ROBUST FDI IN REDUNDANT MEMS-IMUS SYSTEMS

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ABSTRACT:

This research presents methods for detecting and isolating faults in multiple Micro-Electro-Mechanical System (MEMS) Inertial Measurement Unit (IMU) configurations. Traditionally, in the inertial technology, the task Fault Detection and Isolation (FDI) is realized by the parity space method. However, this approach performs poorly with low-cost MEMS-IMUs, although, it provides satisfactory results when applied to tactical or navigation grade IMUs. In this article, we propose a more complex approach to detect outliers that takes into account the shape and size of multivariate data. The proposed method is based on Mahalanobis distances. Such approach has already been successfully applied in other fields of applied multivariate statistics, however, it has never been tested with inertial sensors. As Mahalanobis distances (as well as the parity space method) is very sensitive to the presence of the same outliers this method aims to detect, we propose using its robust version. The performances of the proposed algorithm are evaluated using dynamical experiments with several MEMS-IMUs and a reference signal provided by a tactical-grade IMU run in parallel. The conducted experiment shows that, for example, the percentage of false alarms is approximately ten times lower when using a method based on Mahalanobis distances as compared to that based on the parity space approach.

1 INTRODUCTION

The use of redundant Micro-Electro-Mechanical System (MEMS) IMU sensors is an economically and ergonomically viable solution to improve navigation performance while enhancing monitoring of individual sensor performance. Therefore, these low-cost inertial sensors are more and more employed in commercial applications. As single MEMS IMUs do not provide sufficiently accurate measurements for standalone inertial navigation, the process of permanent sensor-error compensation during navigation becomes mandatory. However, the precision of the navigation parameters can be considerably improved when several MEMS-IMU's triads are used in parallel provided faulty measurements are detected and isolated from the estimation process (Guerrier, 2008; Waegli et al., 2008). The resulting cost and volume of such systems still remain very attractive.

FDI algorithms applied to inertial sensors were traditionally used in safety critical operations such as in the control of military or space aircrafts. However, multiple MEMS-IMUs systems are not yet used for safety critical applications. In this context, FDI aims at detecting and removing gross errors to enhance navigation performances and to improve the stability of the filters used in the GPS/INS integration. The most commonly used FDI approach for inertial sensors is the parity space method (Gai et al., 1979a,b; Sturza, 1988), but other approaches such as artificial neural networks have also been examined, e.g. Krogmann (1995). As previously mentioned, the parity space method gives satistifactory results when applied to IMUs of higher accuracy, however it performs poorly with low-cost sensors (Guerrier, 2008; Waegli et al., 2008). Indeed, the complexity of implementation of an efficient FDI system is increased when using MEMS-type IMUs. The error-characteristics of these sensors (*i.e.* noise density variations in time and among sensors, large systematic measurement errors compared to the random errors) often create false alarms and increase the possibility of misdetection of faulty measurements. Additionally, it has been shown in Guerrier (2008, 2009) that MEMS-IMUs error characteristics are strongly influenced by variation of the environmental conditions (e.g. increase vibrations or temperature variations).

On the contrary to the parity space method, other methods developed for FDI purpose allow taking into account the correlation and the variance differences between residuals of the synthetic IMU (Filzmoser et al., 2005; Gertler, 1990; Rousseeuw and Van Zomeren, 1990; Beckman and Cook, 1983). These methods are based on Mahalanobis distances and consider the shape of the data cloud quantified by the covariance matrix. In this research, we proposed a generalization of the parity space approach based on Mahalanobis distances (and its robust version) to detect erroneous measurements.

In this article, the theoretical aspects of FDI algorithms are first presented. Then, robustness issues in FDI are discussed. Finally, experimental results are presented.

2 FAULT DETECTION AND ISOLATION WITH MEMS-IMUS

In this section, we present two FDI approaches, namely the classical parity space method and a method based on Mahalanobis distances. These two methods are based on statistical tests designed to detect multivariate outliers. Indeed, the fault detection can be viewed as a choice between two hypotheses concerning the absence (*i.e.* the null hypothesis H_0) or the presence (*i.e.* the alternative hypothesis H_1) of erroneous measurements. Furthermore, the test statistic of the two methods have an approximate

χ^2 distribution under H_0 when assuming multivariate normally distributed residuals.

2.1 Parity space method

The parity space method is based on the statistic D which is computed as follows (Sturza, 1988):

$$
\begin{aligned}
D &= \mathbf{v}^T\mathbf{v} &(1)\\
\mathbf{v} &= \mathbf{Z}\mathbf{y} &(2)\\
\mathbf{Z} &= \mathbf{I}_m - \mathbf{H}(\mathbf{H}^T\mathbf{H})^{-1}\mathbf{H}^T &(3)
\end{aligned}
$$

where m is the number of measurements, $\mathbf{H}$ the design matrix which transforms the state space to the measurement space, $\mathbf{y}$ represents the vector of measurements and $\mathbf{v}$ the vector of measurement residuals after least squares adjustment.

This method is based on the assumption sometime irrealistic that the residuals used in FDI are distributed as:

$$\mathbf{v} \sim N_m(\mathbf{0}, \sigma_x^2\mathbf{I}_m) \tag{4}$$

where σ_x^2 is the variance of the measurements. Under this assumption, the statistic D is the squared sum of independent identically normally distributed random variables. In consequence, it implies that it follows the distribution:

$$\frac{D}{\sigma_x^2} \sim \chi^2_{m-p} \tag{5}$$

where p is the number of independent parameters.

2.2 Mahalanobis distance

To generalize the parity space method, we propose applying a method based on the Mahalanobis distances. This approach also relies on the assumption that the residuals follow a multivariate normal distribution while no assumptions are made about the center or the form of the covariance matrix of this distribution.

The Mahalanobis distances are defined as:

$$
\begin{aligned}
MD &= \sqrt{\mathbf{w}^T\mathbf{w}} &(6)\\
\mathbf{w} &= \mathbf{S}^{\star^{-1/2}}(\mathbf{v}^\star - \bar{\mathbf{v}}^\star) &(7)
\end{aligned}
$$

where $\mathbf{v}^\star$ are $m-3$ residuals taken from $\mathbf{v}$ and $S^\star$ the covariance matrix of $\mathbf{v}^\star$.

The Mahalanobis transformation (*i.e* $\mathbf{w}$ in Eq. 7) eliminates the correlation between the variables and standardizes the variance of each variable (Mahalanobis, 1936; Kent and Bibby, 1979). By definition, it is clear that: $\mathbf{w}^T\mathbf{w}$ (*i.e.* MD^2 in Eq. 6) has a χ^2_{m-p} distribution under the assumption of multivariate normally distributed residuals.

Like D (Eq. 1), the Mahalanobis distances (Eq. 6) estimate how extreme a measurement is with respect to others. Indeed, by setting the (squared) Mahalanobis distances equal to a certain constant, *i.e.* to a certain quantile of a χ^2_{m-p} distribution, it is possible to define an (hyper) ellipsoids with all the points having the same Mahalanobis distances from the centroid. FIG. 1 illustrates this concept with simulated bivariate-normally distributed data. The ellipses represent the quantiles 0.50, 0.75 and 0.99 of a χ^2_2 distribution. Hence, this approach takes into account the shape of the data cloud. On the contrary, the parity space method assumes uncorrelated errors. In such a case, all the points having the same D values can be represented by circles (dashed line in FIG. 1). For this reason, the parity space approach can produce unreliable results when the residuals are correlated (or heteroscedastic).

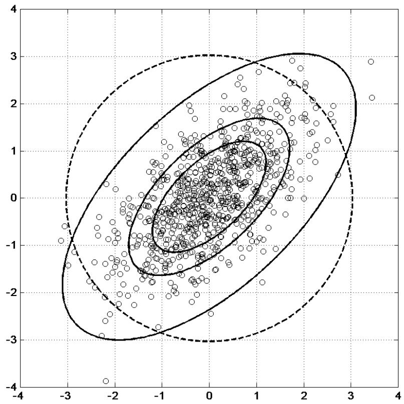

Figure 1: Simulated standard bivariate-normally distributed data with a correlation of 0.60. The ellipses represent the 0.50, 0.75 and 0.99 quantiles of the χ^2_2 distribution. The dashed line corresponds to the 0.99 quantile of the χ^2_2 distribution under the assumption of uncorrelated data (as done in the parity space method).

3 ROBUST APPROACH TO FDI

The FDI process corresponds to identifying the residuals lying outside the circle (parity space) or the ellipse (Mahalanobis distance) of a pre-defined quantile of a χ^2_{m-p} distribution.

However, the Mahalanobis distances (alike D) are very sensitive to the presence of outliers (Rousseeuw and Van Zomeren, 1990). A few extreme observations departing from the main data structure can have a severe influence on the applied distance measure (FIG. 2) due to the non-robust estimation of the covariance matrix $\mathbf{S}$. Consequently, the Mahalanobis distances are heavily affected by the outliers it aims to detect. This is called the "masking effect", by which multiple outliers do not necessarily have large Mahalanobis distances (Rousseeuw and Van Zomeren, 1990). To mitigate such masking, we used a robust estimators for the location and for the scatter of the multivariate distribution of the residuals (Rousseeuw, 1985). The minimum covariance determinant (MCD) estimator is often employed in practice although many other more sophisticate robust estimators have been introduced (for a review see Maronna and Yohai (1998)). When the MCD estimator is used to compute Mahalanobis distances (as defined in Eq. 6), it leads to the so-called robust distances defined as:

$$RD = \sqrt{(\mathbf{v}^\star - \hat{\mu}_{MCD})^T\hat{\mathbf{\Sigma}}^{-1}_{MCD}(\mathbf{v}^\star - \hat{\mu}_{MCD})} \tag{8}$$

where $\hat{\mu}_{MCD}$ and $\hat{\mathbf{\Sigma}}_{MCD}$ are the MCD location and scatter estimates.

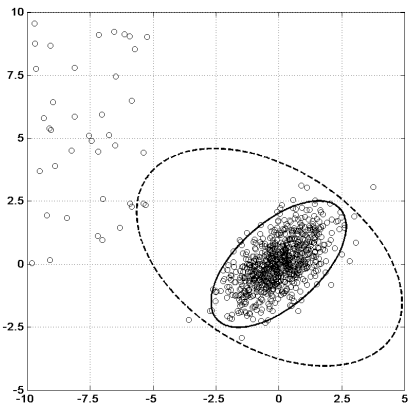

Figure 2: Simulated standard bivariate-normally distributed data with a correlation of 0.60 and with 5% of pertubated data (outliers). The non-robust estimation of the covariance matrix is represented by the dashed ellipse (0.975 quantile of χ^2_2) with an associated correlation coefficient of -0.416. The robust estimation produces the solid ellipse (0.975 quantile of χ^2_2) and estimates a correlation of 0.597.

To isolate a fault we propose using a slightly modified version of the isolation algorithm that is based on maximum likelihood presented in Sturza (1988). This algorithm considers (like the parity space approach) that the covariance matrix of residuals is an identity matrix which is not necessarily the case. Therefore, we incorporated the estimated (robust) covariance matrix also into the original formula (Eq. 3). Thus, the isolated erroneous measurement correspond to the highest values of the ratio $\frac{v_i^2}{Z_{ii}}$ where Z_{ii} correspond to the diagonal elements of the matrix defined by:

$$\mathbf{Z} = \mathbf{I}_m - \mathbf{H}(\mathbf{H}^T \hat{\mathbf{\Sigma}}_{MCD}^{-1} \mathbf{H})^{-1} \mathbf{H}^T \hat{\mathbf{\Sigma}}_{MCD}^{-1} \quad (9)$$

4 EXPERIMENTAL RESULTS

The results presented here are based on an experiment realized in a vehicle. A regular tetrahedron consisting of 4 *Xsens MT-i* MEMS-IMUs was mounted on a rigid structure together with a tactical grade IMU used for reference (*LN200*).

After computing the least squares residuals for the gyros as defined in Eq. 2, we explore the structure of the associated covariance matrix. FIG. 3 shows a partial representation of this matrix (which is from $\Re^{12\times 12}$). It illustrates that the assumptions of the parity space method are not satisfied since the residuals are first correlated (e.g. $corr(V_{x_1}, V_{x_2}) \approx -0.60$), second, have different variances (e.g. $\sigma^2_{V_{x_1}} \approx 2\sigma^2_{V_{z_4}}$) and third, are not centered in zero (e.g. V_{x_4}). Hence, the assumption of strictly Gaussian white noise errors in the parity space approach is unrealistic with MEMS-IMUs.

The performances of the different detection and isolation algorithm are summarized in Table 1.

The results of the robust method and the method based on Mahalanobis distances are very similar. This can be explained by the small proportion of outliers in this experimental data set. Consequently, the maximum likelihood estimation and the robust estimation yield similar results, thus, $\hat{\mathbf{\Sigma}}_{MCD}^{-1} \approx \mathbf{S}^{-1}$ and $\hat{\mu}_{MCD} \approx \bar{\mathbf{v}}$. However, simulations with additional outliers have shown that

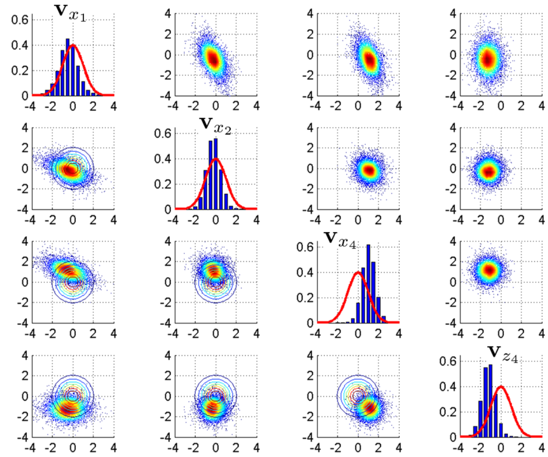

Figure 3: Partial representation of the covariance of the residuals. The diagonal elements compares the histograms of the residuals with distribution assumed in the parity space method. The upper elements show the scatter plots of the residuals and the lower elements compare the scatter plots with the assumed bivariate distribution.

	FA	MD	SDE	IA	MIA
Classic	4.99%	1.06%	0.06%	80%	80%
Mahalanobis	0.45%	0.89%	0.24%	78%	91%
Robust	1.01%	0.95%	0.17%	75%	92%

Table 1: Performance comparison for FDI between different approaches based on the quantile 0.999 of a χ^2_9 *(FA = False alarm, MD = misdetections and SDE = successfully detected errors, IA = performance of the classical isolation algorithm, MIA = performance of the modified isolation algorithm)*

the method based robust Mahalanobis distance is significantly more reliable.

5 CONCLUSION

In this paper, we focused on three FDI algorithms. We have shown that a FDI algorithm based on Mahalanobis distances or robust distances give better results than the parity space method when applied to MEMS-IMUs measurements. Hence, such FDI approaches are considered valuable for detecting gross errors, which elimination in turn enhances the navigation performances and improves the stability of the filters used in the GPS/INS integration. We also demonstrated that the performance of the isolation algorithm can be slightly improved by incorporating the (robustly estimated) covariance matrix. Nevertheless, the percentages of undetected errors, as well as the level of false alarms, remains relatively high and, consequently, shows the need for more complex FDI models and/or improved modeling of MEMS-IMU systematic errors.

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