LINEAR RECOVERY OF THE EXTERIOR ORIENTATION PARAMETERS IN A PLANAR OBJECT SPACE

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ABSTRACT:

The Direct Linear Transformation (DLT) model requires 3-D object space control points to estimate the full set of its parameters that can be used to recover exterior orientation parameters. The use of DLT in a planar object space leads to a rank deficient model. This rank deficient model leaves the DLT defined up to a 2-D projective transformation, which makes the explicit solution of the exterior orientation parameters or a single photo-resection represented by the camera position and orientations, a non-trivial task. This paper presents a new closed form solution to a single photo-resection in a planar object space based on homogenous coordinate representation and matrix factorization. Homogenous coordinate representation offers a direct matrix correspondence between the parameters of the 2-D projective transformation and the collinearity model. This correspondence lends itself to a direct matrix factorization to solve the photo-resection problem. The matrix factorization starts by recovering the elements of the rotation matrix and then solving for the camera position. It will be shown that an incomplete representation of the rotation matrix is captured by the 2-D projective parameters but the actual physical parameters of the rotation matrix still can be recovered explicitly and without any ambiguity. These elements were used to build a complete rotation matrix, which in turn facilitates the correct solution of the camera position. The developed solution can serve as a complementary companion to the classical DLT model. In this paper, a detailed derivation of the proposed solution will be presented, accommodated with two simulated examples to demonstrate its validity. The first example simulates aerial photography and the second one simulates close range photogrammetric application.

1. INTRODUCTION

The DLT model was introduced to the photogrammetric community by (Abdel-Aziz and Karara, 1971). It has gained a wide popularity in close-range photogrammetry, computer vision, robotics, and biomechanics. The wide popularity of the DLT is due to the linear formulation of the relationship between image and object coordinates. Namely the following characteristics are associated with the DLT model: image coordinates can be expressed in a non-orthogonal system with unequal scales, the position of the coordinate system is arbitrary, and the principal distance can be unknown and vary from image to image, see (Kraus, 1997).

The DLT model requires 3-D object space control points to estimate the full set of its parameters. The use of DLT in a planar object space leads to a rank deficient model. This rank deficient model leaves the DLT defined up to a 2-D projective transformation, which makes the explicit recovery of the photo resection parameters, represented by the camera attitudes and position, a non-trivial task.

Several approaches are used to relate the parameters of the 2-D projective transformation to the physical parameters of the collinearity model using a term-wise correspondence formulation, which essentially leads to a set of sequential equations to recover the exterior orientation parameters, see (Kobayashi and Mori, 1997; Melen, 1993). We argue that these methods are not practical. For example, in (Melen, 1993) many special cases have to be considered, otherwise ambiguous solutions will show up.

This paper presents a new linear approach to recover the exterior orientation parameters in a planar object space. This approach is based on homogenous coordinate representation and matrix factorization. Homogenous coordinate representation offers a direct matrix correspondence between the parameters of the 2-D projective transformation and the collinearity model. This correspondence lends itself to a direct matrix factorization to solve for the exterior orientation parameters. Algorithmically, the developed approach is a cooperative strategy between the 2-D projective transformation and a linear version of the collinearity model. The 2-D projective transformation supplies the orientation parameters, which serve as input in the linear version of the collinearity model to determine the camera position.

The proposed approach assumes that the interior orientation parameters of the camera are known, which is a safe assumption in many practical applications. The presented approach can accommodate horizontal and vertical planes, but with minimal effort this approach can be extended to tilted planes.

This paper is organized as follows. Section 2 presents the proposed approach. Section 3 presents the experimental results and some discussions. Finally, section 4 concludes the paper.

2. THE PROPOSED APPROACH

This section starts with a quick review for the projective and collinearity models and then proceeds to develop the proposed approach.

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The following mathematical model can capture the relationship between a planar object space and image space using homogeneous coordinates:

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} L_1 & L_2 & L_3 \\ L_4 & L_5 & L_6 \\ L_7 & L_8 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix} \qquad (1)$$

where
x, y: are the image coordinates.
X, Y: are the object space coordinates.
$L_1...L_8$: are the projective transformation parameters.

A linear version of the system of equations depicted in (1) can be represented by the following equations:

$$x = XL_1 + YL_2 + L_3 - (x - e_x)XL_7 - (x - e_x)YL_8 + e_x \qquad (2)$$

$$y = XL_4 + YL_5 + L_6 - (y - e_y)XL_7 - (y - e_y)YL_8 + e_y \qquad (3)$$

where e_x and e_y are the true unknown errors associated with the image coordinates measurements.

We should denote that, the representation of the 2-D projective transformation presented in equations (2) and (3) induces a stochastic non-linearity, but the model retains its linearity since very good approximations can be obtained by

The rotation matrix can be captured by the following representation:

$$r_{i,j} = \begin{bmatrix} c_2 c_3 & c_1 s_3 + s_1 s_2 c_3 & s_1 s_3 - c_1 s_2 c_3 \\ -c_2 s_3 & c_1 c_3 - s_1 s_2 s_3 & s_1 c_3 + c_1 s_2 s_3 \\ s_2 & -s_1 c_2 & c_1 c_2 \end{bmatrix} \qquad (6)$$

where s_1: $\sin \omega$, s_2: $\sin \varphi$, s_3: $\sin \kappa$
c_1: $\cos \omega$, c_2: $\cos \varphi$, c_3: $\cos \kappa$
ω: is the primary rotation around the x-axis.
φ: is the secondary rotation around the y-axis.
κ: is the tertiary rotation around the z-axis.

In a homogenous coordinate representation equation (4) and (5) can be captured by the following system of equations:

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \lambda \underbrace{\begin{bmatrix} 1 & 0 & xp \\ 0 & 1 & yp \\ 0 & 0 & \dfrac{-1}{f} \end{bmatrix}}_{K} \begin{bmatrix} r_{11} & r_{21} & -(r_{11}X_o + r_{21}Y_o + r_{31}Z_o) \\ r_{12} & r_{22} & -(r_{12}X_o + r_{22}Y_o + r_{32}Z_o) \\ r_{13} & r_{23} & -(r_{13}X_o + r_{23}Y_o + r_{33}Z_o) \end{bmatrix} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix} \qquad (7)$$

where
λ: is the scale factor.
K: is the calibration matrix.

From equation (1) and (7) we can write the following equivalency:

$$\begin{bmatrix} L_1 & L_2 & L_3 \\ L_4 & L_5 & L_6 \\ L_7 & L_8 & 1 \end{bmatrix} = \lambda K \begin{bmatrix} r_{11} & r_{21} & -(r_{11}X_o + r_{21}Y_o + r_{31}Z_o) \\ r_{12} & r_{22} & -(r_{12}X_o + r_{22}Y_o + r_{32}Z_o) \\ r_{13} & r_{23} & -(r_{13}X_o + r_{23}Y_o + r_{33}Z_o) \end{bmatrix} \qquad (8)$$

From equations (8) we can deduce the following:

neglecting the stochastic component at the beginning and then proceed iteratively.
Now, we shift our focus to the collinearity model. First, we show the representation of the collinearity model for the horizontal and vertical planes. Then, we write a homogenous coordinate version of the collinearity model, which is a corner stone in our development.

A horizontal plane along the XY plane of the object space coordinate system using the collinearity model can be stated as follows:

$$x = x_p - f \frac{r_{11}(X - X_o) + r_{21}(Y - Y_o) + r_{31}(-Z_o)}{r_{13}(X - X_o) + r_{23}(Y - Y_o) + r_{33}(-Z_o)} + e_x \qquad (4)$$

$$y = y_p - f \frac{r_{12}(X - X_o) + r_{22}(Y - Y_o) + r_{32}(-Z_o)}{r_{13}(X - X_o) + r_{23}(Y - Y_o) + r_{33}(-Z_o)} + e_y \qquad (5)$$

where x_p, y_p: are the coordinates of the principal point.
r_{ij}: elements of the rotation matrix.
f: is the focal length of the camera.
X_o, Y_o, Z_o: is the camera position.
e_x and e_y: are the true unknown errors associated with the image coordinate measurements.

We should denote that, in the horizontal version of the collinearity model, the Z value is either equal to zero or a constant and similarly the vertical version is nothing but Y is equal to zero or a constant.

$$\begin{bmatrix} r_{11} & r_{21} \\ r_{12} & r_{22} \\ r_{13} & r_{23} \end{bmatrix} = \frac{1}{\lambda} K^{-1} \begin{bmatrix} L_1 & L_2 \\ L_4 & L_5 \\ L_7 & L_8 \end{bmatrix} \qquad (9)$$

Analyzing the information content of the right hand side matrix depicted in (9) one can easily see that the effect of the calibration matrix is cancelled out, but still there is scale factor that needs to be eliminated. A simple normalization process can provide a complete remedy to eliminate the scale factor

Then, the final representation of system of equations depicted in (9) can be written as:

$$\begin{bmatrix} r_{11} & r_{21} \\ r_{12} & r_{22} \\ r_{13} & r_{23} \end{bmatrix} = \begin{bmatrix} \dfrac{C_{11}}{\sqrt{C_{11}^2 + C_{21}^2 + C_{31}^2}} & \dfrac{C_{12}}{\sqrt{C_{12}^2 + C_{22}^2 + C_{32}^2}} \\ \dfrac{C_{21}}{\sqrt{C_{11}^2 + C_{21}^2 + C_{31}^2}} & \dfrac{C_{22}}{\sqrt{C_{12}^2 + C_{22}^2 + C_{32}^2}} \\ \dfrac{C_{31}}{\sqrt{C_{11}^2 + C_{21}^2 + C_{31}^2}} & \dfrac{C_{32}}{\sqrt{C_{12}^2 + C_{22}^2 + C_{32}^2}} \end{bmatrix} \qquad (10)$$

where

$$C = K^{-1} \begin{bmatrix} L_1 & L_2 \\ L_4 & L_5 \\ L_7 & L_8 \end{bmatrix} \qquad (11)$$

The rationale behind the normalization process can be explained as follows:
  a- After multiplying by the inverse of the calibration matrix, the information content of the projective

transformation contained in (9) are the elements of the rotation matrix coupled with a scale factor.

b- Taking the norm of the first or the second column of the matrix on the right hand side of (9) can be stated mathematically as follows:

$$\left\| \frac{1}{\lambda^2} (r_{11}^2 + r_{12}^2 + r_{13}^2) \right\|,$$

but $(r_{11}^2 + r_{12}^2 + r_{13}^2) = 1$, due to the normality constraints of the rotation matrix. Then:

$$\left\| \frac{1}{\lambda^2} (r_{11}^2 + r_{12}^2 + r_{13}^2) \right\| = \frac{1}{\lambda}.$$

From the above analysis, the normalization process is nothing more than the computation of the scale factor.

By relating the elements of the matrix in (10) to the elements of the rotation matrix presented in (6), the camera attitudes can be retrieved explicitly. For instance, the element of the rotation matrix for the horizontal plane can be derived as follows:

$$\varphi = \sin^{-1}\left(\frac{C_{31}}{\sqrt{C_{11}^2 + C_{21}^2 + C_{31}^2}} \right) \qquad (12)$$

$$\omega = \sin^{-1}\left(\frac{-C_{32} \sec \varphi}{\sqrt{C_{12}^2 + C_{22}^2 + C_{32}^2}} \right) \qquad (13)$$

$$\kappa = \sin^{-1}\left(\frac{-C_{21} \sec \varphi}{\sqrt{C_{11}^2 + C_{21}^2 + C_{31}^2}} \right) \qquad (14)$$

Similar relationships can be derived for the vertical plane.

From equations (4) and (5) a linear version of the collinearity model was derived as listed below:

$$(xr_{13} + fr_{11})X_o + (xr_{23} + fr_{21})Y_o + (xr_{33} + fr_{31})Z_o = (xr_{13} + fr_{11})X + (xr_{23} + fr_{21})Y + (xr_{33} + fr_{31})Z \qquad (15)$$

$$(yr_{13} + fr_{12})X_o + (yr_{23} + fr_{22})Y_o + (yr_{33} + fr_{32})Z_o = (yr_{13} + fr_{12})X + (yr_{23} + fr_{22})Y + (yr_{33} + fr_{32})Z \qquad (16)$$

In the linear version of the collinearity, the only unknowns are the camera position since the rotation matrix will be recovered from the 2-projective transformation.

The basic steps of the proposed approach can be stated as follows:
1. Compute the parameters of the projective transformation using equations (2) and (3).
2. Compute the normalized C matrix using equation (10).
3. Compute the elements of the rotation matrix using equations (12), (13) and (14).
4. Form the complete rotation matrix using equation (6).
5. Compute the camera translations using equations (15) and (16).

3. EXPERIMENTAL RESULTS AND DISCUSSIONS

The developed approach was tested on two examples that simulate two typical photogrammetric applications. The first example simulates normal aerial photography and the second one simulates close range application. Tables 1, 2, and 3 show the relevant information to the aerial example. Tables 7, 8, and 9 show the relevant information of the close range example. The corresponding image coordinates are obtained by a simple back-projection using equations (4) and (5). The image coordinates are treated as error free data.

X m	Y m	Z m	X m	Y m	Z m
-200	-200	100	2200	-200	100
-200	2200	100	2200	1000	100
2200	2200	100	200	1000	100

Table 1: The ground control points used to simulate the aerial example.

X_O	Y_O	Z_O	ω	φ	κ
1000 m	1000 m	2000 m	7 deg	4.5 deg	11 deg

Table 2: The true exterior orientation parameters used in the aerial example.

f	x_p	y_p
150 mm	0. mm	0. mm

Table 3: The interior orientation parameters used in the aerial example.

L1=79.9080   L2=16.2309 L3=-87.8867
L4=-15.5325 L5=79.4086   L6= -84.7415
L7=-0.0427   L8=0.0661

Table 4: The computed parameters of the projective transformation for the aerial example

The computed rotation matrix
0.9786   0.1988
-0.1902   0.9725
0.0785   -0.1215
The true rotation matrix
0.9786   0.1988   -0.0532
-0.1902   0.9725   0.1345
0.0785   -0.1215   0.9895

Table 5: The computed and true parameters of the rotation matrix.

Xo	Yo	Zo

1000	1000	2000
ω	φ	κ

7.0000	4.5000	11.0000

Table 6: The full solution of the exterior orientation parameters of the aerial example.

X m	Y m	Z m	X m	Y m	Z m
0	0	0	2	0	1
0	0	1	2	0	2
0	0	2	2	0	3
0	0	3	3	0	0
2	0	0	3	0	1

Table 7: The ground control points used to simulate the close range example.

f	x_p	y_p
6.8 mm	0.0 mm	0.0 mm

Table 8: The interior orientation parameters used in the close range example

X_o	Y_o	Z_o
4 m	-15 m	1.52 m
ω	φ	κ
82 deg	-40 deg	2.5 deg

Table 9: The true exterior orientation parameters used in the close range example.

L1=0.5820 L2= 0.1017 L3=-9.7440
L4=-0.0254 L5=0.7526 L6=0.8707
L7=0.0726 L8= -.0119

Table 10: The computed parameters of the projective transformation for the close range example.

The computed rotation matrix		
0.7619	0.1331	
-0.0333	0.9854	
-0.6468	0.1061	
The true rotation matrix		
0.7619	-0.6338	0.1331
-0.0333	0.1670	0.9854
-0.6468	-0.7552	0.1061

Table 11: The computed and true parameters of the rotation matrix

Xo	Yo	Zo
4.0000	-15.0000	1.5200
ω	φ	κ
82.0000	-40.3000	2.5000

Table 12: The full solution of the exterior orientation parameters of the close range example.

In order to compute the exterior orientation parameters, in the aerial and close range example, the process started by the recovery of the 2-D projective transformation parameters, using equations (2) and (3), then followed by the computation of the normalized C matrix, using equation (10). The explicit retrieval of rotation parameters was done by using equations (12), (13), and (14). The rotation parameters were used to form the complete rotation matrix. In the aerial example, the first two columns of the rotation matrix were recovered, but in the close range example, the first and the third columns were recovered. The complete rotation matrix was used in a system of linear equations composed from a linear version of the collinearity model, as depicted in equations (15) and (16), to recover the camera position. The developed algorithm was successfully tested in the above two examples and the true camera parameters were recovered as presented in tables 6 and 12.

4. CONCLUSIONS

In this paper, we proposed a new linear approach to a single photo-resection in planar object space.
The proposed approach can be considered as a restricted DLT in 2-D. It is evident in this approach that the normalization process is the most critical step for the final solution. The proposed approach can be considered as a very practical and important tool for the provision exterior orientation parameters for camera calibration in a 2-D test filed and 3-D object reconstruction. Algorithmically, the proposed approach is a cooperative strategy between the 2-D projective transformation and a linear version of the collinearity model. The 2-D projective transformation supplies the rotation matrix and the collinearity supplies the camera position. The proposed approach assumes that the interior orientation parameters of the camera are known, which is a safe assumptions in many practical applications. The presented approach can accommodate horizontal and vertical planes, but with a minimal effort it can be extended to tilted planes.

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