A UNIFIED CALIBRATION APPROACH FOR GENERIC CAMERAS

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ABSTRACT:

The classic perspective projection is mostly used when calibrating a camera. Although this approach is fairly developed and often suitable, it is not necessarily adequate to model any camera system like fish-eyes or catadioptrics. The perspective projection is not applicable when field of views reach 180° and beyond. In this case an appropriate model for a particular non perspective camera has to be used. Having an unknown camera system a generic camera model is required. This paper discusses a variety of parametric and generic camera models. These models will be validated subsequently using different camera systems. A unified approach of deriving initial parameter guesses for subsequent parameter optimisation is presented. Experimental results prove that generic camera models perform as accurate as a particular parametric model would do. Furthermore, there is no previous knowledge about the camera system needed.

1. INTRODUCTION

Camera calibration is a fundamental task in photogrammetry to derive accurate measurements in object space from camera images. The perspective projection is the camera model used most and therefore well-established (Brown, 1971; Tsai, 1992). This classic approach models most common types of cameras exactly enough to yield sub-pixel accuracy in image space. Despite that, there are many camera systems that cannot be calibrated at all or not precisely enough using a perspective camera model. However, these particular cameras can be calibrated by approximating the deviation from perspective projection by using a distortion model. Many different types of distortion models have been developed (Brown, 1971; Kraus 2004; Luhmann 2006). Originally, these distortion models were implemented to compensate lens errors caused by physical effects or other manufacturing issues. Nevertheless, distortion models are potentially applicable to model non-perspective cameras accurately by using them in addition to the classic perspective camera model adjustment. Although this extended perspective projection yields sufficient accuracy at the expense of additional distortion parameters it will probably fail having wide field of views. So called catadioptric or fish-eye lenses have FOV's beyond 180° and cannot be modelled using a perspective projection. However, there are parametric models that approximate these types of lenses and other non-perspective lenses precisely (Rahul et al., 2000; Schneider et al. 2009; Zhang, 2000). When calibrating a general camera system with an unknown lens it is difficult to decide in advance which particular model fits the real type of camera projection best. To avoid the decision for a proper camera model, one single model is needed that approximates most common types of projection. Such a general camera model was introduced and evaluated by different authors for particular applications (Basu et al., 1995; Claus et al., 2005; Gennery, 2006; Heikkilä, 2006; Orekhov, 2007). This generic model is suitable to calibrate a wide variety of commonly used camera systems including perspective and non-perspective types of projection. Despite the generic character of such a model an extension by an adequate distortion model is often necessary to compensate additional lens errors.
The classic calibration approach uses corresponding object and image points. These correspondences are used to estimate the model's parameters. A maximum likelihood estimation (MLE) algorithm is used to derive the final model parameters (Markwardt, 2008). Due to the nonlinearity of the proposed generic models initial parameter guesses are needed for initialisation of the estimation algorithm. In contrast, a goniometer or collimator setup is used to derive camera model's parameter directly (Grießbach et al., 2010).

In this paper a closed-form DLT approach (Abdel-Aziz et al., 1971) is used to derive a very basic guess of the model's parameter. Afterwards this basic guess is improved by fitting an equisolid-angle camera model. These refined parameters are excellent initial parameter values that can directly be used to derive final parameter values. This yields a unified calibration procedure that does not need any prior camera information like focal length or type of projection. Absolutely no parameters have to be set up in advance. Furthermore, there is no particular calibration target needed. Either a planar or a 3D target is suitable. This whole calibration approach will be validated using real camera systems ranging from normal, wide-angle, fish-eye to catadioptric lenses. Additionally this unified approach is compared to the results of an industrial calibration software and results of Zhang (Zhang, 2000).

2. CAMERA MODELS

This section introduces the perspective, other parametric and generic camera models used for the calibration approach. Figure 1 displays the classic perspective camera model. An object point

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P is projected onto an image plane. The corresponding image point P', the object point P and the projection centre O' form a straight line. The total set of parameters can be subdivided into parameters of exterior orientation (red colour) and interior orientation (blue colour). The essential elements θ, φ, r defining the type of projection are marked green in the figure. The depicted relationship between parameters can be used to derive a general projection equation.

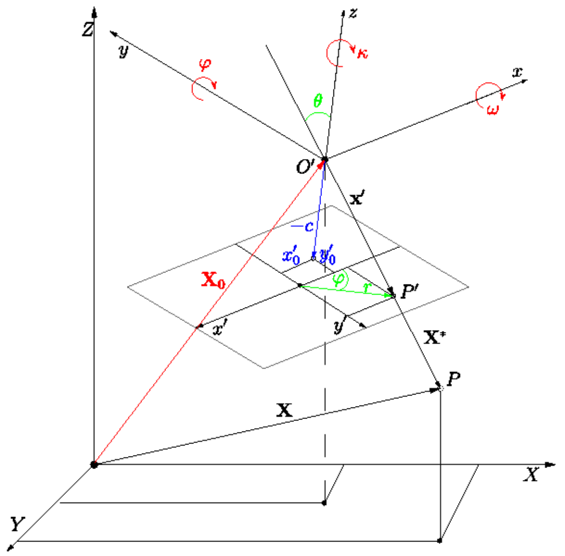

Figure 1. Perspective Camera Model

An image point may be expressed in polar coordinates:

$$\begin{aligned} x' &= r \cdot \cos(\varphi) \\ y' &= r \cdot \sin(\varphi) \end{aligned} \tag{1}$$

Using the function *atan2* to yield the azimuthal angle φ ensures the correct quadrant. This function is defined as follows:

$$\varphi = \operatorname{atan2}(X, Y) = \arg(X + iY) = \frac{1}{i} \ln\left(\frac{X + iY}{\sqrt{X^2 + Y^2}}\right) \tag{2}$$

The inclination angle θ is the angle between the negative z axis and the incoming object ray. It is defined as:

$$\theta = \operatorname{acos}\left(\frac{Z}{\sqrt{X^2 + Y^2 + Z^2}}\right) \tag{3}$$

The radial distance r is a function of θ and further parameters Ω (e.g. focal length). It is given by:

$$r = f(\theta, \Omega) \tag{4}$$

Transformation and substitution of (4), (3), (2) in (1) yield a general projection equation as follows:

$$\begin{aligned} x' &= f\left(\operatorname{acos}\left(\frac{Z}{\sqrt{X^2+Y^2+Z^2}}\right), \Omega\right) \cdot \frac{X}{\sqrt{X^2+Y^2}} \\ y' &= f\left(\operatorname{acos}\left(\frac{Z}{\sqrt{X^2+Y^2+Z^2}}\right), \Omega\right) \cdot \frac{Y}{\sqrt{X^2+Y^2}} \end{aligned} \tag{5}$$

2.1 Perspective camera model

The perspective camera model is characterised by the following radial distance function:

$$r = c \cdot \tan\theta \tag{6}$$

By inserting equation (6) into (5) and simplifying it, it equals the commonly known collinearity equations:

$$\begin{aligned} x' &= c \cdot \frac{x}{z} \\ y' &= c \cdot \frac{y}{z} \end{aligned} \tag{7}$$

where

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{pmatrix} \begin{pmatrix} X - X_0 \\ Y - Y_0 \\ Z - Z_0 \end{pmatrix}$$

is the exterior orientation. The rotation matrix is a function of three angles ω, φ, κ.

2.2 Parametric camera models

To incorporate other camera models the radial distance function (6) has to be accordingly replaced. The following non-perspective camera models are often used:

- Stereographic projection: $r = 2c \tan\frac{\theta}{2}$
- Equidistant projection: $r = c\theta$
- Equisolid-angle projection: $r = 2c \sin\frac{\theta}{2}$
- Orthogonal projection: $r = c \sin\theta$

Figure 2 shows the plot of these parametric models. The radial distance r is plotted over the inclination angle θ. The perspective and orthogonal projection have an inappropriate characteristic concerning wide inclination angles. Nevertheless, each model may be used when calibrating a particular camera.

2.3 Generic camera models

To circumvent the decision for a particular parametric model and to calibrate cameras which do not follow one of the mentioned parametric models a general model is needed. This model is a generic function (4) which approximates the parametric models. A polynomial with odd powers is appropriate to serve as a generic camera model since it approximates trigonometric functions (Kannala, 2006; Heikkilä et al., 2006). Therefore it is able to model the mentioned parametric models. The polynomial may be defined as follows:

$$r(\theta) = \sum_{i=1}^{p} k_i \theta^{2i-1} \tag{8}$$

To derive the final camera model, equation (8) is to be substituted in (5). Fixing the polynomial degree to $p = 5$ yields a generic camera model that is able to replace many types of projection (see 2.2). Furthermore, field of views may exceed the problematic 90° inclination angle limit. The parameter k_0 corresponds to the classic focal length c.

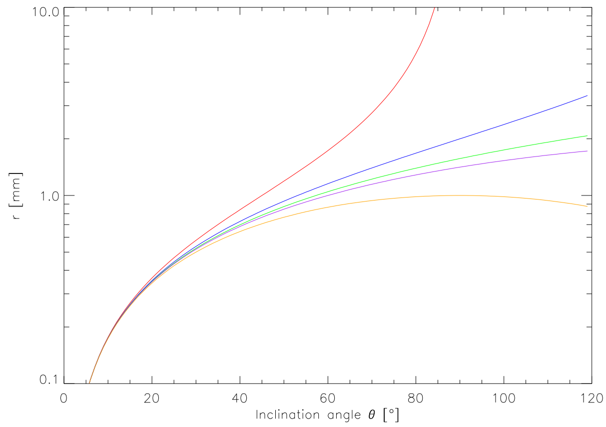

Figure 2. Parametric Camera Models: perspective – red line; stereographic – blue line; equidistant – green line; equisolid-angle – purple line; orthogonal – orange line

Another approach of approximating the parametric camera models is a direct trigonometric one. The proposed models may be merged to the following expression (Gennery, 2006):

$$r(\theta) = c \cdot \frac{\sin L\theta}{L\cos(\max(0, L\theta))} \tag{9}$$

Which can be reformulated to cope with $L \approx 0$:

$$r(\theta) = c \cdot \begin{cases} \dfrac{\sin L\theta}{L} & L < 0 \\ \theta & L = 0 \\ \dfrac{\tan L\theta}{L} & L > 0 \end{cases} \tag{10}$$

These two generic approaches should perform equally when approximating the proposed trigonometric functions in 2.2. Since the polynomial model already incorporates a radial component there is no need for further radial distortion compensation. Thus, the polynomial and trigonometric camera model base on an equally numbered parameter set.

2.4 Distortion model

To account for the difference between the actual camera projection and the camera model which is caused by lens effects, a distortion model is needed (Brown, 1971). In general such a model consists of four major elements:

- Principle point shift

$$\begin{aligned} u &= x' + x'_0 \\ v &= y' + y'_0 \\ r &= \sqrt{u^2 + v^2} \end{aligned} \tag{11}$$

- Radial distortion

$$\begin{aligned} x'_{\text{rad}} &= u\left(K_1 r^2 + K_2 r^4 + K_3 r^6\right) \\ y'_{\text{rad}} &= v\left(K_1 r^2 + K_2 r^4 + K_3 r^6\right) \end{aligned} \tag{12}$$

- Tangential distortion

$$\begin{aligned} x'_{\tan} &= P_1\left(2u^2 + r^2\right) + P_2 2uv \\ y'_{\tan} &= P_2\left(2v^2 + r^2\right) + P_1 2uv \end{aligned} \tag{13}$$

- Affinity and shear

$$\begin{aligned} x'_{\text{aff}} &= B_1 u + B_2 v \\ y'_{\text{aff}} &= 0 \end{aligned} \tag{14}$$

In this case, the radial distortion is fixed to the 6$^{\text{th}}$ degree while the tangential distortion is fixed to the 2$^{\text{nd}}$ degree. In most cases a higher degree of distortion yields no significant gain in accuracy.

3. PARAMETER ADJUSTMENT

To calibrate a camera system its model parameters have to be determined. Often a maximum likelihood estimator (MLE) is used to carry out the final parameter estimation. Since the proposed models are non-linear the camera model has to be linearised in order to run a MLE. The initial parameter values have to be accurately enough to linearise the camera model. The total set of parameter values consists of parameters
of exterior orientation

$$P_{\text{ext}} = \left\{X_0, Y_0, Z_0, \omega, \phi, \kappa\right\} \tag{15}$$

of interior orientation

$$P_{\text{int}} = \left\{x'_0, y'_0, c\right\} \tag{16}$$

of polynomial or trigonometric model

$$P_{\text{pol}} = \left\{k_1 \ldots k_i\right\} \quad P_{\text{tri}} = L \tag{17}$$

and a set of distortion model parameters

$$P_{\text{dist}} = \left\{K_1, K_2, K_3, P_1, P_2, B_1, B_2\right\} \tag{18}$$

A Direct Linear Transformation (DLT) yields the essential initial values.

3.1 Direct Linear Transformation

A direct linear correspondence between a 3D object and image point is given by:

$$x' = \frac{a_1 X + a_2 Y + a_3 Z + a_4}{c_1 X + c_2 Y + c_3 Z + 1} = \frac{\mathbf{aX} + a_4}{\mathbf{cX} + 1}$$
$$y' = \frac{b_1 X + b_2 Y + b_3 Z + b_4}{c_1 X + c_2 Y + c_3 Z + 1} = \frac{\mathbf{bX} + b_4}{\mathbf{cX} + 1} \quad (19)$$

These DLT parameters equal the parameter set of a perspective camera model extended by a scaling and shearing parameter:

$$x' = x_0 - c\frac{\mathbf{i}^T(\mathbf{X}-\mathbf{X}_0)}{\mathbf{k}^T(\mathbf{X}-\mathbf{X}_0)}$$
$$y' = y_0 - b_1 \cdot c\frac{\mathbf{j}^T(\mathbf{X}-\mathbf{X}_0)}{\mathbf{k}^T(\mathbf{X}-\mathbf{X}_0)} - b_2 \cdot c\frac{\mathbf{i}^T(\mathbf{X}-\mathbf{X}_0)}{\mathbf{k}^T(\mathbf{X}-\mathbf{X}_0)} \quad (20)$$

where $\{\mathrm{i},\mathrm{j},\mathrm{k}\}$ = unit vectors of the rotation matrix
X_0 = is the projection centre
$\{b_1, b_2\}$ = scaling and shearing parameters

This approach is applicable if there is no error at all: $0 = \mathbf{Ax} - \mathbf{l}$. In reality there is an error incorporated (Kraus, 1996):

$$0 \approx \mathbf{Ax} - \mathbf{l} \rightarrow \mathbf{v} = \mathbf{Ax} - \mathbf{l} \quad (21)$$

Assuming an error v, equation (19) becomes:

$$x' + v_x = \frac{\mathbf{aX} + a_4}{\mathbf{cX} + 1} \rightarrow x + v_x(\mathbf{cX} + 1) = \mathbf{aX} + a_4 - \mathbf{cX}x$$
$$y' + v_y = \frac{\mathbf{bX} + b_4}{\mathbf{cX} + 1} \rightarrow y + v_y(\mathbf{cX} + 1) = \mathbf{bX} + b_4 - \mathbf{cX}y \quad (22)$$

In that case, the direct linear transformation becomes non-linear and has to be solved iteratively. Here $(\mathbf{cX}+1)$ is treated as a constant. Starting with $(\mathbf{cX}+1) = 1$ it can be calculated in every step and be used in the subsequent one. This iteration is to be repeated until the parameter set converges. This iteration approach is particularly essential when dealing with non-perspective camera systems since they differ from perspective projection to much.
The result of this initial parameter estimation is a set of the following parameters:

$$P_{\mathrm{DLT}} = \{X_0, Y_0, Z_0, \omega, \phi, \kappa, x_0', y_0', c, b_1, b_2\} \quad (23)$$

In case of a polynomial based camera model k_1 corresponds to c. All other parameters $\{k_2 \ldots k_p\}$ will be set initially to zero. In case of trigonometric camera model the parameter L is set to an equisolid-angle projection (see 3.2). All other distortion parameters are initially set to zero.
In case of a planar target where the z axis is zero, equation (19) reduces to:

$$x' = \frac{a_1' X + a_2' Y + a_3'}{c_1' X + c_2' Y + 1} = \frac{\mathbf{a}'\mathbf{X} + a_3'}{\mathbf{c}'\mathbf{X} + 1}$$
$$y' = \frac{b_1' X + b_2' Y + b_3'}{c_1' X + c_2' Y + 1} = \frac{\mathbf{b}'\mathbf{X} + b_3'}{\mathbf{c}'\mathbf{X} + 1} \quad (24)$$

3.2 Refinement Step

The DLT-derived initial parameter values do not suffice for strong non-perspective cameras (e.g. fish-eye). Therefore, a refinement step is added where the DLT parameters are fitted to a parametric camera model. At the first glance, the trigonometric model (9) seems to be adequate since it is potentially able to model all proposed parametric projections. However, experiments proved that this approach does not converge in every case. That is why the equisolid-angle model was chosen. Accurately enough initial parameter values for subsequent final parameter estimation can be achieved even in perspective case. Furthermore, it yields a stable solution in case of inclination angle reaches 90° and beyond.
The actual refinement is carried out using a LM algorithm (see 3.3)

3.3 Final Adjustment

The total parameter set of the camera model is adjusted using a MLE. An implementation of the often used Levenberg-Marquardt-Algorithm (LMA) (Markwardt, 2008) was chosen. This optimisation algorithm has been applied in many adjustment tasks throughout the literature and proved to be robust and efficient. In this case the LMA was used to minimise the total image point rms of the camera model. The rms is a practical measure to compare different models and to judge the overall accuracy of the camera model.

4. CAMERA SYSTEMS AND EXPERIMENTAL SETUP

In order to validate the overall accuracy and general applicability of the proposed calibration approach 17 different camera systems have been used. All these camera systems differ in sensor sizes, lenses and the amount of control points and images that were taken.
Additionally, some of these camera systems were mounted into weather protection cases. The panes of these cases can cause additional refractions of imaging rays and therefore alter the camera's true type of projection. Depending on the particular setup these cases may prevent a successful calibration using parametric camera models. Table I displays the used camera systems: The professional SLR Nikon D2Xs, different off-the-shelf consumer class cameras (Cannon G3, Pentax Optio W60 and Panasonic FZ8), three GigE professional cameras (DBK 8.5mm lens, DBK 4.8mm lens and DBK 3.5mm lens), three protection case mounted camera systems (DBK 8 mm lens (no. 4), DBK 5mm lens (no. 9) and MVcam 3.8mm lens (no. 11)), two proprietary camera systems (DLRcam 7mm lens and DLRcam 2.2mm lens), one Webcam, a fish-eye lens camera system (no. 14) and two catadioptric camera systems (no. 15 and no. 16). None of the chosen camera systems follow the orthogonal (or sine-law) projection. Although, no system fits the orthogonal projection best this camera model can still be used in some cases to derive an accurate calibration (no. 8, 9 and 12).
To compare this paper's calibration approach with other authors, Zhang's data set has been evaluated (no. 17) as well. It was chosen because his data set is publically accessible. Additionally, the commercial calibration software *Australis* was used as a reference. As *Australis* incorporates the perspective camera model it was not able to successfully calibrate every evaluated camera system.

No.	Specifications		
	Camera/Lens	***Type of Projection***	***C [mm]***
1	D2Xs/Nikkor	Perspective	24.5
2	G3/Cannon	Stereographic	9.4
3	DBK/Cosimcar	Stereographic	8.5
4	DBK/Computar	Perspective	8.0
5	Optio W60/Pentax	Stereographic	7.3
6	DLRcam	Perspective	7.0
7	DMC FZ8/Leica	Perspective	6.1
8	Logitech/Webcam	Stereographic	5.1
9	DBK/Computar	Perspective	4.9
10	DBK/Cosimcar	Stereographic	4.9
11	MVcam/Computar	Perspective	3.8
12	DBK/Kowa	Equisolid-angle	3.6
13	DLRcam	Equisolid-angle	2.2
14	DBK/Fujion	Equidistant	1.8
15	PicSight/Computar	Stereographic	0.9
16	CF-2000	Stereographic	0.6
17	PULNiX	Stereographic	6.2

Table 1. Camera System Specifications

A commercial bundle adjustment using the D2Xs with tripod, proper lightning and calibrated scale bars yielded the points of the calibration target. The target consists of black circled markers on a white wall. The room is air-conditioned providing a stable temperature of about 22°C. 120 images were taken counting about 175 image points each. The camera and the scales are certified both by the DKD*. This ensures a proper setting and results in an overall object point sigma of 20µm. Figure 3 shows the calibration target in the corner of the room.

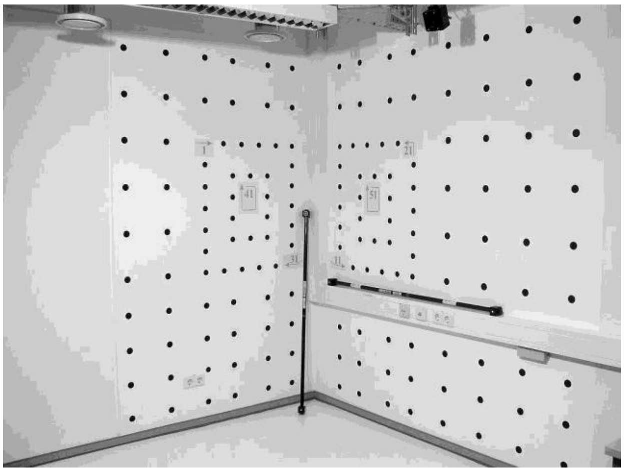

Figure 3. Calibration Target

Taking each camera system from table 1 between 8 and 50 images were taken at a distance of approximately 3-5m from that target. This leads to a range of 300 up to 7500 control points per camera system. Least squares ellipse fitting was used to measure the corresponding points in the images (Heikkilä, 2000). According to Heikkilä the accuracy of centre point determination depends on the particular camera but is usually clearly sub-pixel accurate. All calibration data is available for evaluation purposes. Do not hesitate to contact the authors for a download link.

5. EXPERIMENTAL RESULTS

In the following section the experimental results of the camera calibrations are presented.
In the beginning initial parameter values have to be derived. The combination of a direct linear transformation followed by a refinement step yield accurately enough initial parameters since the final adjustment converges in every case successfully. The initialisation step relies on a set free of erroneous control points. Since the DLT approach is very liable to gross errors the control points set has to be chosen carefully. A prior step of checking for correspondence errors has to be implemented if needed. In this work all calibration data sets were manually verified. After the successful parameter initialisation all camera systems were calibrated using one of the following three camera models:

- The best fitting (see table 1) parametric camera model extended by a full distortion model (see 2.4). Therefore, this *PAR+FD* model has 10 parameters in total.
- A generic trigonometric based camera model (10) extended by a full distortion model. This *TRI+FD* model incorporates 11.
- The polynomial based generic camera model (8) extended by a reduced distortion model. The radial term of distortion is fully covered by the polynomial camera model and can be neglected. Thus, this model also has 11 parameters.

Note, that these parameter amounts exclude 6 parameters of exterior orientation for each image. The distortion model is not needed in every case. It often hardly reduces the rms values at the expense of seven additional parameters. However, in some cases the distortion model is useful to compensate significant lens effects. In terms of generality a distortion model is mandatorily added. There are attempts to choose the complexity of a distortion model automatically (Orekhov, 2007). This approach is not further considered in this paper.
As a reference calibration the commercial calibration software *Australis* was used to calibrate every camera system. Since this software uses a perspective projection model extended by the same distortion model used in this work (AUS) it has 10 parameters plus 6 parameters of exterior orientation for each image, like the *PAR+FD* model has. Table 2 lists all resulting rms values in image space and pixel. Firstly, all tested camera systems achieve sub pixel accuracy no matter what model was used. Secondly, both of the generic models perform quite equally. In addition, they outperform the reference clearly when type of projection becomes strongly non-perspective. Furthermore, the generic model proved to accurately approximate all proposed parametric camera models. Despite some cases an accuracy of 10^{th} a pixel is possible. Zhang achieved an rms error of 0.335px using data set no. 17. Even reducing the POL+RD to the same number of parameters Zhang used (by neglecting the distortion model = 7 parameters), an rms value of 0.238px can be achieved. The reason why the catadioptric system no. 16 performs comparably bad is the strong chromatic aberration, resulting in a very poor image quality. The reason why the other catadioptric system fails to achieve rms values as good as the other systems could not be found. Nevertheless, sub pixel accuracy could be achieved.

* Deutsche Kalibrierdienst (accredited calibration service)

No.	PAR+FD	TRI+FD	POL+RD	AUS
1	0.082	0.082	0.082	0.082
2	0.203	0.201	0.202	0.21
3	0.029	0.029	0.029	0.03
4	0.086	0.087	0.086	0.06
5	0.118	0.110	0.088	0.136
6	0.015	0.015	0.015	0.015
7	0.162	0.163	0.163	0.173
8	0.043	0.043	0.042	0.044
9	0.080	0.081	0.078	0.042
10	0.058	0.058	0.058	0.071
11	0.059	0.055	0.052	fail
12	0.051	0.051	0.051	0.303
13	0.061	0.052	0.052	2.95
14	0.093	0.093	0.092	33.71
15	0.517	0.505	0.509	16.42
16	0.498	0.387	0.321	fail
17	0.109	0.109	0.109	0.112

Table 2. Experimental Rms Values

6. SUMMARY AND CONCLUSION

The experimental results indicate that the proposed generic camera models are applicable to calibrate a wide variety of camera types. Catadioptric, fish-eye, wide-angle and common focal length camera systems were calibrated using comparatively simple and well known means. Absolutely no previous knowledge about the camera system that is to be calibrated is needed. Initially, a direct linear transformation was used to derive initial values for the model's parameters. Due to the non-perspective characteristics of many of the examined camera systems a further refinement step was introduced. The initial parameter values are fitted to an equisolid-angle model. This model accepts inclination angles greater than 90°. The subsequent final adjustment of parameters using the proposed LMA yields acceptable errors (table 2) in terms of sub pixel accuracy. A precision up to $1/10^{th}$ a pixel is achievable. With exception of the catadioptric camera systems the achieved image rms for all tested systems are about one order below sub pixel accuracy in image space. Furthermore, table 2 emphasises the applicability of the generic non-parametric models as substitution for the proposed parametric models. In addition, the overall accuracy of the generic models is comparable to the accuracy of the suitable particular parametric model. Even though camera cases were used, which caused additional refraction of light rays at media interfaces, an accurately enough modelling is possible. Future work will focus on accurately modelling non-perspective projections like catadioptrics with the proposed generic approach. Those lenses have not yielded satisfying results in terms of image rms.

References

Abdel-Aziz, Y. and Karara, H.M, 1971. Direct linear transformation from comparator coordinates into object space coordinates in close range photogrammetry. *ASP Symposium on Close-Range Photogrammetry*, pp. 1-18.

Basu, A. and Licardie S.: 'Alternative models for fish-eye lenses', Pattern Recogn. Lett., 1995, 16, (4), pp. 433-441

Brown, D.C.: 'Close-range camera calibration', Photogrammetric Engineering, 1971, 37, (8), pp. 855-866

Claus D. and Fitzgibbon A. W.: 'A Rational Function Lens Distortion Model for General Cameras'. Proceedings of the 2005 IEEE Computer Society Conference on Computer Vision and Pattern Recognition (CVPR'05) - Volume 1 - Volume 012005 pp. Pages

Gennery D. B.: 'Generalized Camera Calibration Including Fish-Eye Lenses', Int. J. Comput. Vision, 2006, 68, (3), pp. 239-266

Grießbach, D., Bauer, M., Scheele, M., Hermerschmidt, A. and Krüger, S.: 'Geometric stereo camera calibration with diffractive optical elements',Proceedings of the EuroCOW 2010, Castelldefels

Heikkilä, J.: 'A Polynomial Camera Model and Calibration Method for Conventional, Wide-Angle, and Fish-Eye Lenses', IEEE Trans. Pattern Anal. Mach. Intell., 2006, 28, (8), pp. 1335-1340

Heikkilä, J.: 'Geometric Camera Calibration Using Circular Control Points', IEEE Trans. Pattern Anal. Mach. Intell., 2000, 22, (10), pp. 1066-1077

Kannala, J., Brandt S., "A Generic Camera Model and Calibration Method for Conventional, Wide-Angle, and Fish-Eye Lenses," IEEE Transactions on Pattern Analysis and Machine Intelligence, pp. 1335-1340, August, 2006

Kraus, K.: 'Photogrammetrie, Band 2' (Duemmler, 1996, 3 edn. 1996)

Kraus, K.: 'Photogrammetrie, Band 1' (de Gruyter, 2004, 7 edn. 2004)

Luhmann, T., Robson, S., Kyle, S., and Harley, I.: 'Close Range Photogrammetry' (Whittles Publishing, 2006. 2006)

Markwardt, C.B.: 'Non-Linear Least Squares Fitting in IDL with MPFIT', in Editor (Ed.)^(Eds.): 'Book Non-Linear Least Squares Fitting in IDL with MPFIT' (ASP Conference Series, 2008, edn.), pp. TBD

Orekhov, V., Abidi, B., Broaddus, C., and Abidi, M.: 'Universal Camera Calibration with Automatic Distortion Model Selection', in Editor (Ed.)^(Eds.): 'Book Universal Camera Calibration with Automatic Distortion Model Selection' (2007, edn.), pp. VI - 397-VI - 400

Rahul, S., and Shree, K.N.: 'Nonmetric Calibration of Wide-Angle Lenses and Polycameras', IEEE Trans. Pattern Anal. Mach. Intell., 2000, 22, (10), pp. 1172-1178

Schneider, D., Schwalbe, E., and Maas, H.G.: 'Validation of geometric models for fisheye lenses', ISPRS Journal of Photogrammetry and Remote Sensing, 2009, 64, (3), pp. 259-266

Tsai, R.Y.: 'A versatile camera calibration technique for high-accuracy 3D machine vision metrology using off-the-shelf TV cameras and lenses': 'Radiometry' (Jones and Bartlett Publishers, Inc., 1992), pp. 221-244

Zhang, Z.: 'A Flexible New Technique For Camera Calibration', IEEE Trans. Pattern Analysis and Machine Intelligence, vol. 22, no. 11, pp. 1330-1337, Nov. 2000