Camera Calibration Considerations for UAV Photogrammetry

Prof. Clive Fraser
CRC for Spatial Information
Dept. of Infrastructure Engineering
University of Melbourne, Australia

Towards Photogrammetry 2020
ISPRS Technical Comm. II Symposium 2018, Riva del Garda, Italy, 3 - 7 June 2018
Cameras for Drones/UAS/UAVs

Vary from off-the-shelf point & shoot, to zoom compact, mirrorless, super-zoom, DSLRs and numerous specially designed cameras

Universal requirement for photogrammetric applications: cameras must be calibrated
UAVs for Spatial Information Generation

Networks can be regular or irregular

Photogrammetric processing must accommodate every configuration
Sensor Calibration, Orientation & 3D Object reconstruction

Initial Network Relative Orientation

(A) Manual to Semi-Automated Image Measurement

(B) Automatic, using coded targets

(C) Automatic with no targets (SfM approach)

Bundle adjustment (+self-calibration) for Refined Network Relative/Exterior Orientation

Bundle Adjustment with Self-Calibration

UAVs/ Drones

Subsequent Determination of 3D Points (generally with final Bundle Adjustment)

(A) Manual or semi-automated referencing or measurement of targetted or untargetted points

(B) Automatic 3D coordinates for targetted points

(C) Automatic generation of dense point cloud – no targets (SfM or FBM approach)
Network Geometry for Self-Calibration

Attributes of successful self-calibration networks (No obj. space control):

- Multi-image, highly convergent network
- Orthogonal camera roll angles
- Depth in the object space (not mandatory, but very desirable)
- Highly redundant network (all pts >6 ray)
- High image measurement accuracy (targets: $\sigma_{xy}$ to 0.03 pixel for monochrome CCD & generally 0.1 pixel for colour; targetless: $\sigma_{xy}$ to 0.3 pixel for FBM/SfM)

Plus

- Unifocal lens (desirable), fixed focus
- Stable interior orientation
FBM/SfM-based orientation with self-calibration

FBM-based approach affords massive redundancy to compensate for potential modest loss in geometric strength

Salzscheider network: 23 images, 46,000 points, RMS $v_{xy} = 0.25$ pixel
On-ground calibration via self-calibration

A very feasible approach for short focal length UAS cameras

<table>
<thead>
<tr>
<th>Camera parameters: $c$, $x_p$, $y_p$, $K_1$, $K_2$, $K_3$, $P_1$, $P_2$</th>
<th>1-moving</th>
<th>2-moving</th>
<th>3-moving</th>
<th>4-Static</th>
<th>5-Static</th>
<th>6-Static</th>
</tr>
</thead>
<tbody>
<tr>
<td>Images</td>
<td>44</td>
<td>62</td>
<td>49</td>
<td>35</td>
<td>44</td>
<td>41</td>
</tr>
<tr>
<td>Points</td>
<td>149</td>
<td>151</td>
<td>144</td>
<td>129</td>
<td>141</td>
<td>139</td>
</tr>
<tr>
<td>RMS vxy (pl)</td>
<td>0.30</td>
<td>0.33</td>
<td>0.41</td>
<td>0.45</td>
<td>0.38</td>
<td>0.28</td>
</tr>
<tr>
<td>Max rays</td>
<td>42</td>
<td>54</td>
<td>47</td>
<td>31</td>
<td>43</td>
<td>33</td>
</tr>
<tr>
<td>Min rays</td>
<td>5</td>
<td>7</td>
<td>7</td>
<td>5</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>$c$ (mm)</td>
<td>8.729</td>
<td>8.733</td>
<td>8.732</td>
<td>8.735</td>
<td>8.735</td>
<td>8.738</td>
</tr>
<tr>
<td>$x_p$ (mm)</td>
<td>-0.000</td>
<td>-0.001</td>
<td>-0.006</td>
<td>0.000</td>
<td>0.001</td>
<td>0.002</td>
</tr>
<tr>
<td>$y_p$ (mm)</td>
<td>-0.026</td>
<td>-0.023</td>
<td>-0.025</td>
<td>-0.017</td>
<td>-0.022</td>
<td>-0.018</td>
</tr>
</tbody>
</table>

Results of 6 separate self-calibrations of a 20mpixel Phantom P4P camera

Note high level of calibration stability of the Phantom 4 PRO camera
Geometric Attributes of UAV Networks for Self-Calibration

Case 1: ‘Standard’ aerial block with limited terrain relief

Oriented UAV image block of 297 images & 480,000 points, 68K pts in >6 images; many in >10.
No convergence, no orthogonal roll, limited depth in obj. space, RMS vxy=0.8pl
Self-calibration of IO not feasible; lens distortion possibly OK, depending upon overlap & control
Geometric Attributes of UAV Networks for Self-Calibration

Brief revisit to mechanism of projective coupling/compensation

- Roll-angles orthogonally diverse (0°, 90°, 270°)
- A strongly convergent imaging configuration
- For comprehensive modelling of distortion, especially lens distortion, the image points should cover the full image format
- A 3D object point field is very useful, though not mandatory for self-calibration

Note: usually one of two aims being pursued through:

i) provide systematic error compensation, not necessarily camera calibration
ii) to provide a scene-independent calibration of the camera(s)

Under-Appreciated Requirement to Minimize Projective Coupling between Calibration Parameters:
Maximise Scale Variation within & between Images
Case 2: ‘Standard’ aerial block with large depth variation in object space

Geometric Attributes of UAV Networks for Self-Calibration

Oriented UAV image block of 127 images & 180,000 points, 31K pts in >6 images; many in >10.
No convergence, no orthogonal roll, but very significant depth in obj. space (H-h varies from 130m to 290m, so scale varies within some images by 50% from the mean), RMS vxy=0.45pl

Full self-calibration of IO & lens distortion feasible, good precision & low projective couplings (no need for object space control)
**Geometric Attributes of UAV Networks for Self-Calibration**

Case 3: Mixed range network (scale variation between images, not within)

UAV image block of 84 images & 30,000 pts in >6 images; many in >10. No convergence, no roll, but significant scale diff. between 3 flying heights, RMS vxy=0.8pl.

Full self-calibration of IO & lens distortion feasible, but strong projective coupling between principal distance & camera stn Z coord. (ideal example of where in-air & on-ground GPS constraints should apply)
Geometric Attributes of UAV Networks for Self-Calibration

Case 4: Scale variation from presence of high buildings

UAV image block of 122 images & >100K points, RMS vxy=0.4pl

No convergence, no roll, but significant scale diff. between the ground & tops of buildings
(Building height >50% of flying height)

Full self-calibration of IO & lens distortion feasible, good precision & low projective couplings
(no need for object space control)
Geometric Attributes of UAV Networks for Self-Calibration

Case 5: Highly overlapping oblique imagery of a 3D object scene (images from FMV)

UAV image block of 56 images & 28K points, 200 with >6 rays, RMS vxy=0.44pl

No convergence, no roll, but significant scale diff. within each image, and 3D

Full self-calibration of IO & lens distortion feasible, moderate precision with moderate projective couplings (espec. related to IO/EO and decentring distortion)

(no requirement for object space control)
Geometric Attributes of UAV Networks for Self-Calibration

Case 5: Highly overlapping oblique imagery of a 3D object scene (images from FMV)

UAV image block of 56 images & 28K points, 200 with >6 rays, RMS vxy=0.44pl
No convergence, no roll, but significant scale diff. within each image, and 3D

Full self-calibration of IO & lens distortion feasible, moderate precision with moderate projective couplings (espec. related to IO/EO and decentering distortion)
(no requirement for object space control)
Geometric Attributes of UAV Networks for Self-Calibration

Case 7: Network for 3D reconstruction of a historic barn

UAV image network of 49 images & 50K points, 6000 with >6 rays, RMS vxy=0.3pl

No convergence, no roll, but significant scale diff. within each image

Full self-calibration of IO & lens distortion feasible, moderate precision with moderate projective couplings (no necessity for object space control)
Geometric Attributes of UAV Networks for Self-Calibration

Case 7: Network for 3D reconstruction of a historic barn

Self-calibration OK due substantial variation in image scale within each image, but note projective linkage between principal point & elevation angle/Z

Recovery of IO is moderately strong (sigmas of a few micrometers)
Geometric Attributes of UAV Networks for Self-Calibration

Case 8: 2-camera ISPRS benchmark network

UAV image block of 224 images & >150K points, RMS vxy=0.4pl

‘Accidental’ convergence, no roll & limited scale variation within the images

Full self-calibration feasible but not strong, high projective coupling espec. between IO & EO

(Camera roll would dramatically improve accurate IO recovery)
In-air versus on-ground calibration via self-calibration of P4P

Recall importance of image scale variation

Obj XYZ discrepancies: <1mm in XY, 3mm in Z for tgts, 5mm for FBM
Affine distortion in object space – a problem?

Differential scale difference between XY and Z
length of vectors at ground level is 6 – 9mm
Bias introduced through different focal length

$\Delta c$ of 0.033mm at scale of 1:1200 $\rightarrow \Delta Z$ of 40mm; actual RMS is 37mm
Self-calibration in a one-level high-overlap near-nadir aerial P4P network with small height variation – definitely not recommended!

Flying Ht= 35m
53 images
32,000 pts
RMSvxy = 0.51 pl

σXY= 5mm
σZ= 10mm

Result: classical doming of terrain plus mean bias of 0.6m in XY & 0.9m in Z

Should not constrain GPS cam. stn coords since accuracy too low (0.5m RMS)
Fixed IO (pre-calibrated) & self-calibration of radial distortion only – also not recommended!

Should not constrain GPS cam. stn coords since accuracy too low (0.5m RMS)

Result: classical doming of terrain, Z-discrepancy range of -0.9m to +0.51m
Three radial distortion profiles for 53-image P4P network

Pre-Calibration
\[ c = 8.758 \text{ mm} \]
\[ dr = +23 \text{ microns} @ 7.5\text{mm} \]

Partial self-calibration (no IO)
\[ c = 8.758 \text{ mm} \]
\[ dr = -46 \text{ microns} @ 7.5\text{mm} \]

Full self-calibration (incl. IO)
\[ c = 8.935 \text{ mm} \]
\[ dr = -46 \text{ microns} @ 7.5\text{mm} \]

All three solutions have same internal closure, RMS vxy = 0.50 pixel

In the absence of accurate cam. stn control, opt for pre-calibration
Constraints in Bundle Adjustment of UAV Image Blocks

**PURPOSE:** To remove both datum & configuration defects

**Precision → Weights**

i) Image coord. obs. $\sigma_{xy} \rightarrow P$

ii) GCPs $\sigma_{XYZ} \rightarrow P2$

iii) GPS camera stns. $\sigma_{X}^{c}, \sigma_{Y}^{c}, \sigma_{Z}^{c} \rightarrow P1$

**Bundle Adjustment**

$$
\begin{pmatrix}
A_1^T P A_1 + P_1 & A_1^T P A_2 & A_1^T P A_3 \\
A_2^T P A_1 & A_2^T P A_2 + P_2 & A_2^T P A_3 \\
A_3^T P A_1 & A_3^T P A_2 & A_3^T P A_3
\end{pmatrix}
\begin{pmatrix}
\delta_1 \\
\delta_2 \\
\delta_3
\end{pmatrix}
+
\begin{pmatrix}
A_1^T P w + P_1 w_1 \\
A_2^T P w + P_2 w_2 \\
A_3^T P w
\end{pmatrix}
= 0
$$

Consider the case where $\sigma_{xy}^* \approx \sigma_{XYZ} \approx \sigma_{X}^{c}, \sigma_{Y}^{c}, \sigma_{Z}^{c}$ & where $\sigma_{xy}^* = \text{Scale No.} \times \sigma_{xy}$

Now consider number of constraints in FBM orientation by self-calib. bundle adj.

<table>
<thead>
<tr>
<th>$\sigma_{xy}$</th>
<th>$\sigma_{XYZ}$</th>
<th>$\sigma_{X}^{c}, \sigma_{Y}^{c}, \sigma_{Z}^{c}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10,000s - 100,000s</td>
<td>10s</td>
<td>100s</td>
</tr>
</tbody>
</table>

The relative magnitudes of $\sigma_{XYZ}$ & $\sigma_{X}^{c}, \sigma_{Y}^{c}, \sigma_{Z}^{c}$ against $\sigma_{xy}^*$ are critically important in self-calibration within UAV image networks
Constraints in Bundle Adjustment of UAV Image Blocks

**Removal of the datum defect**

i) Delete 7 rows/columns from the normal equations (usually related to $\delta_2$)

ii) Assign 7 $\sigma_{XYZ}$ or $\sigma_{X^cY^cZ^c}$ values to ‘zero’ (eg 2 GCPs fixed in XYZ & 3rd in Z)

iii) Free-Network adjustment (via Helmert bordering or less useful pseudo inverse)

iv) Assign appropriately ‘tight’ values to $\sigma_{XYZ}$ & $\sigma_{X^cY^cZ^c}$

Optimal precision of XYZ ground point coordinates will only be attained when computational base is assigned zero variance, ie (i) – (iii) with $\delta_2$ parameters only

**Important to remember that for non-minimal constraint**

\[
\begin{pmatrix}
A^T PA + k^2I \\
\end{pmatrix} \delta = \begin{pmatrix}
(A^T Pw + k^2Iw^*) \\
\end{pmatrix} \\
\]

\[
k^2I = \begin{pmatrix}
P_1 \\
P_2 \\
0
\end{pmatrix}
\]

**Note that for a moderate range of $k$**

\[
\delta = (A^T PA)^+(A^T Pw) \approx (A^T PA + k^2I)^{-1}(A^T Pw + k^2Iw^*)
\]

**but**

\[
C_{\delta} = (A^T PA)^+ \neq (A^T PA + k^2I)^{-1}
\]

**can result in seriously inflated variances & covariances of the parameters $\delta$**
Constraints in Bundle Adjustment of UAV Image Blocks

Removal of the configuration defect

i) Utilise a network geometry that supports self-calibration (especially of IO parameters of \( c, x_p, y_p \))

And/or

ii) Impose constraints \( \sigma_{XYZ} \land \sigma_{Z} \ll \sigma_{xy}^* \) such that projective coupling of IO & EO parameters is sufficiently suppressed, while at the same time the imposed constraints lead to a normal equation matrix of full rank.

Options for (i) centre upon introduction of significant scale variation within and/or between images.

Hence the method of ‘mixed range’ or multi-scale calibration in aerial block adjustment
Example of mixed range calibration for a UAV camera

- UAV: DJI Phantom 3 with 12 mpixel camera of 3.8mm focal length
- 84-images & 30,000 FBM ground points (all with 6 or more rays)
- 3 Flying Heights: 10m, 20m & 30m; 31, 39 & 14 images, respectively
- 7 GCPs, ‘true’ standard error of $\sigma_{XYZ} = 0.015$m

Results of free-network self-calibration

- RMS $v_{xy} = 0.89$ pixel
- RMS $v_{XY}^{c} = v_{Z}^{c} = 0.48$m
- RMS $v_{XY} = 0.011$m
- $\bar{\sigma}_{XY} = 0.004$m, $\bar{\sigma}_{Z} = 0.009$m
Quality of GPS UAV camera station coordinates

- 84-images & 30,000 FBM points
- 3 Flying Heights: 10m, 20m & 30m; 31, 39 & 14 images, respectively

Results of free-network self-calibration

- RMS $v_{xy} = 0.89$ pixel
- RMS $v_{x'y'} = 0.48m$ $v_{z'} = 0.51m$
- RMS $v_{xy} = 0.011m$ $v_{z} = 0.013m$
- $\bar{\sigma}_{xy} = 0.004m$ $\bar{\sigma}_{z} = 0.009m$
- $\bar{\Delta}_{xy} = 0.001m$ $\bar{\Delta}_{z} = 0.028m$
### Results of Multi-Scale Self-Calibration: varying $\sigma_{X,Y,Z}^{c}$ & No GCPs

<table>
<thead>
<tr>
<th>a priori constraint</th>
<th>RMS $v_{xy}$ (pixel)</th>
<th>$\bar{\sigma}_{XY}$</th>
<th>$\bar{\sigma}_Z$</th>
<th>RMS $v_{X,Y}^{c}$</th>
<th>RMS $v_{Z}^{c}$</th>
<th>RMS $\Delta_{XY}$</th>
<th>RMS $\Delta_{Z}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Free-net</td>
<td>0.88</td>
<td>0.004m</td>
<td>0.009m</td>
<td>0.48m</td>
<td>0.51m</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Min CTRL 51,46,70</td>
<td>0.88</td>
<td>0.011</td>
<td>0.030</td>
<td>0.60</td>
<td>0.59</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma_{X,Y,Z}^{c}$ 0.1m</td>
<td>0.89</td>
<td>0.47</td>
<td>0.48</td>
<td>0.48</td>
<td>0.51</td>
<td>0.003m</td>
<td>0.15m</td>
</tr>
<tr>
<td>$\sigma_{X,Y,Z}^{c}$ 0.01m</td>
<td>0.89</td>
<td>0.035</td>
<td>0.040</td>
<td>0.47</td>
<td>0.48</td>
<td>0.02</td>
<td>0.16</td>
</tr>
<tr>
<td>$\sigma_{X,Y,Z}^{c}$ 0.001m</td>
<td>0.90</td>
<td>0.005</td>
<td>0.010</td>
<td>0.39</td>
<td>0.43</td>
<td>0.06</td>
<td>0.62</td>
</tr>
<tr>
<td>$\sigma_{X,Y,Z}^{c}$ 0.0001m</td>
<td>2.33</td>
<td>0.008</td>
<td>0.020</td>
<td>0.20</td>
<td>0.23</td>
<td>0.13</td>
<td>0.17</td>
</tr>
</tbody>
</table>

**Distortion in object space for $\sigma_{X,Y,Z}^{c} = 0.0001$m**  
Largest error vector (red) = 0.59m
Results of Multi-Scale Self-Calibration varying $\sigma_X^c, \sigma_Y^c, \sigma_Z^c$ & No GCPs

Calibration results not projectively equivalent, affine distortion introduced through changing $c$ resulting from varying $\sigma_X^c, \sigma_Y^c, \sigma_Z^c$
Multi-Scale Self-Calibration results: varying $\sigma_{XYZ}$ & $\sigma_{X_Y Z}^c = 0.1m$

<table>
<thead>
<tr>
<th>a priori constraint</th>
<th>RMS $v_{xy}$ (pixel)</th>
<th>$\bar{\sigma}_{XY}$</th>
<th>$\bar{\sigma}_Z$</th>
<th>RMS $v_{X_Y Z}^c$</th>
<th>RMS $v_{Z}^c$</th>
<th>RMS $\Delta_{XY}$ (v. 7 GCPs)</th>
<th>RMS $\Delta_{Z}$ (v. 7 GCPs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Free-net</td>
<td>0.88 0.004m 0.009m 0.48m 0.51m</td>
<td>(0.011)</td>
<td>(0.009)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Min CTRL 2pts in XYZ, 1 in Z</td>
<td>0.88 0.004 0.011 0.48 0.54</td>
<td>(0.018)</td>
<td>(0.131)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma_{XYZ}$ 0.1m</td>
<td>0.89 0.313 0.306 0.48 0.51</td>
<td>0.009m (0.018)</td>
<td>0.006m (0.132)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma_{XYZ}$ .01m</td>
<td>0.88 0.079 0.092 0.48 0.53</td>
<td>0.01 (0.017)</td>
<td>0.12 (0.019)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma_{XYZ}$ .001m</td>
<td>0.87 0.009 0.014 0.48 0.53</td>
<td>0.02 (0.012)</td>
<td>0.13 (0.009)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma_{XYZ}$ .0001m</td>
<td>0.87 0.004 0.009 0.49 0.54</td>
<td>0.02 (0.005)</td>
<td>0.14 (0.001)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- RMS $v_{xy}$ basically invariant with changing $\sigma_{XYZ}$
- Large inflation in $\bar{\sigma}_{XY}$ & $\bar{\sigma}_Z$ with decreasing weight of GCP constraints
- RMS fit to camera stations largely invariant with changing GCP weights
- RMS $\Delta_{Z}$ increasing with increasing GCP weight due to bias in calibration params.
Multi-Scale Self-Calibration results: varying $\sigma_{XYZ}$ & $\sigma_{XYZ}^c$ = 0.1m

Radial Distortion for $\sigma_{EO}$ = 0.1m

Calibration results are quite consistent
Multi-Scale Self-Calibration results: varying $\sigma_{XYZ}$ & $\sigma_X^{c}, \sigma_Y^{c}, \sigma_Z^{c} = 0.1m$

$\sigma_{XYZ} = 0.1m$ Largest $\Delta = 0.07m$

$\sigma_{XYZ} = 0.01m$ Largest $\Delta = 0.18m$

$\sigma_{XYZ} = 0.001m$ Largest $\Delta = 0.19m$

$\sigma_{XYZ} = 0.0001m$ Largest $\Delta = 0.20m$

Plots of distortion in object space for different GCP weights
Self-Calibration from Constrained Single-Scale UAV Image Network

- 39-images & 33,000 FBM points
- Flying Height of 20m
- All points in 4 or more images
- Same GCP & Camera Stn data as 3-level 84-stn network

**Free-network BA with fixed calibration (from 84-stn self-cal)**

- RMS $v_{xy} = 0.85$ pixel
- RMS $v_{x'}^{c} = 0.50m$ $v_{z'}^{c} = 0.43m$
- RMS $v_{xy} = 0.011m$ $v_{z} = 0.013m$
- $\bar{\sigma}_{xy} = 0.006m$ $\bar{\sigma}_{z} = 0.015m$

**GENERALLY NOT RECOMMENDED!**
### Constrained Single-Scale Self-Calibration: varying $\sigma_{XYZ}$ & $\sigma_{XYZ} = 0.1\text{mm}$

<table>
<thead>
<tr>
<th>a priori constraint</th>
<th>RMS $v_{xy}$ (pixel)</th>
<th>$\bar{\sigma}_{XY}$</th>
<th>$\bar{\sigma}_Z$</th>
<th>RMS $v_{XYZ}^c$</th>
<th>RMS $v_{Z}^c$</th>
<th>RMS $\Delta_{XY}$</th>
<th>RMS $\Delta_Z$ (GCPs)</th>
<th>RMS $\Delta_Z$ (GCPs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Free-net, fixed calibration</td>
<td>0.85</td>
<td>0.006 m</td>
<td>0.015 m</td>
<td>0.50 m</td>
<td>0.43 m</td>
<td>0.011 m</td>
<td>0.013 m</td>
<td></td>
</tr>
<tr>
<td>$\sigma_{XYZ} = 0.1\text{m}$</td>
<td>0.85</td>
<td>0.006</td>
<td>0.015</td>
<td>0.49</td>
<td>0.18</td>
<td>0.008 m</td>
<td>0.08 m</td>
<td>0.006</td>
</tr>
<tr>
<td>$\sigma_{XYZ} = 0.01\text{m}$</td>
<td>0.85</td>
<td>0.006</td>
<td>0.015</td>
<td>0.48</td>
<td>0.13</td>
<td>0.008 m</td>
<td>0.08 m</td>
<td>0.006</td>
</tr>
<tr>
<td>$\sigma_{XYZ} = 0.001\text{m}$</td>
<td>0.86</td>
<td>0.006</td>
<td>0.014</td>
<td>0.42</td>
<td>0.13</td>
<td>0.017 m</td>
<td>0.10 m</td>
<td>0.007</td>
</tr>
<tr>
<td>$\sigma_{XYZ} = 0.0001\text{m}$</td>
<td>1.6</td>
<td>0.010</td>
<td>0.026</td>
<td>0.14</td>
<td>0.11</td>
<td>0.13 m</td>
<td>0.19 m</td>
<td>0.033</td>
</tr>
</tbody>
</table>

For $\sigma_{XYZ} = 0.1\text{ mm}$, there is basically no impact on object point precision and accuracy when varying $\sigma_{XYZ}$, except for the case of camera stn. coords. being very tightly constrained $\sigma_{XYZ} = 0.1\text{ mm}$.
Constrained Single-Scale Self-Calibration: varying $\sigma_X^{\epsilon}, \sigma_Y^{\epsilon}, \sigma_Z^{\epsilon}$ & $\sigma_{XYZ} = 0.1\text{mm}$

Note the significant biases in the estimated focal length!
Constrained Single-Scale Self-Calibration: varying $\sigma_{X,Y,Z}^c$ & $\sigma_{XYZ} = 0.1\text{mm}$

- $\sigma_{X,Y,Z}^c = 0.1\text{m}$, Largest $\Delta = 0.30\text{m}$
- $\sigma_{X,Y,Z}^c = 0.01\text{m}$, Largest $\Delta = 0.30\text{m}$
- $\sigma_{X,Y,Z}^c = 0.001\text{m}$, Largest $\Delta = 0.37\text{m}$
- $\sigma_{X,Y,Z}^c = 0.0001\text{m}$, Largest $\Delta = 0.46\text{m}$

Plots of very large distortion in object space for varying camera station weights.
Constrained Single-Scale Self-Calibration: varying $\sigma^c_{XY}^c$, $\sigma^c_Z$ & $\sigma_{XYZ} = 1$mm

<table>
<thead>
<tr>
<th>a priori constraint</th>
<th>RMS $v_{xy}$ (pixel)</th>
<th>$\bar{\sigma}_{XY}$</th>
<th>$\bar{\sigma}_Z$</th>
<th>RMS $v_X^c$</th>
<th>RMS $v_Y^c$</th>
<th>RMS $v_Z^c$</th>
<th>RMS $\Delta_{XY}$</th>
<th>RMS $\Delta_Z$</th>
<th>RMS $\Delta_{XY}$ (GCPs)</th>
<th>RMS $\Delta_Z$ (GCPs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Free-net, fixed calibration</td>
<td>0.85</td>
<td>0.006m</td>
<td>0.015m</td>
<td>0.50m</td>
<td>0.43m</td>
<td></td>
<td>0.011m</td>
<td>0.013m</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma^c_{XY}^c$ 0.1m</td>
<td>0.84</td>
<td>0.011</td>
<td>0.019</td>
<td>0.48</td>
<td>0.14</td>
<td></td>
<td>0.008m</td>
<td>0.10m</td>
<td>0.013m</td>
<td>0.053</td>
</tr>
<tr>
<td>$\sigma^c_{XY}^c$ .01m</td>
<td>0.84</td>
<td>0.010</td>
<td>0.018</td>
<td>0.48</td>
<td>0.13</td>
<td></td>
<td>0.009</td>
<td>0.10</td>
<td>0.014</td>
<td>0.053</td>
</tr>
<tr>
<td>$\sigma^c_{XY}^c$ .001m</td>
<td>0.86</td>
<td>0.008</td>
<td>0.016</td>
<td>0.42</td>
<td>0.12</td>
<td></td>
<td>0.09</td>
<td>0.07</td>
<td>0.079</td>
<td>0.043</td>
</tr>
<tr>
<td>$\sigma^c_{XY}^c$ .0001m</td>
<td>1.6</td>
<td>0.010</td>
<td>0.028</td>
<td>0.13</td>
<td>0.11</td>
<td></td>
<td>0.21</td>
<td>0.90</td>
<td>0.18</td>
<td>0.71</td>
</tr>
</tbody>
</table>

For $\sigma_{XYZ} = 1$mm, significant impact on object point planimetric precision and accuracy occurs when $\sigma^c_{XY}^c, \sigma^c_Z \leq \sigma_{XYZ}$; impact on vertical accuracy occurs for all constraint values.
Constrained Single-Scale Self-Calibration: varying $\sigma^c_{X^c} \sigma^c_{Y^c} \sigma^c_{Z^c}$ & $\sigma_{XYZ} = 1\text{mm}$

Radial Distortion for varying $\sigma_{EO}$ and $\sigma_{GCPs} = 1\text{mm}$

Note the significant biases in the estimated focal length!
Constrained Single-Scale Self-Calibration: varying $\sigma_{X,Y,Z}^c$ & $\sigma_{XYZ} = 1\text{mm}$

Plots of very large distortion in object space for varying camera station weights
Concluding Remarks

• The network geometry rules regarding self-calibration apply to UAV/UAS networks – there are no shortcuts!

• Consider comprehensive pre-calibration & assessment of calibration stability; multi-scale networks preferred over constrained single-scale

• Not only IO calibration is problematic for near-nadir UAS networks w/o EO constraints, self-calibration of radial distortion also a problem

• GPS camera stn. constraints in self-calibration are generally useful in UAS networks only when GPS accuracy is cm level – not yet common

• Biases & distortions in point positioning are not necessarily removed through the use of moderately to tightly constrained GCPs

• Covariance matrices only realistic when datum & configuration defects removed – helped by control sigmas << image point sigmas*

• Object point biases & distortions can be >> magnitude indicated by object point XYZ sigmas
To ensure successful self-calibration: Maximise image scale variation within and between images forming the network ... and utilise roll angle variation.