed of the sensor related to the object surveyed. Thouth it is difficult

to provide Doppler frequency of every object point, they can be acquired by assuming that it is a fixed value, or acquired by linear or polynomial model in side-looking radar images. While the discrepancy of doppler frequency may rapid increased by polynomial model in front or squint side-looking radar images, it may influence the precision of image positioning.

Researchers in the field of optical photogrammetry expected that the same orientation parameters as those in optical image orientation could be used in radar image positioning, thus collonearity equation could be introduced into radar images accordingly. Because of the differences in imaging mechanisms between optical and radar sensor, the precision of radar image positioning is often low with collonearity equation. Many scholars had made modifications on the model[3], but the image equations are more complicated. By now, the major application of collonearity equation in radar images is geocoding, and complex photogrammetric processing such as in stereo positioning, especially in block adjustment is still uncommon.

2. COPLANARITY EQUATION

2.1 The principle of orientation with range-coplanarity equation

The model is established with the range-coplanarity equation by satisfying the range condition and coplanarity condition.

The range between the sensor and ground object point is equal to the calculation values by image column coordinate which is same as range condition in R-D model.

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All ground points related to one row images and sensor antenna are in same radar beam planarity which is determined by the sensor state vector and attitudes related to shoot time or the image row coordinate.

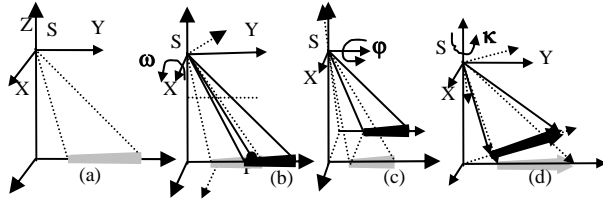## 2.2 Analysis on the influence of attitude orientation

Figure 1. The influence of attitude on image positioning.

The effect of independently changing attitude angle on image positioning was analyzed as follows.

(1) Rolling angle: The existence of rolling angles resulted in changes of the aiming range. But for the same ground point, neither the distance between antenna and ground point nor the photographing time changed. Therefore, rolling angles barely influenced coordinates of the image point (fig.1b).

(2) Pitch angle: The existence of pitch angles made the aimed object shift to pre or post direction of airline, the time moment when photos were taken on the same ground point changed, and the distance between sensor antenna and ground point increased. Therefore, the row and line of the ground point in image were also changed (Fig.1c).

(3) Yaw angle: The existence of yaw angles made the aiming range of antenna rotate, the moment when photos were taken on the same ground point changed and the distance between sensor antenna and ground point increased. Therefore, the row and line of the ground point in image were also changed (Fig.1d).

2.3 Choice of rotation angle system and coplanarity equation

Three Euler angles are not independent parameters because the latter rotation angle is formed on the basis of the former one. Whether the roll(ω) angle will affect radar imaging positioning or not is determined by the adopted rotation angle system. There are 6 rotation angle orders. The ω-φ-κ system is generally employed in the field of optical imaging positioning, however, it is not the best rotation angle system to build the equation of radar beam scanning plane. The following discussion will provide a proof of this conclusion.

(1) Coplanarity equation derived by ω-φ-κ system
The rotation matrix which can transform the sensor vector coordinate system of ω-φ-κ rotation angle system to attitude reference frame is shown as follows:

$$R_b^A = R_X(\omega)R_Y(\varphi)R_Z(\kappa) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\omega & -\sin\omega \\ 0 & \sin\omega & \cos\omega \end{bmatrix} \begin{bmatrix} \cos\varphi & 0 & \sin\varphi \\ 0 & 1 & 0 \\ -\sin\varphi & 0 & \cos\varphi \end{bmatrix} \begin{bmatrix} \cos\kappa & -\sin\kappa & 0 \\ \sin\kappa & \cos\kappa & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\varphi\cos\kappa & -\cos\varphi\sin\kappa & \sin\varphi \\ \cos\omega\sin\kappa+\sin\omega\sin\varphi\cos\kappa & \cos\omega\cos\kappa-\sin\omega\sin\varphi\sin\kappa & -\sin\omega\cos\varphi \\ \sin\omega\sin\kappa-\cos\omega\sin\varphi\cos\kappa & \sin\varphi\cos\kappa+\cos\omega\sin\varphi\sin\kappa & \cos\omega\cos\varphi \end{bmatrix} \quad (1)$$

The normal line of the scanning plane is consistent with the X axis of sensor coordinate system after the attitude is rotated. Thus its unit vector is:

$$\vec{i} = R_b^A \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \cos\varphi\cos\kappa \\ \cos\omega\sin\kappa + \sin\omega\sin\varphi\cos\kappa \\ \sin\omega\sin\kappa - \cos\omega\sin\varphi\cos\kappa \end{bmatrix}$$

And the coplanarity condition equation is:

$$\vec{i}(\bar{P} - \bar{S}) = 0$$

Namely,

$$i_X(X - Xs) + i_Y(Y - Ys) + i_Z(Z - Zs) = 0 \quad (2)$$

or:

$$(X - Xs)\cos\varphi\cos\kappa + (Y - Ys)(\cos\omega\sin\kappa + \sin\omega\sin\varphi\cos\kappa) + (Z - Zs)(\sin\omega\sin\kappa - \cos\omega\sin\varphi\cos\kappa) = 0 \quad (3)$$

It is seen that, ω-φ-κ system requires three attitude angles to constitute an expression if the scanning plane is to be obtained.

(2) Coplanarity equation derived by φ-κ-ω system:
The rotation matrix which can transform the sensor vector coordinate system of φ-κ-ω rotation angle system to attitude reference frame is shown as follows:

$$R_b^A = R_Y(\varphi)R_Z(\kappa)R_X(\omega)$$

$$= \begin{bmatrix} \cos\varphi\cos\kappa & -\cos\omega\cos\varphi\sin\kappa+\sin\omega\sin\varphi & \sin\omega\cos\varphi\sin\kappa+\cos\omega\sin\varphi \\ \sin\kappa & \cos\omega\cos\kappa & -\sin\omega\cos\kappa \\ -\sin\varphi\cos\kappa & \cos\omega\sin\varphi\sin\kappa+\sin\omega\cos\varphi & -\sin\omega\sin\varphi\sin\kappa+\cos\omega\cos\varphi \end{bmatrix} \quad (4)$$

The unit vector of normal direction of radar scanning beam plane is:

$$\vec{i} = R_b^A \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \cos\varphi\cos\kappa \\ \sin\kappa \\ -\sin\varphi\cos\kappa \end{bmatrix} \quad (5)$$

And the coplanarity condition equation can be easily obtained:

$$(X - Xs)(\cos\varphi\cos\kappa) + (Y - Ys)\sin\kappa - (Z - Zs)(\sin\varphi\cos\kappa) = 0 \quad (6)$$

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(3) Coplanarity equation derived by κ-φ-ω system

In the same way, the expansion of the coplanarity condition equation is derived as:

$$(X - Xs)(\cos\kappa\cos\varphi) + (Y - Ys)(\sin\kappa\cos\varphi) + (Z - Zs)(-\sin\varphi) = 0 \quad (7)$$

It can be concluded that the coplanarity condition is independent of ω angle when the κ-φ-ω or φ-κ-ω rotation angle system is used.

3. RANGE-COPLANARITY EQUATIONS IN DIFFERENT COORDINATE SYSTEMS

3.1 The imaging equation in tangent orthogonal coordinate system

It is suitable for airborne or small area spaceborne radar images to orient in tangent orthogonal coordinate system. Thereinafter, imaging equations and image processing model will be derived based on the scanning plane confirmed by φ-κ-ω system.

The distance equation based on range-coplanarity equation is the same as the one based on range-Doppler model. For slant range images, the equation is as follows:

$$(X - Xs)^2 + (Y - Ys)^2 + (Z - Zs)^2 = (y_s M_y + D_s)^2 \quad (8)$$

Thus, the range-coplanarity equations are obtained:

$$\begin{cases} (X - Xs)(\cos\varphi\cos\kappa) + (Y - Ys)\sin\kappa - (Z - Zs)(\sin\varphi\cos\kappa) = 0 \quad (9) \\ (X - Xs)^2 + (Y - Ys)^2 + (Z - Zs)^2 = (y_s M_y + R_0)^2 \end{cases}$$

3.2 Imaging Equation in Geocentric Orthogonal Coordinate System (GOCS)

The origin O of orbit coordinate system is the exposure station in the sensor orbit. The mathematic definitions of three axes in the coordinate system are as follows:

$$\begin{cases} \vec{Z}_o = [(Z_o)_X, (Z_o)_Y, (Z_o)_Z] = \vec{P}(t)/\|\vec{P}(t)\| \\ \vec{Y}_o = [(Y_o)_X, (Y_o)_Y, (Y_o)_Z] = (\vec{Z}_o \times \vec{V})/\|(\vec{Z}_o \times \vec{V})\| \\ \vec{X}_o = [(X_o)_X, (X_o)_Y, (X_o)_Z] = \vec{Y}_o \times \vec{Z}_o \end{cases} \quad (10)$$

The normal vector of the scanning plane is:

$$\vec{i} = R_o{}^E R_b{}^o \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} i_X \\ i_Y \\ i_Z \end{bmatrix}$$

And the coplanarity condition equation is:

$$i_X(X - Xs) + i_Y(Y - Ys) + i_Z(Z - Zs) = 0$$

namely：

$$\left(\begin{bmatrix} (X_O)_X & (Y_O)_X & (Z_O)_X \\ (X_O)_Y & (Y_O)_Y & (Z_O)_Y \\ (X_O)_Z & (Y_O)_Z & (Z_O)_Z \end{bmatrix} \begin{bmatrix} \cos\varphi\cos\kappa \\ \sin\kappa \\ -\sin\varphi\cos\kappa \end{bmatrix} \right) \bullet \begin{bmatrix} X - Xs \\ Y - Ys \\ Z - Zs \end{bmatrix} = 0 \quad (11)$$

where

$$\begin{cases} i_X = \cos\varphi\cos\kappa(Xo)_X + \sin\kappa(Yo)_X - \sin\varphi\cos\kappa(Zo)_X \\ i_Y = \cos\varphi\cos\kappa(Xo)_Y + \sin\kappa(Yo)_Y - \sin\varphi\cos\kappa(Zo)_Y \\ i_Z = \cos\varphi\cos\kappa(Xo)_Z + \sin\kappa(Yo)_Z - \sin\varphi\cos\kappa(Zo)_Z \end{cases} \quad (12)$$

Thus, the range- coplanarity equation in GOCS is:

$$\begin{cases} i_X(X - Xs) + i_Y(Y - Ys) + i_Z(Z - Zs) = 0 \\ (X - Xs)^2 + (Y - Ys)^2 + (Z - Zs)^2 - (y_s M_y + R_0)^2 = 0 \end{cases} \quad (13)$$

3.3 The Range-coplanarity Equation with Coordinates of Image Point as Explicit Function

In order to express the above equation as a function of coordinates of the image point, the left-side of the coplanarity equation of the formula (9) is multiplied by a function F, with the value of coplanarity equation still being 0. Let F be:

$$F = \frac{-f}{\sin\varphi(X - Xs) + (Z - Zs)\cos\varphi} \quad (14)$$

Where f is called equivalent focal length whose value does not affect the tenability of the equation, but affects the number of iterations while the equation is solved as well as the fixed weight of the observed values. f can be obtained by the following formula according to the geometric relation among pixel size, ground resolution and focal length:

$$f = \frac{Hs \cdot \mu_0}{GSDx} \quad (15)$$

where μ_0 is the value of equivalent pixel, Hs is the height of the sensor, and $GSDx$ is ground resolution of corresponding azimuth of the pixel.

Meanwhile, the range equation of the formula (9) is also transformed into a function of y, that is, the range-coplanarity equation of SAR image positioning with coordinates of image point (x,y) as the explicit function can be obtained:

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$$\begin{cases} x = -f \dfrac{\cos\varphi\cos\kappa(X-Xs)+\sin\kappa(Y-Ys)-\sin\varphi\cos\kappa(Z-Zs)}{\sin\varphi(X-Xs)+(Z-Zs)\cos\varphi} = 0 \\ y = [\sqrt{(X-Xs)^2+(Y-Ys)^2+(Z-Zs)^2}-R_0]\mu_0/M_y \end{cases}$$
(16)

Similarly, the range-coplanarity equation of explicit function of coordinates of image point in the geocentric coordinate system can also be obtained.

It is concluded from the above equation that the mature method based on the collinearity equation model of optical images can be easily applied in the processing of photogrammetric data of SAR images.

4. THE CORRECTION AND GEOCODING OF RADAR IMAGERY

4.1 The preprocessing of attitude angles

The φ-κ-ω system is adopted in the R-Cp model this paper, while generally ω-φ-κ system is adopted in the original attitude measurements. Before three values of attitude angles in ω-φ-κ system are used, they should be converted into the values in φ-κ-ω system.

Since different attitude systems have the same translation matrix, that is:

$$R(\omega,\varphi,\kappa)=R(\varphi,\kappa,\omega)$$
(17)

Set $R(\omega,\varphi,\kappa)=[a_{ij}]$, ($i,j=0,1,2$), and a_{ij} is matrix element which is obtained from formula(1).

Thus, φ,κ,ω value of φ-κ-ω system can be calculated by a_{ij}.

$$\begin{cases} \varphi = -\arctan(a_{30}/a_{00}) \\ \kappa = -\arcsin(a_{10}) \\ \omega = \arctan(a_{12}/a_{11}) \end{cases}$$
(18)

4.2 Calculation of coordinates of ground point using image point

The coordinates of ground point can be obtained using the corresponding coordinates of the image point in combination with distance condition, coplanarity condition and earth ellipsoid equation. The coordinate（X Y Z）can be acquired by the following three equations:

$$\begin{cases} F_1 = \dfrac{(X-Xs)^2+(Y-Ys)^2+(Z-Zs)^2}{Rj^2}-1 = 0 \\ F_2 = i_X(X-Xs)+i_Y(Y-Ys)+i_Z(Z-Zs) = 0 \\ F_3 = \dfrac{X^2+Y^2}{(a+H)^2}+\dfrac{Z^2}{(b+H)^2}-1 = 0 \end{cases}$$
(19)

（Where $Rj = j\bullet M_y + R_0$, Rj is the slant range of the object point, j is the column coordinate, H is geodetic altitude）

According to the least squares method, the coordinate (X, Y, Z) of the ground point can be solved iteratively.

4.3 Refinement of Orientation parameters [4]

The range-coplanarity equation of formula (16) is linearized, therefore the error equation can be obtained:

$$\begin{cases} v_x = f_{1Xs}d_{Xs}+f_{1Ys}d_{Ys}+f_{1Zs}d_{Zs}+f_{1\kappa}d_\kappa+f_{1\varphi}d_\varphi-l_1 \\ v_y = f_{2Xs}d_{Xs}+f_{2Ys}d_{Ys}+f_{2Zs}d_{Zs}+f_{2\kappa}d_\kappa+f_{2\varphi}d_\varphi-l_2 \end{cases}$$
(20)

Where $f_{..}$ represents linear coefficient and $l_.$ represents constant.

Correction model of observation data of exterior orientation elements expressed by low order polynomial of time parameter t :

$$\begin{cases} dp_i = b_{i0}+b_{i1}t+b_{i2}t^2 \quad (dp_i = dXs,dYs,dZs) \\ da_i = c_{i0}+c_{i1}t+c_{i2}t^2 \quad (da_i = d\varphi,dk) \end{cases}$$
(21)

In order to overcome the singularity of normal equation and satisfy adjustment with sparse control points, combined adjustment error equations can be formed using EO observing data along with virtual observing values of R-Cp:

$$\begin{cases} V_y = B_{Rb}b \qquad\qquad -L_R.......P_y \\ V_x = B_{Cb}b+ \quad B_{Cc}c-L_C........P_x \\ V_b = E_b b \qquad\qquad -L_b........P_b \\ V_c = \qquad\qquad E_c c-L_c.........P_c \end{cases}$$
(22)

Where V_y, V_x, V_b, V_c are corrections of range equation, coplanarity equation, polynomial model parameters of EO lines and attitudes, respectively; b,c are polynomial coefficients for exterior orientation model.

The above formula can be incorporated as:

$$V=BX-L \quad ...... \quad P$$
(23)

The solutions of orientations are：

$$X=(B^TPB)^T(B^TPL)$$
(24)

5. EXPERIMENT

The experimental material was an ALOS/PALSAR image (70Km×60Km, with pixel resolution of 3.189m in velocity direction and 9.368m in range direction), covering the mountainous areas in Shanxi province in the west of China.

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There were 21 image points with ground points coordinates surveyed from 1:10,000 scale relief map.

5.1 Experiment 1: Direct Geocoding to the Earth Ellipsoid

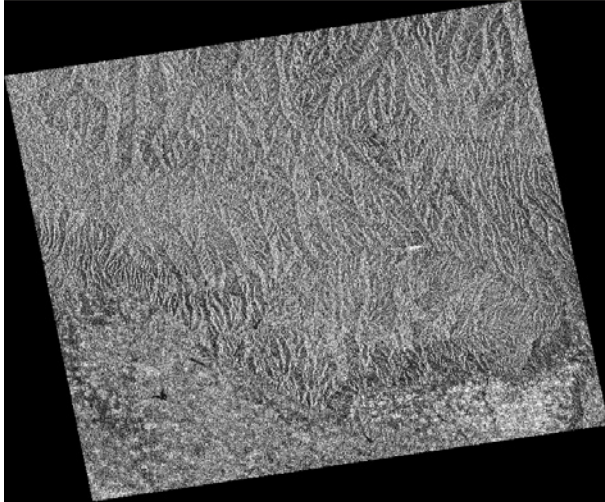The ephemeris and attitude data acquired from supplementary file of the image were used, and Gauss projection was also adopted.

Figure2.Direct geocoding to the earth ellipsoid

Since the misaligned angle of sensor axis relative to satellite body axis was not available to the public, the three attitude angles were all set to 0; meanwhile the DEM data were used due to significant change of the terrain. The precision of direct geocoding was acquired by comparing coordinates of 21 homonymy points from geocoding image and the relief map points, and the results were listed in the "direct geocoding" column of table 1.

The results also indicated that all the check points had obvious systemic errors both in azimuth and range directions.

5.2 Experiment 2: Orthophoto correction with sparse GCPs and DEM data.

In this experiment, two surveying points were selected as control points (distributed on the top right and bottom left corners of the image) and 19 as check points so as to refine the image orientation elements, and the images were corrected with refined orientation elements and DEM data.

Through comparing calculation results and the known data of surveying object, errors of ground control points (GCPs) and check points (CPs) were listed in the "Orthophoto correction" column of table 1.

Exp content	Direct Geocoding	Orthophoto Correction	
Unit: (pixel)	21CPs	2GCPs	19CPs
RMS_x(azimuth)	3.53	0.44	1.26
RMS_y(range)	4.71	0.40	1.27
RMS_xy	**5.88**	**0.59**	**1.79**
MAX_error(x)	7.07	0.11	3.72
MAX_error(y)	5.77	-0.12	2.97

Table 1   The statistics of precision of check points

6. CONCLUSIONS

The collinearity equation model of radar images has such shortcomings as poor rigorousness and practicality; Range-Doppler model does not apply to the existing processing algorithms of photogrammetric data. With elements of exterior orientation as the orientation parameters, in this study a new imaging equation of radar image was constructed ---Range-Coplanarity equation. The imaging equation reflected the imaging mechanism of the radar images in the direction of range and the direction of orientation, and manifested the attribute of the image point coordinate as independent observed value of photogrammetry so that the positioning of radar image are able to easily adopt the mature algorithms of photogrammetric data processing of optical image.It also has simple form and requires less orientation parameters than R-D model and collinearity equation.

In the years past, techniques of sensor positioning and attitude determination technique had been greatly developed. With the utilization of them, the image mapping without GCPs will become a trend in future. Therefore, this image equation will bring convenience for radar image rectification, stereo positioning, block adjustment, and InSAR baseline calculation in photogrammetry. At present, related studies are still being kept on by us.

7. ACKNOWLEDGEMENT

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