MULTI IMAGE MATCHING OF STRAIGHT LINES WITH GEOMETRIC CONSTRAINTS

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ABSTRACT:

Automatic multi image matching of linear features still remains to be a problem in both the photogrammetric and computer vision communities. Changes in the illumination conditions, camera position, and surrounding features challenge the automation of stereo matching of linear features. Moreover, linear feature matching becomes tougher when attempting to match linear features in more than two images. Hence, most researchers perform the multi-photo matching in a pair wise mode. This ignores the benefits of the multi-image geometry and illumination. This research presents an alternative algorithm that can be used in matching straight lines simultaneously, using any number of photos. All possible corresponding sets are generated and a matching cost is computed for each set. Both geometric and radiometric properties are utilized in computing the cost value for each set. The set with the minimum matching cost is then selected. Finally, a constrained least squares adjustment model is used to compute the object space parameters of the 3D lines. These constraints impose parallel and perpendicular relationships between the lines. The coordinates of the end points for each line are computed using the calculated 3D line parameters. These coordinates are compared with those calculated through point-based photogrammetric techniques. The algorithm is tested on the city hall of Zurich building dataset provided on the ISPRS website. An average absolute difference in the coordinates of the end points of three centimeters is observed.

1. INTRODUCTION

In the past few years, there has been a tremendous interest by photogrammetric researchers in utilizing linear features in various photogrammetric activities (Habib et al., 2004 and Akav et al., 2004). However, straight line matching is one of the challenging problems in photogrammetry and computer vision. Factors such as: changes in the radiometric characteristics between different views, complete or partial occlusion of the lines, shortage of 2D line extraction algorithms, and failure to extract the end points of the straight lines, affect the matching results. Another problem associated with the straight line matching is the absent of well defined techniques for the computation of image correlation between different straight lines (Hartley and Zisserman, 2004). In addition, unlike point features straight lines have weak geometric constraints (Schmid and Zisserman, 2000). The stated reasons support the need to solve the straight line matching problem using more than one pair of images.

Several line matching algorithms have been introduced in previous and current research. These algorithms are mainly divided to two types; either matching individual line segments or matching group of line segments (Baillard et al., 1999). Matching groups of line segments increase the geometric and radiometric information and reduce the uncertainty in the matching process. However, the algorithms based on matching groups of line segments are more complex. Researchers usually utilize the geometric and radiometric attributes of the matched lines to reduce the matching ambiguity.

The approach in (Schmid and Zisserman, 1997) utilizes two attributes of the linear features in the matching process. The first attribute is the intensity of neighboring pixels. The epipolar geometry between the images is used to provide point to point corresponding along the line segment. The second attribute is the use of the epipolar geometry, together with the matched lines, to restrict the possible projective transformation between the images to a one parameter family. Hence, this family is used to solve for the neighbourhood mapping.

The approach in (Manuel et al., 1995) presents is used to track lines in image sequences. The line parameters used are the midpoint position, the line length, and the direction of the line. Three independent Kalman filters are used for tracking, one for each parameter. The matching cost is the normalized Mahalanobis distance or the geometric constraints. This algorithm has several weak points if implemented in senses with occlusion. The midpoint of the lines might not match, and the line length might change from one view to the other.

Some algorithms are based on geometric information only without incorporation of any image intensities. For example the algorithm presented in (Heuel and Forstner, 2001) uses a geometric method for matching 2D line segments from multiple images and reconstructing 3D line segments and grouping 3D line segments to corners. The 2D geometric entities points, lines and planes are represented in homogeneous coordinates and new 3D entities are constructed. Each 3D entity is associated with a propagated uncertainty value. Hence, the reconstruction is performed directly or as estimation. In addition, relations such as incidence, equality, parallelity and orthogonality between points, lines and planes are tested statistically and used to support the reconstruction process. Beder (2004) presented a framework for automatic reconstruction of points and lines from multiple oriented images. The framework uses graphs induced by relational geometric properties, which can be handled in a rigorous statistical manner.

Another strategy for matching features in multiple images is presented in (Shao et al., 2000). The process starts from initial matches, which are validated in multiple images using multi-image constraints. These initial matches are then filtered through a relaxation procedure and are subsequently used to locally predict additional features that might well be extracted using different thresholds for the feature extraction process. The relaxation labeling is simultaneously performed among multiple

images, as opposed to the usual case of just two images. The overall process has been applied to segment matching of both aerial and close range imagery.

2. STRAIGHT LINES ATTRIBUTES

In this research, straight lines in a 2D space are characterized by two parameters (Mikhail et al., 2001); p (the length of the perpendicular from the origin to the line), and α (the angle measured counter-clockwise from the positive x-axis to the perpendicular). The following subsections present the method used to construct the matching window and the representation of the linear feature in the ground coordinate system.

2.1 Matching windows for linear features

Matching windows for image points are constructed in a straight forward manner. However, for linear features this is not an easy task due to the changes in the orientations and lengths of the lines. In this research, 1D matching window is used. The gray values for each element in this window are computed. For each 2D line, a number of parallel sections are constructed. These sections have the same (α). However, the (p) parameter for each section differs than the (p) parameter of the next section by one. For each section, all belonging pixels are located. A pixel is assumed to belong to a certain section, if it passes two tests. First the pixel has to satisfy the homogenous line equation for the (α and p) parameters of this section. In addition, the pixel should lay in the surrounding area of the section as shown in Figure 1. The gray values of these pixels are then used to compute an average gray value for this section. This will provide a 1D array of average intensities for each line. This array is used as the matching window in the correlation process. The advantage of this method is its applicability to use with any line orientation.

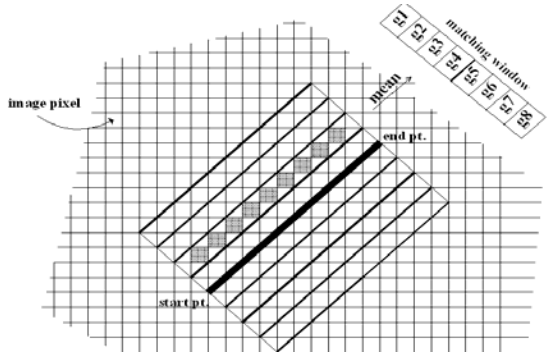

Figure 1. The matching array

2.2 Linear feature 3D representation

For each 2D straight line laying in an image, a plane in the space could be constructed using the perspective center of the image and the line. This plane will be used in the matching process. The plane is represented using three independent parameters. However, in this research, four dependent parameters are used to represent the plane as shown in Equation (1). Since only three parameters are sufficient to represent any plane there is a need to add one constraint between the four parameters as presented in Equation (2). This form of the plane is used since the plane is represented using one point, i.e. the exposure station, and a vector, i.e. the image space line.

$$AX + BY + CZ + D = 0 \tag{1}$$

$$D = -\sqrt{A^2 + B^2 + C^2} \tag{2}$$

where A, B, C, D = plane parameters
X, Y, Z = coordinates of any point in the plane

The plane parameters for each 2D line are computed from the 2D line parameters, and the exterior and interior orientation parameters, Figure 2. The process is summarized in the following steps:

1. Compute the vectors T_1 and T_2 using the 2D line parameters (α and p) and the interior orientation parameters of the camera (x_o, y_o, and f) as follows:

$$T_1 = \begin{bmatrix} p\cos(\alpha) - x_o \\ p\sin(\alpha) - y_o \\ -f \end{bmatrix} \qquad T_2 = \begin{bmatrix} \sin(\alpha) \\ -\cos(\alpha) \\ 0 \end{bmatrix}$$

2. Rotate the vectors T_1 and T_2 from image space to object space $T_{1g}=RT_1$ and $T_{2g}=RT_2$, where R is the rotation matrix from image space to object space.

3. Convert the vector T_{1g} to a unit vector.

4. Compute the cross product of T_{1g} and T_{2g}.

5. The values of A, B, and C are the elements of the unit vector of the cross product of the vectors T_{1g} and T_{2g}, while the value of D is computed using the ground coordinates of the exposure station (X_c, Y_c, and Z_c) as follows:

$$D=-AX_c-BY_c-CZ_c$$

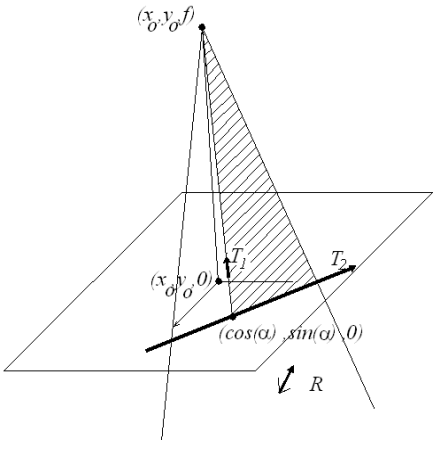

Figure 2. 3D plane for an image space straight line

3. MULTI-IMAGE LINEAR FEATURE MATCHING

In this section, the steps of matching the linear features across several images are described. All possible linear features matching hypotheses are generated. Each matching hypothesis is evaluated, both geometrically and radiometricly. The

algorithm can be applied using *n* images as follows. For one linear feature in one image, there exist one plane between the camera exposure station and the image of the 3D line. Hence, for *n* images there exist n planes. The 3D line is represented as the geometric intersection of these planes.

3.1 Geometric matching cost

The process of multi-image linear feature matching is presented mathematically as a multi-plane intersection. As shown in the section 3.2, any plane is described by four independent parameters. Each plane contributing in the matching process has its own four parameters. According to (Kankok, 1995), any linear feature can be described using six dependent parameters. These parameters represents two vectors, the vector along the line direction (*A*) and the vector passing through the origin, perpendicular to the line direction (*B*). Figure 3 shows the geometry of the two vectors. Two constraints are applied for the six parameters: the two vectors are perpendicular and the vector along the line is a unit vector.

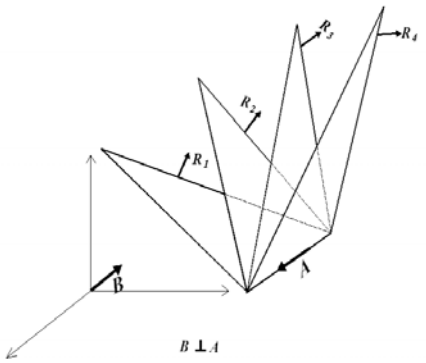

Figure 3. 3D straight line representation and matching

In order to find the six parameters of the intersecting line, the least squares adjustment with constraints is used (Mikhail, 1976). The inputs to the model are the plane parameters for the intersecting planes. Unknown parameters are the six parameters of the linear feature. The model is represented by Equation (3).

$$\begin{aligned} Av + B\Delta &= h \\ Cg &= 0 \end{aligned} \tag{3}$$

where v = residual vector
A = coefficient matrix of the residuals
B = coefficient matrix of the unknowns
Δ = correction vector for the unknowns
h = coefficient vector of the observation equations
C = coefficient matrix of the unknowns in the constraint equations
G = coefficient vector of the constraint equations

For any number of images, the elements of the (i and i+1) rows of the two matrices A, B, and vector h are written using the elements of the two vectors (A) and (B), and the four plane parameters (a_i, b_i, c_i, and d_i) for plane (i) as follows:

$$\begin{bmatrix} B_i \\ B_{i+1} \end{bmatrix} = \begin{bmatrix} a_i & b_i & c_i & 0 & 0 & 0 \\ 0 & 0 & 0 & a_i & b_i & c_i \end{bmatrix}$$

$$\begin{bmatrix} h_i \\ h_{i+1} \end{bmatrix} = \begin{bmatrix} -(a_i a_x + b_i a_y + c_i a_z + d_i) \\ -(a_i b_x + b_i b_y + c_i b_z) \end{bmatrix}$$

The matrix C and the vector g for any number of images are computed using the elements of the two vectors (A) and (B) as follows:

$$C = \begin{bmatrix} b_x & b_y & b_z & a_x & a_y & a_z \\ 0 & 0 & 0 & 2b_x & 2b_y & 2b_z \end{bmatrix}$$

$$g = \begin{bmatrix} -(a_x b_x + a_y b_y + a_z b_z) \\ -(b_x b_x + b_y b_y + b_z b_z - 1) \end{bmatrix}$$

The least squares solution for the system is then solved as follows:

$$N = B^T (AA^T)^{-1} B$$
$$t = B^T (AA^T)^{-1} h$$
$$M = \begin{bmatrix} -N & C^t \\ C & Z \end{bmatrix}$$
$$t_1 = \begin{bmatrix} t \\ g \end{bmatrix}$$
$$\Delta = M^{-1} t_1$$

where Z = zero matrix of two rows and two columns.

The residual vector is then computed. The geometric matching cost for each matching hypothesis is computed as the value of the quadratic form $V^t V$.

3.2 Radiometric matching cost

Each 2D line is associated with a matching array that represents the average gray values on both sides of the line. For each 2D line pair the correlation coefficient is computed between their corresponding gray level arrays using Equation (4).

$$C_{ij} = \frac{\sum_{k=1}^{n} (G_k^i - G^{im})(G_k^i - G^{jm})}{\sqrt{\sum_{k=1}^{n} (G_k^i - G^{im})^2 \bullet \sum_{k=1}^{n} (G_k^i - G^{jm})^2}} \tag{4}$$

where C_{ij} = correlation coefficient between two lines in images i and j
n = total number of sections
G_{im}, G_{jm} = mean intensity values for the two arrays in images i and j respectively

G_k^i, G_k^i = intensity values of element k in the matching arrays in image i and image j respectively

Each matching hypothesis includes *n* 2D lines, hence there exists *n* *(*n* -1)/2 correlation coefficients computed between the *n* pairs. The matching cost for each matching hypothesis is computed as the average of the correlation coefficients between the *n* pairs. Different numbers of sections on both sides of the linear feature were tested. In this research, it was found that the correlation coefficients are stable using four to five sections on both sides of the line.

3.3 Optimum matching set

For each matching hypothesis there exist two costs. The quadratic form computed in section 3.1 and the average correlation coefficient computed in section 3.2. The two costs are combined using equation (5). The equation is based on the idea presented in Pilu (1997).

$$M = e^{-(1-C)} \times e^{-(V^t V)} \tag{5}$$

where C = normalized average correlation for a matching hypothesis

$V^t V$ = quadratic form value for the same hypothesis

M = final cost of the hypothesis

All possible matching sets are generated. Each matching set consists of all available line matching hypothesis. The total cost for each set is computed as the sum of the geometric and radiometric matching costs of all its hypotheses. If the planes of a certain hypothesis do not intersect then its geometric matching cost is given a high value. For some hypotheses, the intersecting plans might not belong to the same 3D line segment, figure 4. Therefore, the matching cost is increased as a relation of the inverse of the average overlapping. After computing the total matching cost for each set, a searching algorithm is used to find the best matching set.

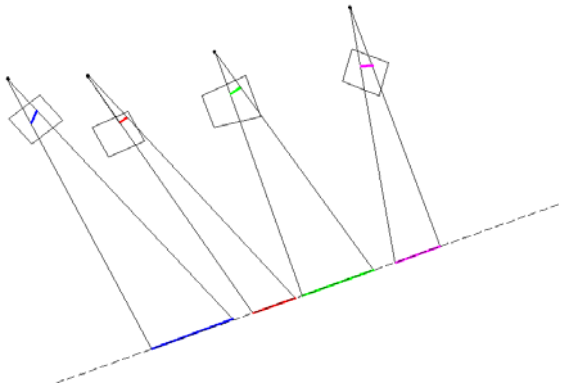

Figure 4. A case where intersecting plans do not correspond to a single 3D line segment

3.4 Refining the parameters of the 3D lines

The results of the previous step provide the best corresponding set of image lines. Each set is provided with the object space parameters of the intersecting 3D lines; however, a refining step should be carried out to refine these parameters through imposing several geometric constraints among the 3D lines. These constraints are as follows:

Constraints for lines parallel to the X axis: If line *i* is parallel to the X axis, then:

$$\begin{bmatrix} A_{iy} \\ A_{iz} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Constraints for lines parallel to the Y axis: If line *i* is parallel to the Y axis, then:

$$\begin{bmatrix} A_{ix} \\ A_{iz} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Constraints for lines parallel to the Z axis: If line *i* is parallel to the Z axis, then:

$$\begin{bmatrix} A_{iy} \\ A_{ix} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Constraints for parallel lines: If two lines *i* and *j* are parallel, then:

$$\begin{bmatrix} A_{ix} - A_{jx} \\ A_{iy} - A_{jy} \\ A_{iz} - A_{jz} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Constraints for perpendicular lines: If two lines *i* and *j* are perpendicular, then:

$$\begin{bmatrix} A_{ix} \\ A_{iy} \\ A_{iz} \end{bmatrix} \begin{bmatrix} A_{jx} & A_{jy} & A_{jz} \end{bmatrix} = 0$$

where A_{ix}, A_{iy}, A_{iz} = the components of the A vector for line i

A_{jx}, A_{jy}, A_{jz} = the components of the A vector for line j

For each line or pair of parallel or perpendicular lines, the constraints are added to the least squares adjustment model presented in equation (3).

3.5 Coordinates of end points

After finding the six parameters for each 3D line, the ground coordinates of the end points for each line are computed using Equation (6).

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 1 & 0 & -u/w \\ 0 & 1 & -v/w \\ 1 & -b_x/b_y & 0 \end{bmatrix}^{-1} \begin{bmatrix} X_c - \dfrac{Z_c u}{w} \\ Y_c - \dfrac{Z_c v}{w} \\ -\left(\dfrac{b_x}{b_y} a_y - a_x\right) \end{bmatrix} \tag{6}$$

where X_c, Y_c, Z_c = ground coordinates of the exposure station
a_x, a_y = elements of the A vector
b_x, b_y = elements of the B vector for the matched line
u, v, w are computed as follows:

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = R \begin{bmatrix} x_p - x_o \\ y_p - y_o \\ -f \end{bmatrix}$$

where x_p and y_p = image coordinates of the end points of the image line
x_o, yo, f = interior orientation parameters of the camera
R = rotation matrix from image space to object space

4. EXPERIMENTS

4.1 Dataset Description

The dataset used in this research is for the city hall of Zurich building. The data is available on the website of the International Society of Photogrammetry and Remote Sensing (ISPRS). The dataset is provided with a complete description of the interior orientation of the used cameras, the images, and the coordinates and description of the reference points measured on the facades of the building by geodetic means (Streilein et al., 1999). The images acquisition was performed using two digital cameras of about ten millimetres focal length. The first camera is an Olympus C1400L camera with 1280x1024 pixels, while the second camera is a Fuji DS 300 camera with 1280x1000 pixels. Nine ground control points are used to compute the exterior orientation parameters of each camera position. The Root Mean Square Errors (RMS) of the check points in the three directions is about seven to eight centimeters.

4.2 Results and analysis

The algorithm is implemented using six images for one faced of the building. The start and end points of 147 image lines are digitized in all images, figure 5, and used to compute the 2D line parameters (α and p) for each line. The values of A, B, C, and D are then computed for each image line. In addition, the matching array for each image line is also evaluated. The matching process then starts using both the geometric and radiometric properties of the image lines. Results showed that all lines are correctly matched and no false matches existed. This is mainly due to the amplifying of the matching cost for non-overlapping lines. It was found that if the matching cost is not modified with respect to the average overlapping percentage, several false 3D lines occur. The reason for such outcome is the geometry of the tested dataset. However, when the algorithm is tested without the radiometric matching cost, only 7 false lines are generated. Although the algorithm is tested in an ideal case where all lines appear in all six images, it can be implemented in other scenarios where the number of lines varies among all images. In such situations, the user can tune the algorithm to generate for example; all *n-2* or *n-3* subsets of 3D lines, then group the final 3D lines.

After finding the corresponding linear features, the least squares adjustment model with constraint is carried out. The inputs to the model are the 2D line parameters (α and p) for each line. The model is then solved iteratively to find the refined set of parameters for the 3D lines.

In order to evaluate the positional accuracy of the line matching process, the end point coordinates are computed using equation (6) through the parameters of the 3D lines. These coordinates are compared with those computed through point-based photogrammetric techniques. Table 1 shows the average absolute differences for the coordinates of the end points of the matched lines. Several cases are evaluated: with and without constraints. The results showed an average absolute difference of about 2 centimeters in the X and Z directions and 3 centimeters in the Y direction. Insignificant differences are observed when the geometric constraints are imposed. Figure 6 shows the matched object space lines using all constraints.

Case	Average coordinate differences (cm)		
	X	Y	Z
no constraints	2.31	3.22	2.64
with constraints for lines parallel to the Z axis only	2.42	3.01	2.54
with constraints for parallel lines only	2.21	3.57	2.32
with constraints for perpendicular lines only	2.53	3.34	2.22
with all constraints	2.15	2.87	2.56

Table 1. RMS for the coordinates of the end points

Figure 5. The 6 images used in the experiment

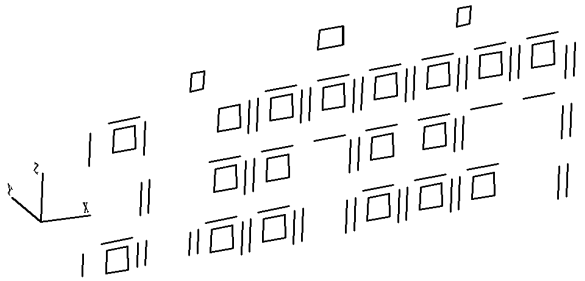

Figure 6. The generated 3D lines

5. CONCLUSIONS

This paper presents an algorithm for solving the multi image straight line matching problem simultaneously. The lines are digitized in the images and a multi plane intersection approach is used to solve the intersection problem. All line matching hypotheses are generated and all matching combination sets are tested. Both geometric and radiometric properties are utilized to obtain the best matching set. In addition, geometric constrains among the 3D linear features are utilized to refine the final object space parameters for each 3D line. Results showed an average absolute difference in the ground coordinates of the end points of the matched lines of about three centimeters using the dataset for the city hall of Zurich building provided on the ISPRS website. Future research will concentrate on using automatically extracted image lines. In addition, the algorithm will be tested in different environments using indoor an outdoor images.

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