NEAR SET INDEX IN AN OBJECTIVE IMAGE SEGMENTATION EVALUATION FRAMEWORK

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ABSTRACT:

The problem considered in this article[*] is how to evaluate image segmentations objectively. An image segmentation is the partition of an image into its constituent parts. Until recently, the evaluation of image segmentations has been largely subjective. Much work has been done in developing metrics for evaluating segmentations with respect to ground truth images. However, little has been done in terms of evaluation without an "expert." An objective technique for evaluating image segmentations with respect to ground truth images called the Normalized Probabilistic Rand (NPR) index has been proposed (Unnikrishnan et al., 2007). This method evaluates a segmented image by way of soft non-uniform weighting of pixel pairs between the segmented images and one or more ground truth images. The NPR index works well in comparing and benchmarking image segmentation algorithms. However, it is not reasonable to assume that online learning image classification systems will always have access to ground truth images. In this article, we propose an objective metric based on near sets (Henry and Peters, 2007, Peters, 2007b) and information content (MacKay, 2003) of a segmentation called the Near Set Index (NSI) for evaluating image segmentations that does not depend on comparison with ground truth images. Information content provides a measure of the variability of pixel intensity levels within an image and takes on values in the interval $[0, \log_2 L]$ where L is the number of grey levels in the image. Near set theory provides a framework for representation of groups of similar pixel windows. Within this framework pixel windows can be approximated and degrees of similarity or nearness can be measured. This is advantageous both because it closely parallels our current views on human perception of complex scenes (Fahle and Poggio, 2002) and is tightly coupled to the preprocessing (*i.e.*, feature extraction) stage of object recognition systems (Duda et al., 2001). The availability of an objective segmentation evaluation measure facilitates object recognition, computer vision, or machine learning used to solve image classification problems. The contribution of this article is the introduction of a Near Set Index (NSI) that provides a basis for automation of an objective segmentation evaluation method that is not dependent on ground truth images. Further, a comparison of the results using the NSI and NPR indices is given.

1 INTRODUCTION

Applications of image segmentation techniques can be seen in areas ranging from photo radar to airport security systems. Many of these systems incorporate learning techniques in order to adapt to their dynamically changing environment. Such systems would benefit from a form of feedback (*i.e.* a signal or reward) representing the quality of segmentation to facilitate on-line learning. Traditionally objective segmentation evaluation has not been a priority (Unnikrishnan et al., 2007), usually a new segmentation technique is presented with a few images and it is left up to the reader to compare the new technique.

Unnikrishnan *et al.* presents an excellent objective technique for evaluating image segmentations with respect to ground truth images called the Normalized Probabilistic Rand (NPR) index (see, also, (Neubert et al., 2006), for other seqmentation evaluation methods). The NPR method evaluates a segmented image by way of soft non-uniform weighting of pixel pairs between the segmented images and one or more ground truth images. The NPR is excellent for comparing/benchmarking image segmentation algorithms. However, it is not reasonable to assume that on-line learning systems will have access to ground truth images. In this article, we propose an objective metric based on near sets and information content called the Near Set Index (NSI) for evaluating an image segmentation without comparison to ground truth images.

Object recognition problems, especially in images (Henry and Peters, 2007), and the problem of the determining the nearness of objects have motivated the introduction of near sets (see, *e.g.*, (Peters, 2007c, Peters, 2007b)). Near sets are a generalization of rough sets introduced by Zdzisław Pawlak (Pawlak, 1981), and later refined in (Pawlak and Skowron, 2007, Peters, 2007a). Near set theory provides a framework for representation of objects characterized by the features that describe them (Orłowska, 1982). Within this framework objects can be approximated and degrees of similarity or "nearness" can be measured. This framework is advantageous both because it is an intuitive method of representing objects (similar to human perception (Fahle and Poggio, 2002)). This framework is also advantageous because characterizing segmentations using features is tightly coupled to the preprocessing (*i.e.* feature extraction) stage of object recognition systems (Duda et al., 2001).

When representing objects by features that describe them, it is useful to have a measure of the variability or uncertainty of those features. In the case of images, features are obtained from functions of pixel values. Information content[1] of an object is maximized when when the probabilities of events are uniform (MacKay, 2003). Thus objects with similar features (*i.e.* small variability among feature values) have a lower information content.

The approach taken in this article is to evaluate segmentations of

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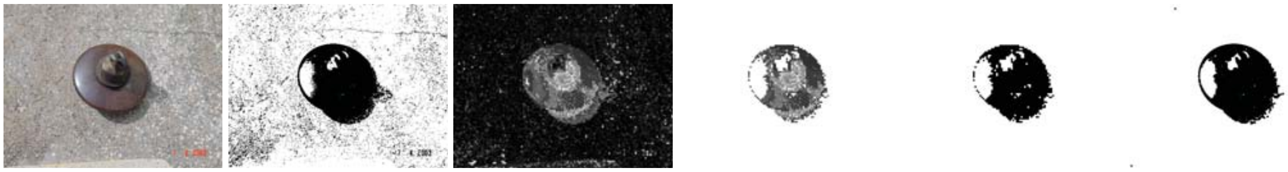
[1]Also known as Shannon entropy. In this article, entropy is used to measure information content of the lower approximation of an image segment rather than information content of sequences of pixel grey levels (Pal and Pal, 1989).



(a) (b) (c) (d) (e) (f)

Figure 1: Example of near sets for segmentation improvement: (a) Original image, (b) segmentation of (a) using Kittler's thresholding technique, (c) equivalence class image created from (a), (d) lower approximation of (b), (e) mask created from (d), and (f) mask created using morphological opening on (b).

images from the Berkeley Segmentation Data Set (Martin et al., 2001) using both the NSI and the NPR index. Segmentations are created using the mean shift segmentation algorithm(Comaniciu, 2002, Christoudias et al., 2002) (Carreira-Perpiñán, 2007), a kernel density estimation algorithm which has become quite popular in the image segmentation community. The basic idea is that for each pixel, the mean shift algorithm iteratively searches for a peak in the local density. Then the pixel is assigned to the region for which all pixels have the same peak (Wang et al., 2004).

The contribution of this article is an objective image segmentation evaluation method that is not dependent on ground truth images. This article is organized as follows. Section 1 provides a brief introduction to near sets. Section 2 combines near set theory and image processing, and Section 3 presents the new method of segmentation evaluation.

Table 1: Notation

Symbol	Interpretation
$\mathcal{O}$	set of objects, *e.g.*, a set of pixels,
$\mathcal{F}$	set of probe functions,
B	$B \subseteq \mathcal{F}$,
ϕ_i	Probe function $\phi_i \in B$,
$\sim_B$	$\{(x, x') \in \mathcal{O} \times \mathcal{O} \mid \forall \phi_i \in B, \Delta\phi_i = 0\}$,
$[x]_B$	$= \{x' \in \mathcal{O} \mid \ x \sim_{B_r} x'\}$, equivalence class,
$\mathcal{O}/\sim_B$	$= \left\{[x]_B \mid \ x \in \mathcal{O}\right\}$, quotient set,

This section describes some background from near sets theory (Peters, 2007c, Peters, 2007b) that provides the foundation for the NSI reported in this paper. Near sets are a generalization of rough sets introduced by Zdzisław Pawlak (Pawlak, 1981) and are used for the approximation and comparison of perceptual objects that are qualitatively near each other[2]. Objects fall into two categories, namely objects that are presented to the senses and those that are knowable to the mind (Murray et al., 1933). We perceive objects by their inherent characteristics and use these characteristics to describe the object to ourselves and others. Consequently, a description of an object is a tuple of characteristic object features, *i.e.* the description of an object x is a tuple $\phi(x)$. Similarly, two objects are considered near each other if they have matching descriptions. For example, two sets X and X' are near to each other when $\phi(x) = \phi(x')$ for elements $x \in X$ and $x' \in X'$, where the predicate "is near" is related to the descriptions of the objects x and x' and not their set membership. Notice that we are moving to an understanding of perception, either by humans or imitated by thinking machines, as a consideration of the appearances of objects characterized by functions representing object features.

Formally, let us define an object x, a set of sample objects $x \in X$, and the universe of objects $\mathcal{O}$ where $X \subseteq \mathcal{O}$. Let $\phi_i : \mathcal{O} \longrightarrow \Re$ such that ϕ_i describes an object (*e.g.*, a sensor reading), and let $\phi_i \in B$ be the choice of functions used to describe an object in $\mathcal{O}$ (known *a priori*). The set B consisting of functions representing object features provides a basis for describing an object. Thus, an object description $\phi : \mathcal{O} \longrightarrow \Re^L$ is a vector such that

$$\phi(x) = (\phi_1(x), \phi_2(x), \ldots, \phi_i(x), \ldots, \phi_L(x)),$$

where $|\phi| = |B| = L$.

An object's description can then be used to define an indiscernibility relation, $\sim_B$, introduced by Zdzisław Pawlak (Pawlak, 1981). Let $x, x' \in \mathcal{O}$ then,

$$\sim_B = \{(x, x') \in \mathcal{O} \times \mathcal{O} \mid \forall \phi_i \in B, \Delta\phi_i = 0\},$$

where $\Delta\phi_i = \phi_i(x') - \phi_i(x)$. If $(x, x') \in \sim_B$ (also written $x \sim_B x'$), then x and x' are said to be indiscernible with respect to all functions in B, or simply, B-indiscernible. Next, let $\xi_B = \mathcal{O}/\sim_B$ denote a partition of $\mathcal{O}$ and let $[x]_B \in \xi_B$ denote an equivalence class[3]. Notice that an equivalence class is a form of information granule since each class is a representation of specific knowledge of the objects in $\mathcal{O}$ based on what we know from the information obtained from the functions in B. In essence these sets offer a perceptive grouping of the objects based on the features that describe them (akin to the way humans group objects). Two sets, X and X', are considered near each other if the sets contain objects with at least partial matching descriptions.

Definition 1 Near Sets (Peters, 2007b)
Let $X,\ X' \subseteq \mathcal{O},\ B \subseteq \mathcal{F}$. Set X is near X' if, and only if there exists $x \in X, x' \in X', \phi_i \in B$ such that $x \sim_{\{\phi_i\}} x'$.

A powerful aspect of near set theory is the provision for set approximation inherited from rough set theory (Pawlak, 1981, Pawlak and Skowron, 2007). In general, approximation the form of replacement of mathematical objects by others that resemble them in certain respects. In particular, set approximation is useful in gaining knowledge about the parts of an image viewed as a set of objects such as pixels or subimages (a collection of pixels that are adjacent to one another). Approximations are carried out in the context of a fundamental approximation space *FAS* $= (\mathcal{O}, \mathcal{F}, \sim_B)$ where $\mathcal{O}$ is the universe of perceived objects, $\mathcal{F}$ is a set of probe functions such that $B \subseteq \mathcal{F}$, and $\sim_B$ is the indiscerniblilty relation. The approximation space is called fundamental because it was the framework used in the original rough set theory (Pawlak, 1981, Orłowska, 1982). Approximation starts with the partition ξ_B of $\mathcal{O}$ defined by the relation $\sim_B$. A set X is then approximated by observing its constituent equivalence classes, *i.e.* observing the relationship between X and the classes $[x]_B \in \xi_B \mid x \in \mathcal{O}$. Approximation of X takes two

[2]Here the phrase "qualitatively near" is associated with the adjective "similar."

[3]By definition each $x \in [x]_B$ has a matching description.

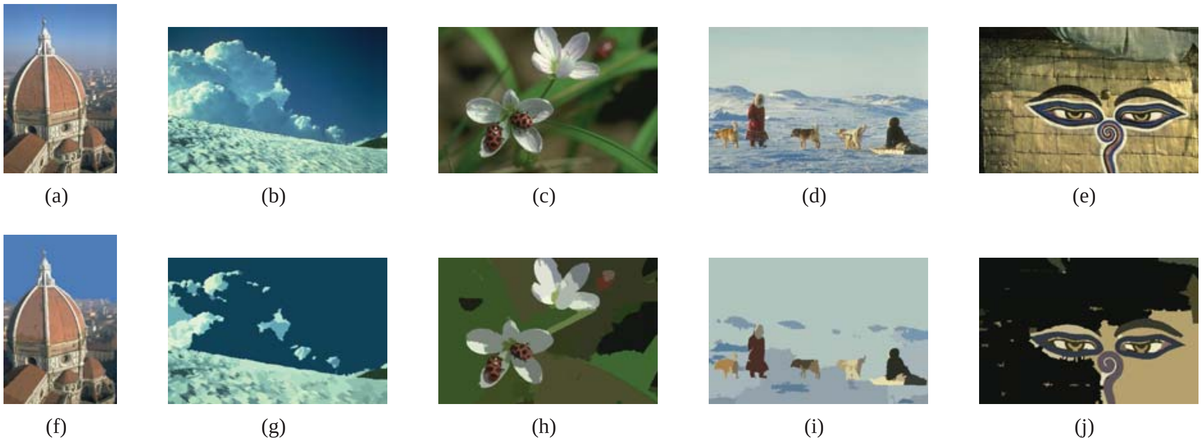


Figure 2: Segmentation of an image using the mean shift algorithm: (a) - (e) Original images, and (f) - (j) segmentations of original images.

forms, the B-lower approximation of X

$$B_*X = \bigcup_{x:[x]_B \subseteq X} [x]_B,$$

which consists of all the equivalence classes $[x]_B \in \xi_B$ that are completely contained in X; and the B-upper approximation of X

$$B^*X = \bigcup_{x:[x]_B \cap X \neq \emptyset} [x]_B,$$

which consists of all the equivalence classes that in some part belong to X.

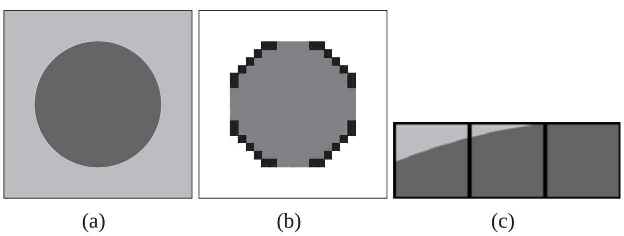


Figure 3: Example of image equivalence classes: (a) Original image, (b) the image formed from assigning grey levels to each equivalence class, and (c) example illustrating need for feature discretization.

2 NEAR SETS AND IMAGE PROCESSING

The near set approach is well suited to images. First, let us define an RGB image as $f : M \times N \longrightarrow \mathbb{N}^3 \mid M, N \in \mathbb{N}$, and let us define $\mathcal{O} = \{f(m,n) \mid (m,n) \in M \times N\}$. Consequently, an object x is considered a pixel in the image f. Similarly, define a subimage as $f_s : P \times Q \longrightarrow \mathbb{N}^3 \mid P \subseteq M, Q \subseteq N$. In the context of this paper, the domain of the functions ϕ_i are subimages. The result is that an object's description will depend on itself as well as all of its neighbours. Examples consist of any function that extracts features from an image, such as RGB or HSV values (see, *e.g.*, (Marti et al., 2001) which contains 34 example features).

As an example of near set theory applied to images, consider the simple monochrome[4] image shown in Fig. 3(a) and let $\phi(x) = B = \{\bar{f}_s\}$, where $\bar{f}_s = \frac{1}{PQ}\sum_{p,q} f_s(p,q)$. Let the set X consist of all the pixels of Fig. 3(a) contained in the grey circle. The image shown in Fig. 3(b) is a result of assigning colours to the partition ξ_B. Further, the grey region in Fig. 3(b) is the lower approximation of the set X since it consists only of equivalence classes that are completely contained in X. Similarly, the upper approximation is obtained by combining the black and grey equivalence classes. Note, the black equivalence class shown in Fig. 3(b) are important for two reasons. First, they highlight the need for discretization. Fig. 3(c) provides a close up of the edge of the circle divided into three different subimages. The value of $\bar{f}_s$ for each of these subimages differs by some small epsilon. As a result, discretization of feature values is used to combine similar features to reduce the number of equivalence classes. Second, it shows the near set concept of the boundary region.

As another example, consider Fig. 1(a), an image of an insulator used to isolate hydro electric transmission lines from the metal towers used for suspension. Notice the image has some features that make it difficult to segment, namely the background is quite detailed, the date is included in the lower right corner of the image, and there is a reflection from the sun on the insulator. Kittler's minimum error thresholding (Kittler and Illingworth, 1986) (an improved version of Otsu's method (Otsu, 1979)) is an fast and easy way to automatically segment an image. However, the method can perform poorly when there is not a clear separation between the modes representing an object and background in the image's histogram. Near sets can be used to solve this problem producing a result similar to that of morphological processing techniques. For example, let us use the definitions of f, f_s, x, and $\mathcal{O}$ from above, and let X be the set of all black pixels in Fig. 1(b). Further, let the functions $\phi_{\bar{I}}(x)$ and $\phi_{\bar{R}}(x)$ respectively denote average intensity value from the HSI colour model (see, *e.g.* (Gonzalez and Woods, 2002)) and the average value of the red component from the RGB colour model in which the domain are subimages f_s containing 16 pixels. Then, define $\phi(x) = B = \{\phi_{\bar{I}}(x), \phi_{\bar{R}}(x)\}$. The resultant equivalence class image is given in Fig. 1(c), where the equivalence classes are again assigned individual grayscale colours (even though the original image was in colour). Similarly, Fig. 1(d) is the lower approximation of the set X using the feature set B. Note, the white background is not included in the lower approximation. Finally, Fig. 1(e) contains only pixels associated with the insulator and is obtained by including all the non-white pixels from Fig. 1(d). Notice that the result in Fig. 1(e) is quite similar to the image in Fig. 1(f). This figure was produced using morphological opening with a structuring element the same size as f_s.

[4] In this case $f : M \times N \longrightarrow \mathbb{N}$ and $f_s : P \times Q \longrightarrow \mathbb{N}$.

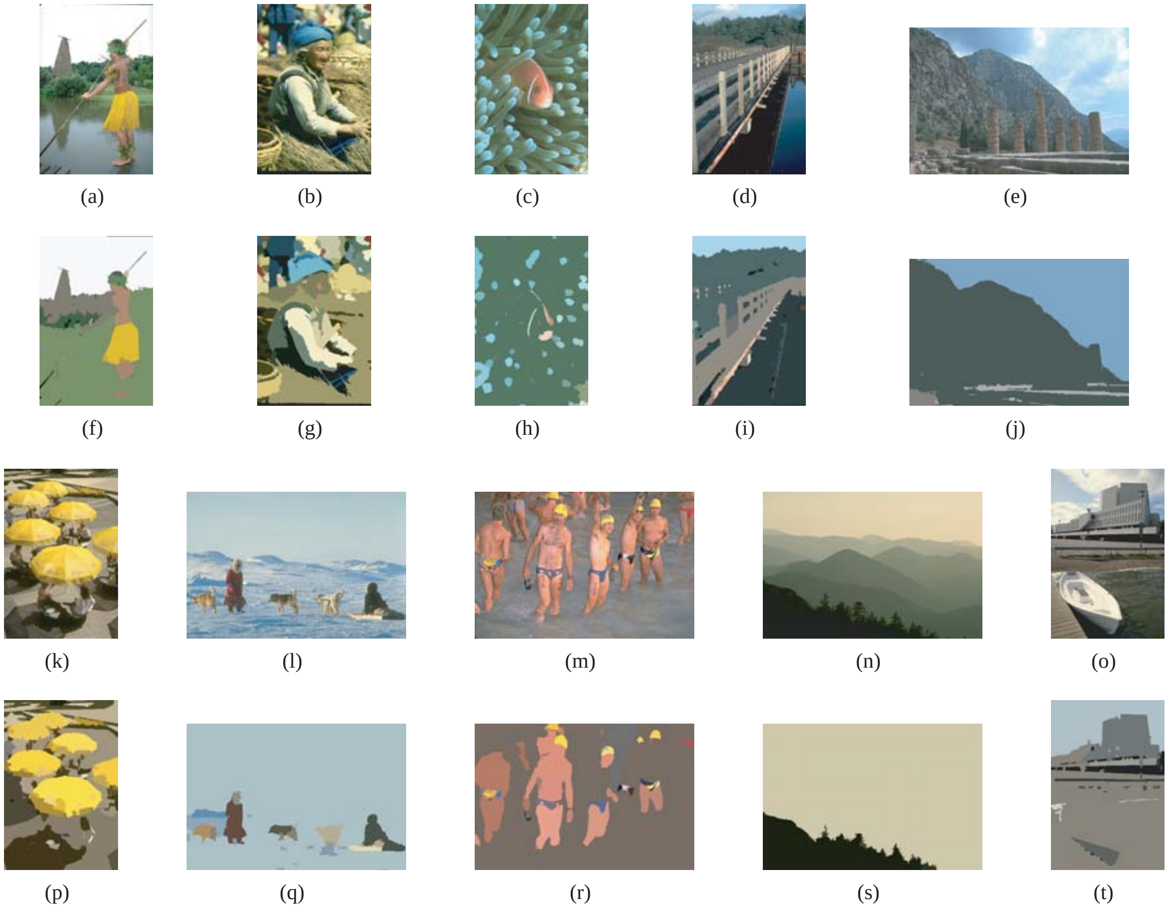


Figure 4: Segmentation of an image using the mean shift algorithm: (a) - (e) and (k) - (o) Original images, and (f) - (j) and (p) - (t) segmentations of original images.

3 SEGMENTATION EVALUATION USING NEAR SETS

Recall segments are supposed to represent components within an image that we generally call objects[5], and that a good segmentation is determined by how well the image component is captured by the segment. Consequently, segmentation evaluation tends to be a subjective process based on our perceptions of the components within the image. As was mentioned in Sect. 1 we can quantify perceptions through the use of probe functions[6] that measure object (and consequently component) features. Moreover, the right feature or combination of features may be able to describe a component entirely. As a result, we propose using information content of the lower approximation obtained from candidate segmentations as the measure for evaluation.

Consider a pixel i in an monochrome image $\mathcal{I}$ representing the lower approximation. Let i be a random variable that can take on any value in the interval $[0, 255]$, and let the image histogram be the probability distribution over the pixels contained in the image. Then the information content of an outcome i is defined by

$$h(i) = \log_2 \frac{1}{P(i)},$$

which is measured in bits, and $h(i = v)$ provides a measure of the information content of the event $x = v$ (MacKay, 2003). Further, the information content of an image is defined as

$$H(\mathcal{I}) = \sum_{i=0}^{255} P(i) \log_2 \frac{1}{P(i)}, \tag{1}$$

where $P(i) = 0$ since $\lim_{\theta \to 0^+} \theta \log_2 1/\theta = 0$ (also measured in bits) (MacKay, 2003). Information content provides a measure of the variability of the pixel intensity levels within the image and takes on values in the interval $[0, \log_2 L]$ where L is the number of grey levels in the image. A value of 0 is produced when an image contains all the same intensity levels and the highest value occurs when each intensity level occurs with equal frequency (Seemann, 2002).

Definition 2 Near Set Index
The NSI is the information content of the lower approximation of a segment.

Using this approach, the problem of segmentation evaluation becomes one of feature selection. For instance let X represent an image component obtained by a manually segmented image (*i.e.* a ground truth segmentation) which represents the best possible segmentation of the component. Next, envision selecting B such that an equivalence class, $[x]_B$, is formed matching the manual segmentation. The resulting lower approximation, B_*X, would contain only one equivalence class and accordingly $H(B_*X) = 0$. Thus, a lower value of information content corresponds to a better segmentation and can be used as a metric to objectively evaluate image segmentations without the need of ground truth

[5] This is not to be confused with the definition of object from near set theory. From here on we will refer to these objects as image components.

[6] Probe functions are used to perform feature extraction.

images. Further, the types of image components are generally known *a priori* when there is a need to perform segmentation evaluation, for instance as a preprocessing stage for object recognition or in the evaluation of image segmentation algorithms. As a result, probe functions can be tailored to produce equivalence classes representative of image components.

As an example of the proposed method, consider Fig. 5 where the segmented image on the left was obtained using the mean-shift-based segmentation algorithm with parameters $h_s = 7$ and $h_r = 11$ (Comaniciu, 2002), and the image next to it is one of it's segments. Using Algorithm 1, the information content of the lower approximation of Fig. 1(b) (*i.e.* Fig. 1(e)) is 4.8735 while the information content of the lower approximation obtained from Fig. 5(b) (using the same $\phi(x) = B$) is 4.5730. These results match the observation that the mean shift segmentation is "better" where this adjective is quantified by the features in B.

Algorithm 1: Algorithm for calculating Near Set Index

Input : $\mathcal{O}$ (original image), B (functions for extracting feature values), X (segment being evaluated)

Output: NSI

1. Divide image into subimages;
2. Assign $\phi(x)$ for each pixel in subimage based on feature values;
3. Create equivalence classes by grouping objects with the same descriptions $\phi(x)$ (*i.e.*, create partition ξ_B);
4. Calculate the B-lower approximation space, B_*X by collecting all equivalence classes that are completely contained in X;
5. For each $[x]_B \in X_*B$, assign the same label to all $x \in [x]_B$;
6. Assign NSI calculated from IC of labels in X_*B;

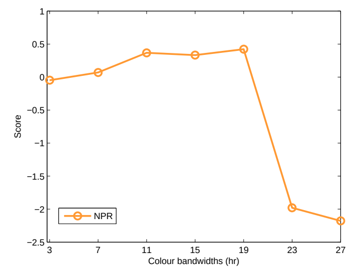
(a) (b)

Figure 5: Segmentation of Fig 1(a): (a) A single segmentation obtained using the using mean shift algorithm with parameters $h_s = 7$ and $h_r = 11$, and (b) mask created from a single segment in (a).

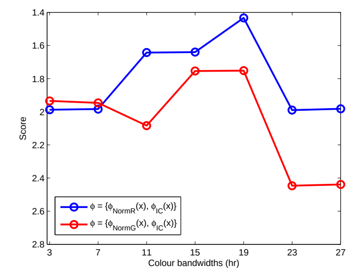
3.1 NSI and Multiple Segments

Segmentation algorithms such as the mean shift (Comaniciu, 2002) partition an image with the result that there is usually more than one segment. In this case the NSI is easily extended. Each segment is evaluated using Eq. 1 and the worst (*i.e.* largest) value is used to rate the entire image. Thus a proposed partition of an image is only as good as it's "worst" segmentation. This can be achieved by applying Algorithm 1 to each segment in the image. The data given in Fig. 6(a) was generated by our own implementation of the NPR index and mirror the data given in (Unnikrishnan et al., 2007). The two different plots in Fig. 6(a) represents two different feature sets (*i.e.* two different sets for B defined in Sect. 1) and are provided to demonstrate the dependence of the NSI method on the selection of image features. As can be seen in Fig. 6 the NSI produces plots with similar characteristics to those of the NPR index with the best segmentation occurring in all three plots at $h_r = 19$.

Further examples comparing the NSI and NPR indexes are given in Fig. 2 & 7. Fig. 7 plots the NSI of the segmentations in Fig. 2 for different feature selections. Notice that Fig. 7(a) & 7(b) are similar except for their evaluation of image 3 (the ladybug picture). The NSI index for this image is poor for all three feature sets. Whether or not this image is the worst is application specific. In this case there exists a segment in the image which produces poor (high) information content with respect to the features selected. More results are given for segmentations shown in Fig. 4. The plot given in Fig. 8(a) is for the segmentations given in Fig. 4. Notice again that the NPR and NSI measures are similar.

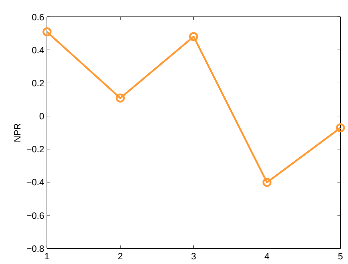


(a)

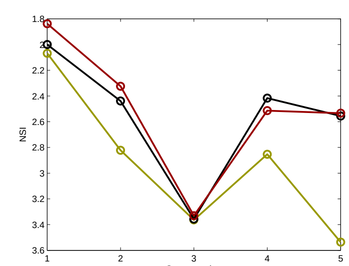


(b)

Figure 6: Results comparing the NPR and NSI indexes: (a) Plot generated by calculating NPR index , and (b) plot generating by the NSI index.

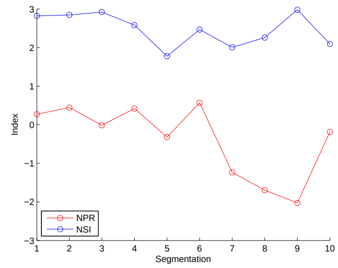


(a)



(b)

Figure 7: Results comparing the NPR and NSI indexes. Segmentation 1 corresponds to image (a), 2 corresponds to image (b), *etc.*: (a) Plot generated by calculating NPR index , and (b) plot generating by the NSI index.



(a)

Figure 8: Results comparing the NPR and NSI indexes: Plot generated by calculating NPR index

4 CONCLUSION

This article introduces the NSI measure for evaluating segmentations. By way of comparison with the NPR, these results suggest that the NSI is a reasonable measure for image segmentation applications. The main advantage of this method is that it is an objective segmentation measure that requires no ground truth images. Not only can it be used both to evaluate segmentation algorithms, but, more importantly, it can be used in unsupervised applications where it is not feasible to have an expert provide ground truth images. This method is dependent on the features selected to describe image components. However, feature selection is usually inherent in any system requiring image segmentation (*e.g.*, object recognition and computer vision systems). Moreover feature selection is desirable over providing ground truth images for systems that that do not employ object recognition, yet still require segmentation evaluation. Also, in many cases, feature selection will be dictated by the application. Future work will include investiging the order of growth of algorithm running time, algorithm optimizations to allow the NSI method to be used in real time (*e.g.*, in adaptive learning) systems, and significant testing in many application areas (including comparison to other evaluation measures).

REFERENCES

Carreira-Perpiñán, M. A., 2007. Gaussian mean-shift is an em algorithm. IEEE Transactions on Pattern Analysis and Machine Intelligence 29(5), pp. 767–776.

Christoudias, C., Georgescu, B. and Meer, P., 2002. Synergism in low level vision. In: Proceedings of the 16th International Conference on Pattern Recognition, Vol. 4, Quebec City, pp. 150–156.

Comaniciu, D., 2002. Mean shift: A robust approach toward feature space analysis. IEEE Transactions on Pattern Analysis and Machine Intelligence 24(5), pp. 603–619.

Duda, R., Hart, P. and Stork, D., 2001. Pattern Classification. 2nd edn, Wiley.

Fahle, M. and Poggio, T., 2002. Perceptual Learning. The MIT Press, Cambridge, MA.

Gonzalez, R. C. and Woods, R. E., 2002. Digital Image Processing. 2nd edn, Prentice-Hall, Toronto.

Henry, C. and Peters, J. F., 2007. Image pattern recognition using approximation spaces and near sets. In: Proc. of the Eleventh International Conference on Rough Sets, Fuzzy Sets, Data Mining and Granular Computer (RSFDGrC 2007), Joint Rough Set Symposium (JRS07), Lecture Notes in Artificial Intelligence, Vol. 4482, pp. 475–482.

Kittler, J. and Illingworth, J., 1986. Minimum error thresholding. Pattern Recognition 19(1), pp. 41–47.

MacKay, D. J. C., 2003. Information Theory, Inference, and Learning Algorithms. Cambridge University Press, UK.

Marti, J., Freixenet, J., Batlle, J. and Casals, A., 2001. A new approach to outdoor scene description based on learning and top-down segmentation. Image and Vision Computing 19, pp. 1041–1055.

Martin, D., Fowlkes, C., Tal, D. and Malik, J., 2001. A database of human segmented natural images and its application to evaluating segmentation algorithms and measuring ecological statistics. In: Proceedings of the 8th International Conference on Computer Visison, Vol. 2, pp. 416–423.

Murray, J. A., Bradley, H., Craigie, W. and Onions, C., 1933. The Oxford English Dictionary. Oxford University Press, London.

Neubert, M., Herold, H. and Meinel, G., 2006. Evaluation of remote sensing image segmentation quality–further results and concepts. In: 1^{st} Int. Conf. on Object-based Image Analysis (OBIA06), pp. 1–6.

Orłowska, E., 1982. Semantics of vague concepts. applications of rough sets. Technical Report 469, Institute for Computer Science, Polish Academy of Sciences.

Otsu, N., 1979. A threshold selection method from gray-level histograms. IEEE Transactions on Systems, Man, and Cybernetics 9(1), pp. 377–393.

Pal, N. and Pal, S., 1989. Entropic thresholding. Signal Processing 106, pp. 97–108.

Pawlak, Z., 1981. Classification of objects by means of attributes. Technical Report PAS 429, Institute for Computer Science, Polish Academy of Sciences.

Pawlak, Z. and Skowron, A., 2007. Rudiments of rough sets. Information Sciences 177, pp. 3–27.

Peters, J. F., 2007a. Classification of objects by means of features. In: Proceedings of the IEEE Symposium Series on Foundations of Computational Intelligence (IEEE SCCI 2007), Honolulu, Hawaii, pp. 1–8.

Peters, J. F., 2007b. Near sets. general theory about nearness of objects. Applied Mathematical Sciences 1(53), pp. 2609–2629.

Peters, J. F., 2007c. Near sets. special theory about nearness of objects. Fundamenta Informaticae 76, pp. 1–27.

Seemann, T., 2002. Digital Image Processing using Local Segmentation. Ph.d. dissertation, School of Computer Science and Software Engineering, Monash University.

Unnikrishnan, R., Pantofaru, C. and Hebert, M., 2007. Toward objective evaluation of image segmentation algorithms. IEEE Transactions on Pattern Analysis and Machine Intelligence 29(6), pp. 929–944.

Wang, J., Thiesson, B., Xu, Y. and Cohen, M., 2004. Image and video segmentation by anisotropic kernel mean shift. In: In. Proc. European Conference on Computer Vision (ECCV'04), Vol. 2, pp. 238–249.